Single Rail Train Dispatch Time Optimization Under Static Demand and Fixed Train Capacity

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Abstract: This paper studies the optimization of an urban single-line metro train dispatch time using static passenger demand while considering the boarding capacity of trains. An origin-destination matrix is used to assign destination of passengers after boarding a station, thus tracking the boarded passengers of trains. Knowing the current capacity of trains improves the actual computation of timetables. Dispatch times of trains are the main objective of this study and a periodic timetable was produced using Advanced Process OPTimizer (APOPT) solver.

Keywords: Transportation, Train Scheduling, Metro Rail Timetabling, Train Dispatch Time

1. INTRODUCTION

The increasing demand for mobility and organization of public transportation becomes more and more important as the urban population increases. Railroad transportation is one of the many modes of transportation that is important to commuters. Railroad planning is a complex process which involves hierarchy of subtasks such as generation of origin-destination (OD) matrices, line planning, train schedule generation and scheduling of rolling stock (Bussieck, 1997).

In the Philippines, the railway system is still sparsely developed. The system spans 79 km in four lines and carries about 1.3 million passengers per day (NEDA, 2014). However, these lines currently have limited resources and are not properly abled to adjust their timetables based from the demand. These greatly affect the efficiency of the system and the passengers get the

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burden of not knowing their estimated time of arrival. An empirical study done by Palmqvist *et.al*. (2017) showed that timetable properties affect punctuality and that it greatly influences the attractiveness and efficiency of the railway system. Shang *et al.* (2019) was able to conduct another empirical study on timetabling on buses with great consideration on passenger satisfaction. Parbo (2016) then gave an empirical evidence that indicates that passengers give more importance to travel time certainty than travel time reductions. With these in mind, this study aims to optimize the timetable problem by establishing the best dispatch times of trains. Furthermore, the proposed model intends to maximize the capacity of the trains in such a way that there are no passengers left after every departure at each station.

The train timetable design for passengers on an urban train line must establish the departure and arrival times at each station for each train. It is dependent to both the passengers and train management. The timetable can be represented in a time-space diagram and can be classified as either periodic or non-periodic (Serafini,1989; Hooghiemstra,1996). A periodic timetable sets the headway between any two successive trains as constant. This makes it easier for passengers to memorize the schedule and ideal with large-scale railway networks. Non-periodic timetables are appropriate for dynamic passenger demand which is the case when a surge of passengers is lining up at the stations during peak hours and small amounts at non-peak hours. Trains in this case could travel with different speeds and stop with different dwell times depending on the demand.

As an essential optimization application in transportation, the problem of train timetabling has received much consideration over the past few decades. Cordeau *et al.* (1998) provided a comprehensive review on routing and scheduling models. The routing models addressed problems for freight transportation and railcar fleet management, while the scheduling models were mainly concerned on the dispatch time of trains and the assignment of locomotives. On the other hand, Caprara *et al*. (2007) completed an extensive survey focusing on passenger train scheduling and modeling. Cacchiani and Toth (2012) conducted a study that primarily dealt with train timetabling for both nominal and robust problems. The study also introduced train track capacity as part of its constraints. Schobel (2011) compiled and introduced various line planning models under public transportation that can be applied for rail planning.

In the problem of timetabling, majority of existing studies tackle the creation of timetables under different conditions and constraints. Odijk (1996) developed his model about constraint generation solution algorithm based from the model of Serafini and Ukovich (1989) on Periodic Event Scheduling Problem which consisted of periodic time window constraints by means of arrival and departure times. Nachtigall and Voget (1997) built a periodic timetable by minimizing the weighted sum of passenger waiting times at stations under the condition of predefined running times and stopping times of trains. Zhou and Zhong (2004) was able to formulate train scheduling models that reflects station headway capacities as limited resources and developed algorithms to minimize both the total passenger travel times and expected waiting times. Cordone and Radaelli (2011) made use of a nonlinear mixed integer model and the branch-and-bound algorithm to maximize the demand under a regular timetable with constant train departure and arrival intervals. Whereas, Shang (2016) developed a mixed integer nonlinear programming model with a spatial branch and bound algorithm to optimize the sum of aggregate passenger waiting time at stations and aggregate riding time on trains. Niu and Zhou (2013) made use of a time-dependent origin-destination (OD) passenger demand matrix to design train timetables for a heavily congested urban rail line by using local improvement and dynamic programming methods for the optimization.

These investigations, however, do not consider the dispatch time of trains as the main objective. In the Philippines, the trains are dispatched before the operating time in order to accommodate the demand coming from all stations in the rail system. Thus, a skip-stop operation is implemented for the first dispatched train until the operation normalizes.

This paper delves into the optimization of a single-line metro timetable under static passenger demand. It focuses on two objectives: maximizing the train capacity of each train in the system and identify the accurate dispatch time of trains in order to accommodate all passengers within the allotted capacity. An objective function was developed and was solved using Advanced Process OPTimizer (APOPT) solver in python. The solution is a single-line metro timetable adapted to static passenger demand with various dispatch times.

The remainder of the paper is structured as follows. Section 2 presents the proposed mathematical model along with the model parameters and corresponding assumptions. Section 3 illustrates the proposed model with arbitrary parameter values. Two experimental cases were presented that varied the train capacity, number of stations, and number of train trips to observe the response of the model to different scenarios. Finally, the conclusions for this study are presented in Section 4.

2. MODEL FORMULATION

2.1 Model Parameters

This section describes the different parameters used in the study as well as the assumptions essential to the formulation of the model. Note that these values are theoretical and do not reflect real time data.

2.1.1 Passenger parameters

The passenger parameters used in the model are listed in Table 1. All values given have been arbitrarily set for testing.

Table 1. Passenger Parameters

Parameter	Units
initial passenger entry	passenger
arrival rate (λ)	passengers/min

riding time ($\rho_t^{s-1,s}$)	min

Initial passenger entry refers to the number of passengers waiting at the platform at time zero. Arrival of passengers at the station is dictated by the arrival rate (λ). It is assumed that the arrival of passengers is linear. Riding time ($\rho_t^{s-1,s}$) is the amount of time a passenger spends aboard train t as it travels between two succeeding stations (s-1 to s). Riding time is assumed to be the same for all passengers. These assumptions are further explained below. All passenger parameters are bounded by the set of positive real numbers (R^+). For this implementation, the passenger count is allowed to be float values.

2.1.2 Station parameters

The station parameters of the model are shown in Table 2. All values have been arbitrarily set for testing, but each parameter is bounded accordingly.

Table 2. Station Parameters

Parameter	Units
number of stations (|S|)	station
distance between adjacent stations ($d^{s-1,s}$)	m

The number of stations (|S|) is constrained to be an element of the set of positive integers (Z^+). Furthermore, the minimum number of stations is set to two because a passenger will not exit the same station where he entered. This ensures that a destination different from the origin is initialized. For the current formulation, the distance between adjacent stations (d) is assumed to be constant and is also bounded by R^+.

2.1.3 Train parameters

Table 3 summarizes the parameters of the train.

Table 3. Train Parameters

Parameter	Units
number of train trips ($\|T\|$)	train
train capacity (C)	passenger
dwell time (ω)	min
minimum headway (h_{min})	min

The number of train trips (|T|) is bounded by Z^+. Train capacity (C) means the maximum number of passengers a train can hold at any given time and is also bounded by Z^+. For this study, all trains are assumed to have the same capacity. The dwell time is the amount of time a train is stationary in a station. It has a lower bound of zero and is also assumed to be the same for all trains and constant throughout. The minimum headway is the minimum separation between two trains. For this case, time headway in minutes is used and the bound is set to be 0.

2.2 Constraints

This section lists the constraints for the model.

2.2.1 Riding time, ρ

The riding time ($\rho_t^{s-1,s}$) in Table 1 is equal to the distance between adjacent stations ($d^{s-1,s}$) in Table 2 divided by the train speed of train t while traversing the said distance. Thus,

$$\rho_t^{s-1,s} = d^{s-1,s} / v_t^{s-1,s} \tag{1}$$

From the station properties, it was assumed that $d^{s-1,s}$ is constant. Furthermore, $\rho_t^{s-1,s}$ is also assumed to be constant. Hence, the train speed can be computed as:

$$v_t^{s-1,s} = d^{s-1,s} / \rho_t^{s-1,s} \tag{2}$$

Since these parameters are assumed constant throughout, the superscripts and subscripts can be omitted.

$$\rho = d/v \tag{3}$$

2.2.2 Departure time, δ_t^s

The departure time of train t at station s (δ_t^s) is the point in time where train t will depart station s. The general equation of δ_t^s is given by:

$$\delta_t^s = \delta_t^{s-1} + \rho + \omega \tag{4}$$

s. t. $\quad \delta_t^s \geq \delta_{t-1}^s + h_{min}$ (5)

where,

ω_t^s : dwell time of train t at station s but since dwell time is constant for all trains at all stations, ω_t^s can be simplified as ω

h_{min} : minimum headway; same with dwell time which is constant all throughout

Depending on which train and in what station, the departure time equation will vary as follows:

▪ *1st station, 1st train*

The first train at the first station does not maintain any headway since it is the first element of the system. Since a previous station does not exist, we assume δ_t^{s-1} to be the train depot and the first train will depart the train depot at the specified dispatch time κ. Hence, $\delta_t^{s-1} = \kappa_1^0$.

$$\delta_t^s = \kappa_1^0 + \rho + \omega \tag{6}$$

▪ *1st station, 2nd train up to the last train*

For all trains at the first station, δ_t^{s-1} does not exist so it assumed that s – 1 is the train depot. Same with the first train, δ_t^{s-1} will be replaced by the dispatch time κ_t^0. However, Equation 2 should now be checked for the follower trains since a minimum headway should be maintained with the train it succeeds. δ_t^s for this case is:

$$\delta_t^s = \kappa_t^0 + \rho + \omega \tag{7}$$

s.t. $\delta_t^s \geq \delta_{t-1}^s + h_{min}$

▪ *2nd station up to the last station, 1st train*

The 1st train at the remaining stations now have a value for δ_t^{s-1}. But will still be the first element of each station, hence no need to check for headway.

$$\delta_t^s = \delta_t^{s-1} + \rho + \omega$$

▪ *2nd station up to the last station, 2nd train up to the last train*

The rest of the trains will now also have a value for δ_t^{s-1} aside from the 1st station. δ_t^s for this case is the general equation.

$$\delta_t^s = \delta_t^{s-1} + \rho + \omega$$

s.t. $\delta_t^s \geq \delta_{t-1}^s + h_{min}$

2.2.3 Passenger entries

This section describes the passenger flow from the station and train entry until passenger exit.

2.2.3.1 Cumulative station entry (M_χ)

The cumulative station entry (M_{χ}^{s}) is defined as the cumulative count of passengers that arrived at station s until time χ. Ideally, each station should have its own cumulative station entry but for simplicity, it is assumed that all stations have the same cumulative station entry equal to M_χ. From the values of the parameters in Section 2.1.1, the cumulative station entry can be computed. It is simply the arrival rate (λ) multiplied by the time elapsed (χ).

2.2.3.2 Train entry (N_t^s) and cumulative train entry ($\sum_{i=1}^{s} N_t^i$)

From being stations entries, passengers should further be assigned to specific trains. For the first train, train entry (N_1^s) at station s is given by:

$$N_1^s = \lambda\delta_1^s \quad (8)$$

For the succeeding trains, passengers that were assigned to the previous trains should be accounted for. Hence:

$$N_t^s = \lambda\delta_t^s - M_{\delta_{t-1}^s}$$

where the time elapsed in Section 2.2.3.1 is equal to the departure time (δ_{t-1}^s = χ),

$$N_t^s = \lambda\delta_t^s - \lambda\delta_{t-1}^s$$

Using the distributive property of multiplication, the equation is now:

$$N_t^s = \lambda(\delta_t^s - \delta_{t-1}^s) \quad (9)$$

Cumulative train entry is the summation of all train entries in a specific train across all stations $\sum_{i=1}^{s} N_t^i$.

2.2.4 Exiting, boarded, and left passengers

The parameter $Exit_t^s$ (X_t^s) is defined as the total number of passengers exiting train t upon arrival at station s. The general equation for the exit is:

$$X_t^s = \sum_{i=0}^{s} G_{i,s} * (B_t^i - B_t^{i-1} + X_t^i) \quad (10)$$

For i = 0, B_t^{i-1} will be B_t^{-1} where it will be equal to zero. The exit at station 0 is also equal to zero, and hence,

$$X_t^s = (G_{0,s} * B_t^0) + \sum_{i=1}^{s} G_{i,s} * (B_t^i - B_t^{i-1} + X_t^i) \tag{11}$$

is the exhaustive form of the equation.

The variable $G_{i,s}$ refers to the percent exit of passengers who came from station i that will exit in station s. The passenger exit distribution should look like Table 4.

Table 4. General Passenger Exit Distribution

Origin / Destination				
Station	1	2	3	4
1	$G_{1,1}$	$G_{1,2}$	$G_{1,3}$	$G_{1,4}$
2	$G_{2,1}$	$G_{2,2}$	$G_{2,3}$	$G_{2,4}$
3	$G_{3,1}$	$G_{3,2}$	$G_{3,3}$	$G_{3,4}$
4	$G_{4,1}$	$G_{4,2}$	$G_{4,3}$	$G_{4,4}$

Take note that a passenger will not exit the station where he entered. A passenger can also no longer exit at the previous stations. Dividing one by the total number available of exits, Table 5 was formed. It is assumed that the percent exits are equal for the remaining stations.

Table 5. Passenger Exit Distribution

Origin / Destination				
Station	1	2	3	4
1	0	0.33	0.33	0.33
2	0	0	0.5	0.5
3	0	0	0	1
4	0	0	0	0

The variable (B_t^s) is defined as the total number of passengers aboard train t upon departure at station s. Boarded is given by:

$$B_t^s = \min(C\,,\ B_t^{s-1} - X_t^s + L_{t-1}^s + N_t^s) \tag{12}$$

The variable (L_t^s) is defined as the total number of passengers unaccommodated by train t upon departure at station s. Passengers left by the train is given by:

$$L_t^s = \max(0,\ L_{t-1}^s + \ N_t^s - (B_t^s - \ B_t^{s-1} + X_t^s)) \tag{13}$$

Depending on which train and in what station, X_t^s, B_t^s, and L_t^s will vary as follows:

- *1st station, 1st train*

Equation (14) states that no passenger will exit at the first station. Boarded passengers from the previous station (B_t^{s-1}) does not exist because this is the first station, therefore equation (15) implies that... Since it is the first train, left passengers from the previous train (L) also does not exist. The equations are now simplified as follows:

$$X_t^s = 0$$

$$B_t^s = \min(C,\ N_t^s)$$

$$L_t^s = N_t^s - B_t^s$$

- *1st station, 2nd train up to the last train*

There will still be no exit and B_t^{s-1} for this case because it is still in the first station. But L_{t-1}^s now exists for the rest of the trains.

$$X_t^s = 0$$

$$B_t^s = \min(\mathrm{C},\ L_{t-1}^s + N_t^s)$$

$$L_t^s = \max(0,\ L_{t-1}^s + \ N_t^s - B_t^s)$$

- *2nd station up to the last station, 1st train*

From the 2nd station onwards, passengers can now exit. But since this is the first train, L_{t-1}^s does not exist.

$$X_t^s = (G_{0,s} * B_t^0) + \sum_{i=1}^{s} G_{i,s} * (B_t^i - B_t^{i-1} + X_t^i)$$

$$B_t^s = \min(\ C\ ,\ B_t^{s-1} - X_t^s + N_t^s)$$

$$L_t^s = \max(0, N_t^s - (B_t^s - \ B_t^{s-1} + X_t^s))$$

- *2nd station up to the last station, 2nd train up to the last train*

For this case, the general equations apply.

$$X_t^s = (G_{0,s} * B_t^0) + \sum_{i=1}^{s} G_{i,s} * (B_t^i - B_t^{i-1} + X_t^i)$$

$$B_t^s = \min(C\ ,\ B_t^{s-1} - X_t^s + L + N_t^s)$$

$$L_t^s = \max(0,\ L_{t-1}^s + \ N_t^s - (B_t^s - \ B_t^{s-1} + X_t^s))$$

2.2.5 Cumulative exit $\sum_{i=1}^{s} X_t^i$

Cumulative exit ($\sum_{i=1}^{s} X_t^i$) is the summation of all passengers that exited a specific train t.

2.3 Objective Function

The objective function was formulated from the equation:

$$\sum_{i=1}^{s} N_t^i - [(\sum_{i=1}^{s} X_t^i) - X_t^s] = C$$

which forces the difference between the cumulative train entry in all stations and the cumulative exit until |S| – 1 to be exactly equal to C. This means that when the train leaves the second to the last station, it is at full capacity. As an additional constraint, L_t^s should be equal to zero for all stations and all trains. Simplifying,

$$\sum_{i=1}^{s} N_t^i - \sum_{i=1}^{s-1} X_t^i = C$$

The model ensures that whenever a train leaves a station, all passengers are accommodated, and it will be at full capacity at the second to the last station to maximize capacity. Therefore, the objective function is written as:

$$\min \quad \sum_{i=1}^{s} N_t^i - \sum_{i=1}^{s-1} X_t^i - C$$

s.t.

$$\rho_t^{s-1,s} = \ d^{s-1,s} / v_t^{s-1,s}$$

$$v_t^{s-1,s} = \ d^{s-1,s} / \rho_t^{s-1,s}$$

$$\rho = d/v$$

$$\delta_t^s = \delta_t^{s-1} + \rho + \omega$$

$N_1^s = \lambda\delta_1^s$

$N_t^s = \lambda(\delta_t^s - \delta_{t-1}^s)$

$X_t^s = \sum_{i=0}^{s} G_{i,s} * (B_t^i - B_t^{i-1} + X_t^i)$

$B_t^s = \min(C, B_t^{s-1} - X_t^s + L_{t-1}^s + N_t^s)$

$L_t^s = \max(0, L_{t-1}^s + N_t^s - (B_t^s - B_t^{s-1} + X_t^s))$

3. COMPUTATIONAL EXPERIMENTS

Tables 7, 8, and 9 identifies the initial parameters used for the succeeding experiments. Note that these are theoretical values and does not reflect real data.

Table 6. Passenger Parameter Values

Parameter	Value
initial passenger entry	0 passenger
arrival rate	1 passenger/min
riding time ($\rho_t^{s-1,s}$)	2 min

Table 7. Station Parameter Values

Parameter	Value
number of stations (|S|)	3 stations
distance between adjacent stations ($d^{s-1,s}$)	1000 m

Table 8. Train Parameter Values

Parameter	Value
number of train trips ($\|T\|$)	5 train trips
train capacity (C)	15 passengers
dwell time (ω)	1 min
minimum headway (h_{min})	0 min

The optimization problem was solved using a program in Python Programming Language. The Gekko Optimization Suite package (Beal *et.al*, 2018) was imported wherein the Advanced Process OPTimizer (APOPT) solver was utilized. For the initial case of three stations and five train trips, the following train dispatches at the train depot were computed.

Table 9. Train Dispatch Times

Train	Dispatch Time at Depot (min)
1	5
2	15
3	25
4	35
5	45

Since the riding time and dwell times are constant, the timetable can be derived as shown:

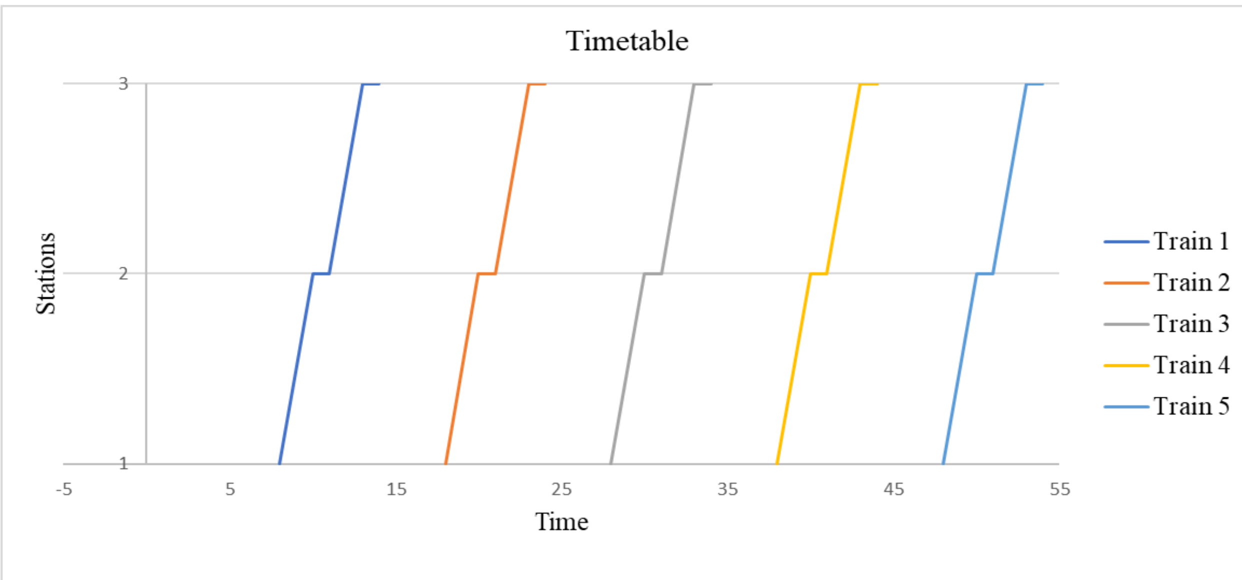

Figure 1. Timetable

Using this timetable, no passengers will be left behind every time a train leaves a station. Also, it is assured that the train is at maximum capacity whenever it leaves the second to the last station as shown in Figures 2 and 3.

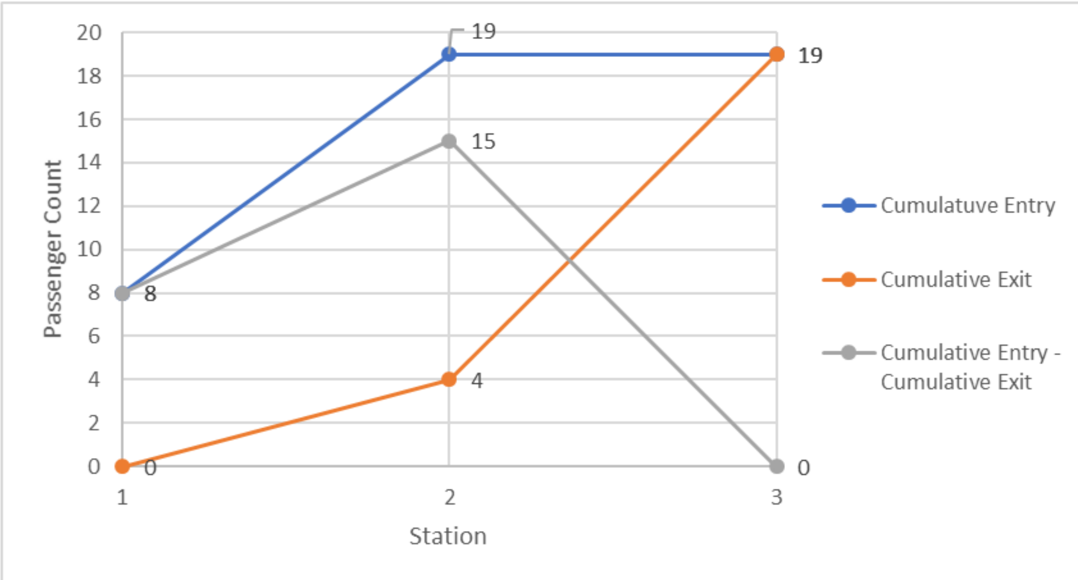

Figure 2. Train 1 Passenger Counts

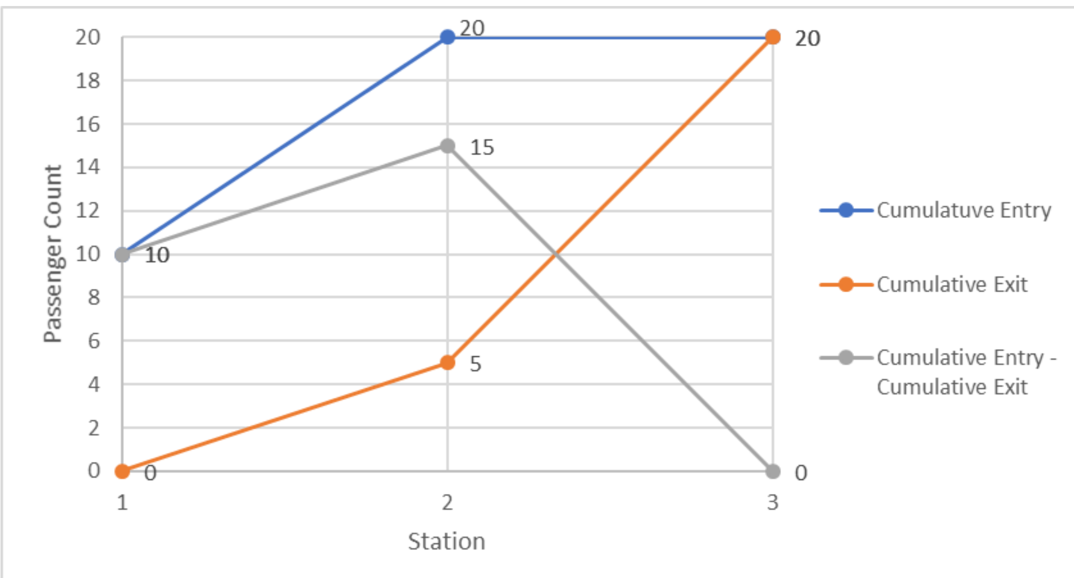

Figure 3. Trains 2-5 Passenger Counts

Decreasing the train capacity to 10 passengers while increasing the total number of stations to four and train trips to 10, the following results were obtained.

Table 11. Train Dispatch Times

Train	Dispatch Time at Depot (min)
1	-1.64
2	3.82
3	9.27

4	14.73
5	20.18
6	25.64
7	31.09
8	36.55
9	42.00
10	47.45

Notice that the train dispatch of train 1 is in negative time. This means that it departed the train depot even before the stations were open for passenger entry at time zero. This was necessary to ensure that there will be no passengers left by train 1 given that the capacity was reduced. The corresponding timetable is shown.

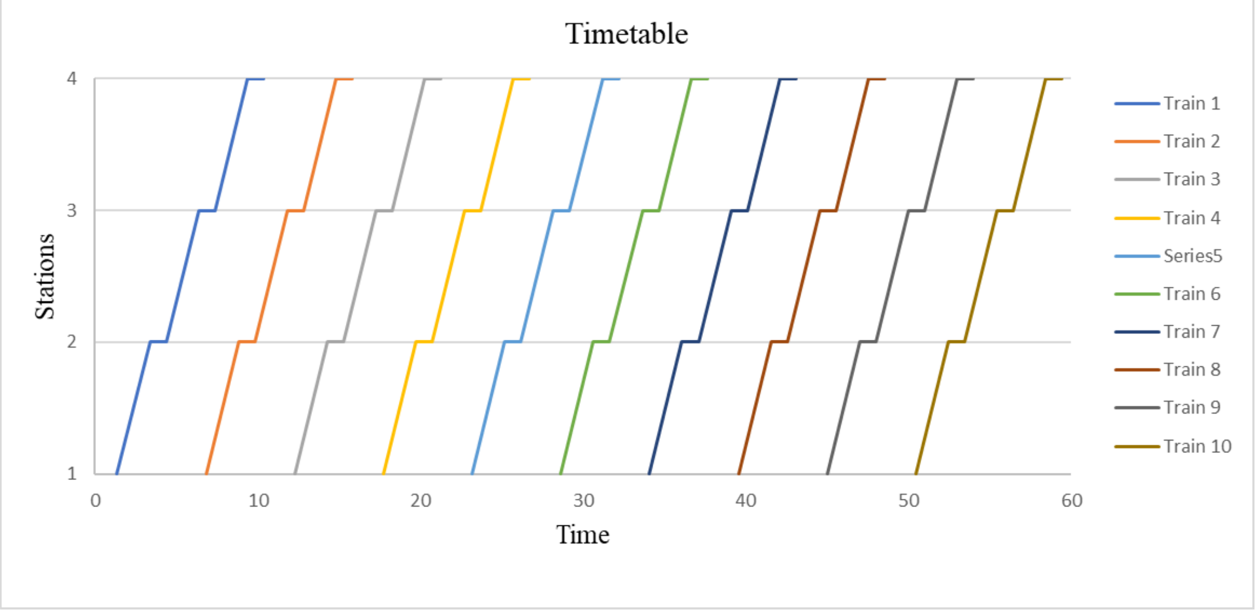

Figure 4. Train Timetable

Again, no passengers will be left behind every time a train leaves a station and it is assured that the train is at maximum capacity whenever it leaves the second to the last station as shown in Figures 5 and 6.

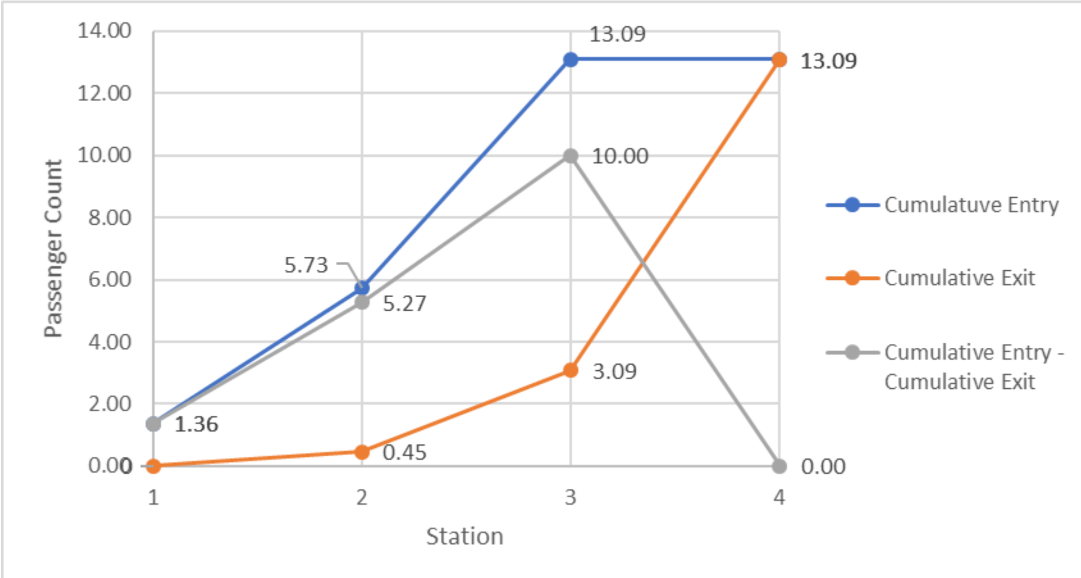

Figure 5. Train 1 Passenger Count

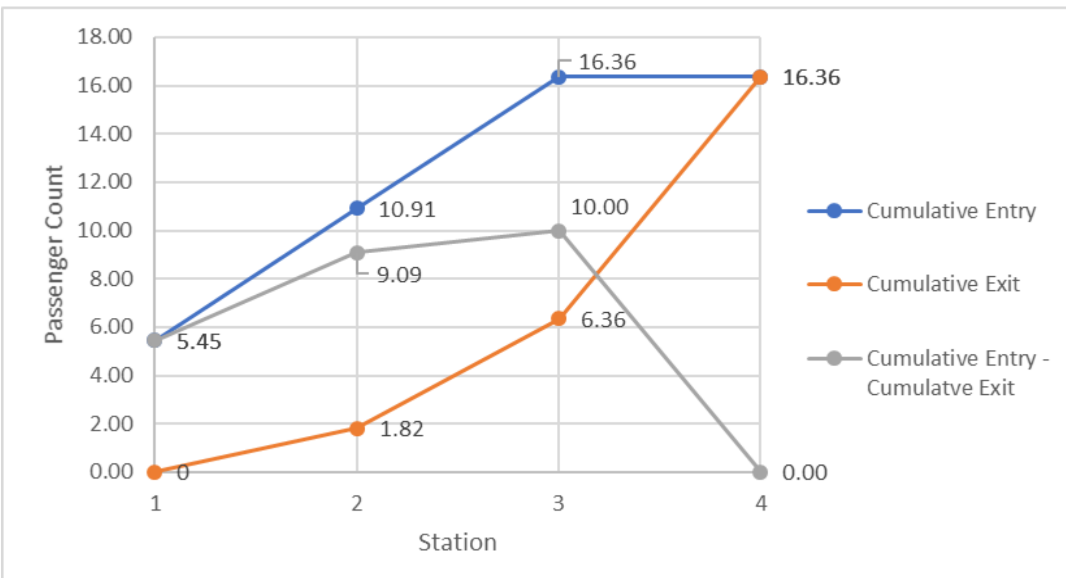

Figure 6. Trains 2 – 10 Passenger Count

4. CONCLUSIONS AND FUTURE DIRECTIONS

A timetabling model that optimizes train dispatch times that considers train capacity was developed in this study. The model ensures that all passengers will be accommodated by the train system upon each departure. Furthermore, a suboptimal solution was portrayed, due to the fact that trains are always at full capacity at the second to the last station.

The next step for future development is to use a dynamic passenger entry demand that changes over time and varies per station. Actual turnstile data can easily be inputted in the program and it will respond to the changes accordingly. These data can usually be requested from train operators. The average waiting time of passengers can also be derived from the model and can be incorporated in the objective function for future studies.

A dynamic OD can also be used to improve realism in modelling train situations that have varying OD matrices within the day. This case is evident with respect to morning versus evening peak between Central Business Districts in the Philippines, such as home to work and work to home.

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