Modified Greedy Algorithm for Game-Theoretic Traffic Assignment

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Abstract: Under microscopic traffic simulation, traffic assignment is the result of multiple travelers concurrently finding their own best path from their origin to their destination paths. This can be considered a game-theoretic problem where the players decide which path to take, with costs assigned to each choice. An objective function derived from Wardrop's Principle can be used to direct the choices of the travelers, converting the game-theoretic problem into an optimization problem. This study attempts to perform traffic assignment using greedy algorithm with backtracking by optimizing the individual decisions of each travelers. The road network used is a 3-route network with one entry and exit node, using LocalSIM as the traffic simulator.A benchmarking methodology was introduced to measure the performance of the greedy algorithm to the global optimal solution. For small number of vehicles, the distribution of the discrepancies among various initial conditions are slowly increasing.

Keywords: Transport, Greedy algorithm, Traffic simulation, Game theory

1. INTRODUCTION

According to a recent study by Japan International Cooperation Agency (JICA), the Philippines loses P 3.5 Billion per day due to traffic congestion in Metro Manila. This amount went higher from a National Economic Development Authority (NEDA) report, citing daily loss of P 2.4 Billion in 2014 (CNN Philippines, 2018).

Long-term solutions for traffic would include expansion of current mass transport infrastructures like the Metro Rail Transit (MRT) and Light Rail Transit (LRT), exploration of additional infrastructures as well as mass transport options like Bus Rapid Transit (BRT). However as these take large amount of time and money, one can look at shorter-term solutions like implementing various traffic policies. In that case, using traffic simulation helps in assessing the impact of these various policies when conducting a dry-run would not be feasible.

One of the key components to effective transport planning is **Travel Forecasting**. This process aims to estimate the future travel demand for certain transportation systems. A four-step model was developed in the late 1950's as a means to perform travel forecasting. Prior to travel forecasting, forecasts about population (demographic and socio-economic factors) and land use are generated first. After that, the modeler goes into the actual four-step model; trip generation, trip distribution, mode choice and traffic assignment (Beimborn, 1995).

Most of the traditional traffic assignment methods are done macroscopically, by considering only the demand estimate for each origin-destination pair and ignoring the minor details about the individual travelers and their interactions (lane changing, homogeneity or heterogeneity of vehicles, driver behavior or preferences, etc.). An alternative to this is to consider traffic demand as a sequence of individual travelers, each with their own behavior. Hence, one can look at microscopic traffic models. As traffic assignment is concerned with how the demand on a given origin-destination (O-D) pair is split into various routes, one can look at this as considering the aggregate route choices of each traveller on that given O-D pair. In a sense, this is the extreme version of incremental traffic assignment where each individual traveler is the increment. From a game-theoretic standpoint, this means each individual traveler on a given O-D pair picks their own strategy (route taken) with experienced travel time as the payoff.

One feature of microscopic traffic model is that the existing models are focused on driver-driver interactions as well as driver-road interactions (Matthew, 2014). In this case, Agent-Based Modeling (ABM) was employed since it is easier to define the rules of interaction rather than determining the equations that govern these interactions. Taking these intricacies into consideration might result to a more realistic traffic simulation.

2. PRELIMINARIES

2.1 Transport Network as Graphs

Assuming that counterflows are not permitted, a **Transport Network** can be represented by a directed graph $G = (V, E)$ with a set of nodes V and a set of directed edges E (called **links**) connecting any 2 distinct nodes ($v_i, v_j \in V\ ; i \neq j$) (Lehman et al., 2010). On actual road networks, nodes can represent junctions while edges can represent roads. For simplicity, it is assumed that every node (v) in the network is connected to at least one other node in a network.

A **source node** is a node in the graph (denoted by $v_o \in V$) where every edge connected to it are outgoing edges while a **sink node** is a node in the graph (denoted by $v_f \in V$) where every edge connected to it are incoming edges (Lehman et al., 2010). In transport networks, vehicles enter the network through source nodes (referred to as origin) while vehicles exit the network through sink nodes (referred to as destination). A transport network can have any number of source node or sink nodes. However, without loss of generality, only traffic networks with singular source and sink nodes will be considered.

A **route** can be defined as a sequence of connected directed edges $a' = \{e_1, e_2, \dots, e_j\}$; $e_i \in E$ that connect v_o and v_j without forming a loop. Let R be the set of routes on G. Define $k = |R|$. Define traffic demand for an origin-destination (O-D) pair to be the number of vehicles that travel from the origin to the destination during a specified time interval.

2.2 Wayfinding Problem

Recall that in the microscopic perspective, an individual traveler engages in 3 levels of decision-making; operational, tactical, and strategic. In general, decision-making under strategic level is similar to solving a Wayfinding Problem. Given a transport network G, Wayfinding Problem concerns with finding a path to traverse from v_a to v_b ; $(v_a, v_b \in V)$. There are two components to Wayfinding Problem; route search and route choice (Bovy et al., 1990).

Route search is a dynamic process that involves searching for routes on a given transport network. This set of route alternatives is called the choice set. The choice set derived from the transport network may vary between individuals and also with respect to time due to several factors (familiarity, convenience, outside factors, etc.) (Bovy et al., 1990). For each individual i, let $A_i \subseteq R$ be its choice set.

After determining the choice set, the traveler now engages in the process called **route choice**. It is a static process in which given a particular trip, a route is chosen from the traveler's current choice set. The route choice process for each individual might vary (Bovy et al., 1990). For an individual i with choice set A_i, let $a_i \in A_i$ be the route picked by the individual i.

Taken individually, each traveler having his/her own preferences performs route search and route choice, picking what he/she believes to be the best route. When these decisions are aggregated, then information on the flow patterns and travel time for each link in the transportation network can be obtained.

2.3 Random Utility Theory

Random Utility Theory postulates that the individual acts rationally. A rational choice has 3 components: perception-rationality, preference-rationality and process-rationality.

Perception-rationality assumes that individuals possess perfect information and are aware of all the alternatives and/or the state in each of those alternatives. Preference-rationality assumes that an individual's sense of utility is well defined. Meanwhile, process-rationality assumes that the individual always picks the choice with the maximum utility that is still subject to constraints (McFadden, 1999).

In addition, the theory also postulates that the individuals in a population are homogenous such that these individuals share the same set of alternatives and face the same constraints (de Dios et al., 2011).

2.4 Traffic Assignment

Traffic assignment is defined as the allocation of a "given set of trip interchanges to a specific transport network or system". Usually, it requires a description of the transport network and an OD matrix as an input, while the output varies but always includes traffic volume estimate and costs on each link (Patriksson, 2015).

2.4.1 Early heuristics

Early heuristics for traffic assignment include *all-or-nothing* schemes, in which all travelers are assigned to the cheapest route for each O-D pair in the network, and the use *diversion curves* derived from empirical studies (Patriksson, 2015). In 1957, efficient shortest route algorithms were developed and an implementation based on Moore's algorithm that finds the minimum time through a network resulted in the first computer-aided traffic assignment (Moore, 1959). This resulted in an improved all-or-nothing scheme. However, the all-or-nothing scheme turned out to be unstable (Patriksson, 2015).

2.4.2 Wardrop's principle

In 1952, Wardrop stated two principles about flow distribution in a traffic network, which is collectively called Wardrop's Principle. Wardrop's first principle, which corresponds to **User Equilibrium** states the following:

> "The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route" (Wardrop, 1952)

This state is reached if the travelers pick routes that minimizes their own travel time. In this case, the travelers are assumed to be selfish. At equilibrium, no traveler can decrease his/her own travel time by picking a different route. This equilibrium is stable and similar to the Nash equilibrium. The equilibrium reached is called the Deterministic User Equilibrium (DUE), and it is one of the most popular traffic assignments in practice (Zhang, 2011).

In 1956, it was shown, using nonlinear optimization theory, that the two Wardrop's principles can be formulated as convex nonlinear optimization problems with linear constraints (Patriksson, 2015). Several iterative solution algorithms for DUE were identified such as Frank-Wolfe algorithm, method of convex combinations, method of successive averages, and Bar-Gera and Boyce's origin-based algorithm (OBA) (Zhang, 2011).

The DUE is rooted on the users making rational choices. Suppose that the perception-rationality assumption is foregone, finding the equilibrium will no longer lead to the DUE. This was solved by introducing a random variable, called the perception error, into the utility function (Daganzo, 1977). The equilibrium obtained, called the Stochastic User Equilibrium, requires iterative solution algorithms to converge and its uniqueness and stability is only confined over certain conditions (Zhang, 2011).

One could also consider the case where the flow rate in each source node is a deterministic function, instead of a constant function. The equilibrium obtained, called Deterministic User Equilibrium, can only be approximated through an iterative process (Merchant et al., 1978 ; Chiu et al., 2011). Finally, an entirely different kind of equilibrium is obtained if a different behavior theory is adapted. For instance, adapting Bounded Rationality will lead to a Bounded Rational User Equilibrium (BRUE), while adapting the Positive Behavior Theory will lead to Behavioral User Equilibrium. However, those equilibriums are undesirable due to issues with uniqueness, stability, and/or complexity (Zhang, 2011).

Meanwhile, the other Wardrop's principle that was later called **System Optimal** states the following:

"The average journey time is a minimum" (Wardrop, 1952)

This state is reached if the users pick make route choices that minimizes the travel time of the entire network. In this case, the users are selfless. But this state is not stable since a traveler can pick a route that would serve him/her more even though it would lead to an increase in the travel time of the entire network (Patriksson, 2015).

In general, the user equilibrium and the system optimal do not coincide, and the difference between the two is called the *price of anarchy*. There are two ways to achieve system optimal when travelers are selfish; by imposing route choices to the travelers, or by implementing a *congestion pricing strategy* (Patriksson, 2015).

2.5 Game Theory

Using Wardrop's Principle for doing traffic assignment requires solving convex non-linear optimization problems (Patriksson, 2015). As an alternative, one can look at traffic assignment through game theory, in which each traveller (referred to as a **player**) picks a route to use as his/her strategy.

Game Theory is defined as the "systematic study of strategic interactions among rational individuals". Games can be written in strategic or **normal form** (N, A, u) and requires specifying the set of players N, the set of actions available to each player A, and a utility or payoff function (could be cost function if minimization is the objective) for each player (Kockesen et al., 2007).

Several transportation studies used game theory to analyze traffic. A study in 2002 used game theory to find the user equilibrium of a system in which the travelers minimize their risk instead of finding the route with the least travel time. It was done by modeling traffic assignment as a $n + m$-player non-cooperative game consisting of n players and m origin-destination (OD)-specific demons maximizing the player's cost by randomly failing links, resulting in a **risk-averse user equilibrium** (Bell et al., 2002).

Meanwhile, another study attempted to find a traffic assignment that balances between user equilibrium (UE) and system optimal (SO). It was done by modeling the traffic system as a Stackelberg game with the leader being the traffic manager and the travelers as the followers in the game. Gradient Projection algorithm was used to improve the efficiency of the traffic assignment (Li et al., 2017). Finally, a 2017 study used Agent-Based Modelling (ABM) to demonstrate the emergence of cooperation in a N-player repeated game on a two-road network (Levy et al., 2017).

2.6 Agent-Based Modeling (ABM)

Suppose that there are N travellers on a given O-D pair. An N-player game can then be defined with route choice process as their strategy and for simplicity, their own experienced travel time as the pay-off. However, this game has several complications. First, the game has a large number of outcomes. If the network has k routes and perfect information for each player/traveller is assumed, then the game has k^N outcomes. Second, there are certain complex behavioral interactions that might arise from vehicle-to-vehicle interactions (i.e.: lane changing) which is captured elegantly by defining rules instead of equations. To address the second complication, Agent-Based Modelling (ABM) would be a suitable tool (Levy et al., 2017).

2.7 Game-Theoretic Traffic Assignment

This paper aims to perform traffic assignment by looking at it as a game where the N travelers are finding their best route. Let $A'_1, \dots A'_n$ be the sequence of travelers that enters a non-empty transport network G with one origin v_o and destination v_f at times $t_1, \dots, t_n$ where $t_1 \leq \dots \leq t_n$.

In this game, each player picks a route to traverse from his/her choice set. A given choice would incur a cost that depends not only on that player's choice of route, but also on the other players route choices. If we let A_i be the choice set for the i^{th} player, then the outcome space of the game can be defined as

$$A = A_1 \times A_2 \times \dots \times A_n = \{\boldsymbol{a} = (a_1, \dots, a_N): a_i \in A_i, i = 1, \dots, N\} \tag{2.1}$$

with the outcome $\boldsymbol{a} = (a_1, \dots, a_N)$ (also called an **action profile**) represented by an n-tuple. The action profile represents the actions made by each of the traveler in the game. Meanwhile, the cost incurred by the i^{th} player depends on the actions of all the players in the game, and can be denoted by $c_i(a_1, \dots, a_N; y)$ or $c_i(\boldsymbol{a}; y)$ where y is the initial condition of the non-empty transport network. In this study, c_i is determined for each player A'_i though Agent-Based Model.

Finally, an objective function $f(\boldsymbol{a})$ is defined that takes the action profile $(\boldsymbol{a})$ as an input and outputs a real number. The objective function will be used to compare any two given action profiles.

3. GREEDY ALGORITHM FOR TRAFFIC ASSIGNMENT

3.1 Greedy Algorithm

Finding the optimal action profile that will fit a given objective function f is intractable given its exponential nature as n increases. One needs to consider using heuristics instead. In this paper, greedy algorithm will be used for this study.

The application of greedy algorithm in this study works as follows. Let $A'_1, \dots A'_n$ be the sequence of travelers that enters a non-empty transport network G. A'_1 will pick the best route depending on the current situation on the transport network, A'_2 will then pick the best route, and so on until A'_n. In this case, the traveler's route choice will be based on the greedy algorithm. Each traveler's best choice will be based on his best experienced travel time given the route choices made by the prior travelers.

Greedy algorithm is simple and easy to implement, as well as the algorithm is fast. However, one problem with greedy algorithm is that it does not always yield optimal solutions (Cormen, 2009). This happens because the greedy algorithm possess two inherent shortcomings; (1) it is short-sighted and (2) it is non-retractable (Edelsbrunner, 2008). In this case, there are two ways to improve the result of the greedy algorithm. Resolving the "short-sightedness" of the greedy algorithm can be done by letting the greedy step be optimizing the decision of b travelers at a time through brute-force method. However, increasing the size of the batch b results to exponential increase in the running time.

An alternative to improving the result of the greedy algorithm is to resolve its "non-retractability". In this case, a backtracking step would be introduced to tweak the result of the greedy algorithm. In the context of this study, the greedy algorithm can be improved by introducing a backtracking step as follows.

Once the greedy algorithm is finished with action profile $(a_1, a_2, \dots, a_n)$, randomly tweak the choice of one of the travelers, say A'_i. After that, re-do the greedy algorithm picking the best route for A'_{i+1}, then picking the best route for A'_{i+2}, and so on. Until the best route for the last traveler A'_n was picked. If the resulting action profile is better than the greedy once, replace the old action profile with the new action profile. Otherwise, retain the old action profile.

The backtracking step can be repeated b times, which on a k-route transport network will require running at most bkn simulations.

3.2 Objective Function

The objective functions that will be used in this paper is based on Wardrop's Principles; User Equilibrium and System Optimal.

3.2.1 User equilibrium

For a real number $p \geq 1$, the L^p-norm of a vector x is defined by

$$||x||_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{1/p} \tag{3.1}$$

Let $\boldsymbol{a}$ be the action profile of n players on a k-route transport network. The objective function based on User Equilibrium (UE) looks at how close the average travel time of each route in the network to the ideal state, where the average travel time in each route is equally distributed. The L^p-norm will be used to measure that closeness. If f_U is the objective function, then

$$f_U(\boldsymbol{a}) = \left\| \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_k \end{bmatrix} - \begin{bmatrix} 1/k \\ 1/k \\ \vdots \\ 1/k \end{bmatrix} \right\|_p \tag{3.2}$$

where α_i is the average travel time of the travelers belonging to the last n_{window} travelers who used route i.

A value of f_U that is closer to 0 implies that the system is close to the ideal state of Wardrop's User Equilibrium.

3.2.2 System optimal

Let $\boldsymbol{a}$ be the action profile of n players on a k-route transport network. The objective function based on System Optimal (SO) is just the average travel time of the n travelers in the transport network. Hence, the objective function f_S can be defined as

$$f_S(a) = \frac{1}{n}\sum_{i=1}^{n} c_i(\boldsymbol{a}) \tag{3.3}$$

where $c_i(\boldsymbol{a})$ is the cost incurred by the i^{th} player.

4. METHODOLOGY

4.1 Experimental Setup

4.1.1 Traffic simulator

For this study, the primary traffic simulator used is LocalSIM, a DOST-PCIEERD funded microscopic traffic simulation software. Compared to other traffic simulators, this software aims to replicate Filipino traffic behavior (Palmiano, 2017).

The car-following model used by the LocalSIM in this study was IDM (Kesting et al., 2013), with the conflict resolution model adapting the model developed by Cruda and Andaca (2016) modified to fit with IDM car-following model. In addition, the flow rate used in the study is 1250 veh/hr, the calibrated road capacity for LocalSIM (Palmiano, 2017). As the study will assume constant entry of vehicles, then the headway used is 2.88 s.

For this study, the travel time is defined includes the time delay due to queuing. In addition, route choice made by the travelers will be determined by the greedy algorithm. Finally, any process that gives random values are turned off so that the simulator outputs will not vary from using the same inputs. Table 4.1 shows the simulator inputs used in the study.

Table 4.1: Simulator Parameters used in the Study

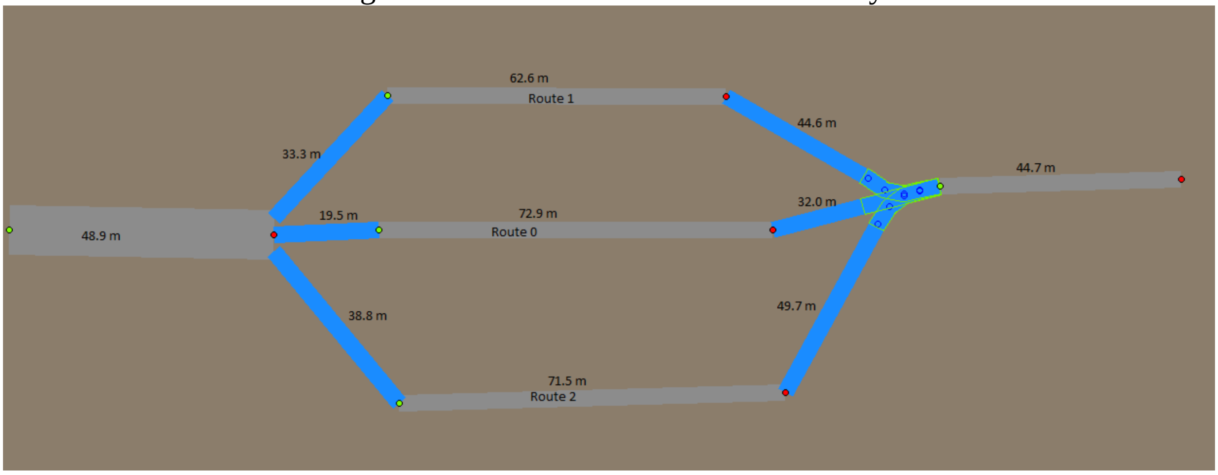
Simulator Parameters	Meaning	Value
v_{max}	Max. velocity	$16.67\ m/s$
a_{max}	Max. acceleration	$3.15\ m/s^2$

Vehicle length		4.81 m
Vehicle width		1.83 m
IDM Parameters	Meaning	Value
v_0	Desired velocity	33.33 kph
T	Safe time headway	1.6 s
δ	Acceleration exponent	4
s_0	Minimum spacing	0

4.1.2 Map

Figure 4.2 shows the road network used for this study. The road network has 3 routes that are coded by "0", "1", and "2", with total lengths of 218 m, 234.1 m, and 253.6 m respectively. The entry lane is a 3-lane road while the exit lane is a 1-lane road, in order to induce congestion in the network as well as to minimize the bias in vehicle conflict resolution due to the architecture of the software used.

Figure 4.2 Road Network used in the study

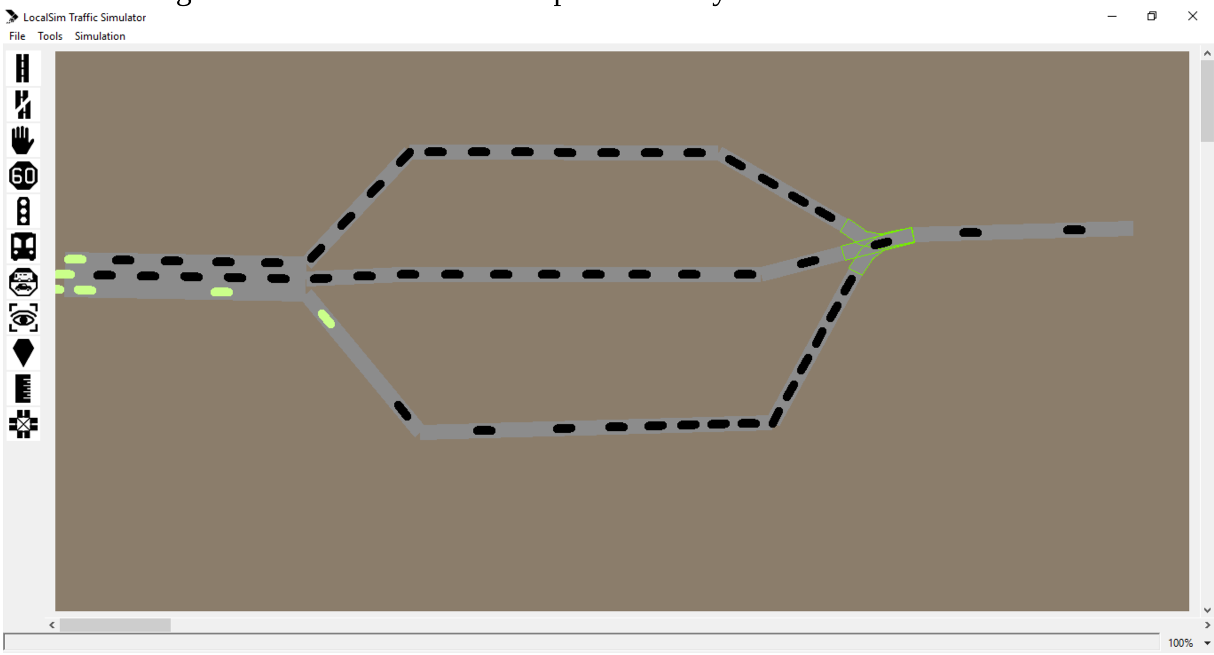


4.1.3 Generating initial conditions

Since the traveler must enter the non-empty transport network, prior to the entry of the first traveler (A_1') the simulation will generate m initial travelers, taking random route choices in order to congest the traffic network. The route choices of the initial travelers will serve as the initial condition of the transport network (y). Define this initial condition to be the **background traffic** of the network.

For this study, the number of travelers in the background traffic is 90, and 30 different background traffic was used. The various generated background traffic can be classified into two; even route choices and uneven route choices. Figure 4.3 shows the scenario upon the entry of the observed travelers (green). The black vehicles represent the background traffic.

Figure 4.3: Network Scenario upon the Entry of the Observed Travelers



4.2 Benchmarking the Greedy Algorithm with the Global Optimum

To determine how close the results of the greedy algorithm with the global optimum, both the greedy algorithm and the global optimum were run over the various background traffic, optimizing the action profile for $1 \leq n \leq 9$ travelers. The data points that would be used for this methodology would be the difference between the greedy algorithm and the global optimum in terms of the value of the objective function (f_U and f_S). For f_U objective function, the parameter value for n_{window} is 50, and for the p-norm, the 1-norm, 2-norm and ∞-norm were all used.

4.3 Finding Optimal Number of Backtracks

Introducing the backtracking step into the greedy algorithm can result in improved results, with the added running time as a penalty. Finding the optimal number of backtrack would mean balancing the gain in value with the increase in running time.

Both the greedy algorithm and greedy algorithm with b backtrack steps will be run among the background traffic, optimizing fixed 100 vehicles. As the backtracking step introduces a random step, the greedy algorithm with b backtrack will be repeated 30 times for each background traffic, with the average value of the 30 trials being used to compare with the greedy algorithm. The objective function to be used is the average travel time (f_S) and for greedy algorithm with b backtracks, the values of b that were tested are $1 \leq b \leq 10$. To determine the optimal number of backtracks, the time improvement ($f_S^{(b)} - f_S$) to running time ratio will be used.

5. RESULTS AND DISCUSSION

5.1 Benchmarking the Greedy Algorithm with Global Optimum

Figure 5.1 Greedy vs Global: Distribution of f Differences

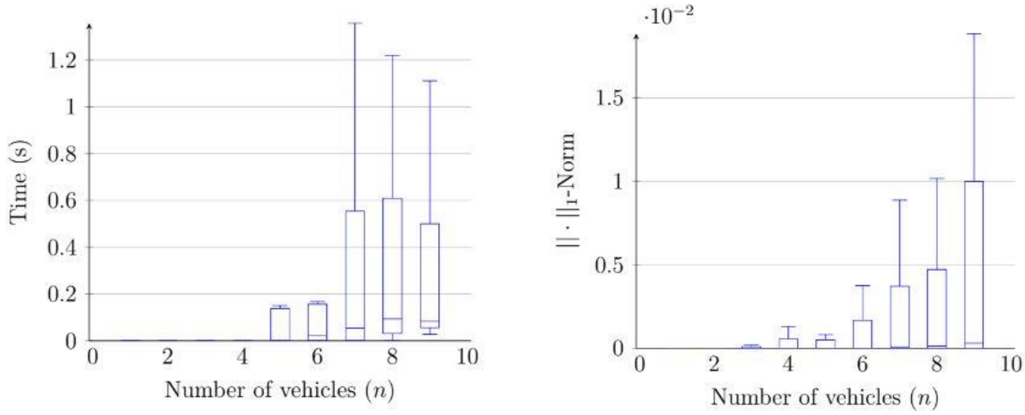
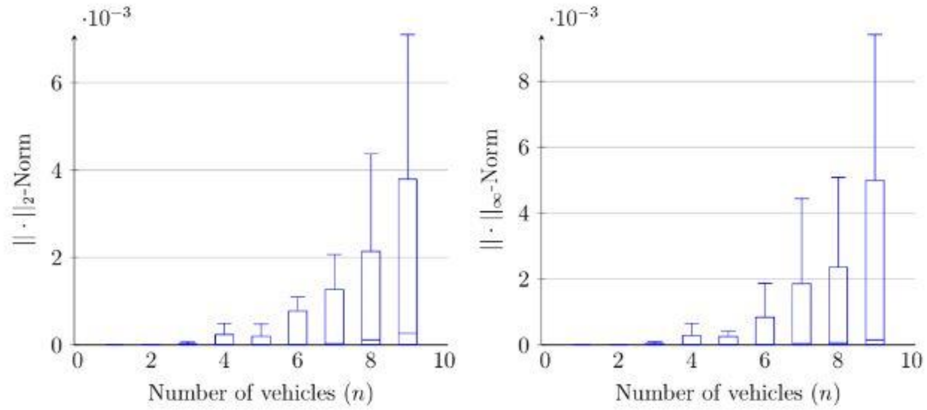


Figure 5.1 shows the distribution of the discrepancies between the greedy algorithm and the global optimum for system optimal f_S (left graph) and user equilibrium, using 1-norm as the objective function f_U (right graph).

In both cases, the discrepancies between the greedy algorithm and the global optimum started to appear at $n \geq 7$. Also, in both cases, it can be noticed that the upper box is longer than the lower box, which means that the median is way closer to the lower quartile than the upper quartile. This implies that for most background traffic, the discrepancies between the greedy algorithm and the global optimum is near 0, albeit increasing, while there are a handful of background traffic that shows high discrepancies between the greedy algorithm and the global optimum as shown by the long whiskers. Hence, it can be said that for most of the initial conditions, the increase in discrepancies is slow. One difference between the two is that for the distribution of differences for f_U, the dispersion and the extreme value seems to continue increasing beyond $n > 9$, while that is not the case for f_S.

For the user equilibrium, similar results are obtained even when the 2-norm or the ∞-norm was used as shown in Figure 5.2 with the 2-norm on the left and the ∞-norm on the right.

Figure 5.2 Greedy vs Global: Distribution of f_U Differences using 2-norm and ∞-norm



5.2 Comparing Greedy Algorithm with Backtrack

After benchmarking the greedy algorithm with the global optimal, the greedy algorithm and greedy algorithm with backtracking will be compared. Table 5.3 shows the average travel time and the normalized running time ratio for various values of b.

Table 5.3 Greedy Algortithm with b Backtracks

b	Time of greedy with b backtracks $f_S^{(b)}$	Time improvement from greedy $f_S^{(b)} - f_S$	running time ratio $r^{(b)}/r$ with $r = 3292.78\ s$
0	158.4346 s	0	1
1	157.8854 s	0.5492 s	1.4803
2	157.6503 s	0.7843 s	2.0294
3	157.3648 s	1.0698 s	2.6320
4	157.2806 s	1.154 s	3.2604
5	157.1879 s	1.2467 s	3.7877
6	157.0389 s	1.3957 s	4.2098
7	157.0262 s	1.4084 s	4.7773
8	156.9526 s	1.482 s	5.3431
9	156.9112 s	1.5234 s	5.9856
10	156.8284 s	1.6062 s	6.5259

Figure 5.4 shows the 95% confidence interval (left graph) and the distribution (right graph) of time improvements from the greedy algorithm for increasing number of backtracks ($1 \leq b \leq 10$). It can be seen that the average travel time improvement, shown by the thick blue line in the left graph, is increasing but has a negative concavity. This affirms the decreasing marginal returns from increasing the number of backtrack. Meanwhile, the right graph shows a more detailed picture of what happens. The discrepancy seems to stay consistent with increasing b, but the extremes sometimes fluctuates. This implies that for some background traffic with already high improvement, increasing the number of backtracks might lead to smaller improvement. Finally, as the lower box is slightly shorter than the upper box in the box plot, the distribution of time improvement seems to be skewed slightly towards 0.

Figure 5.4 Plot of Time Improvements for Increasing Number of Backtracks (b)

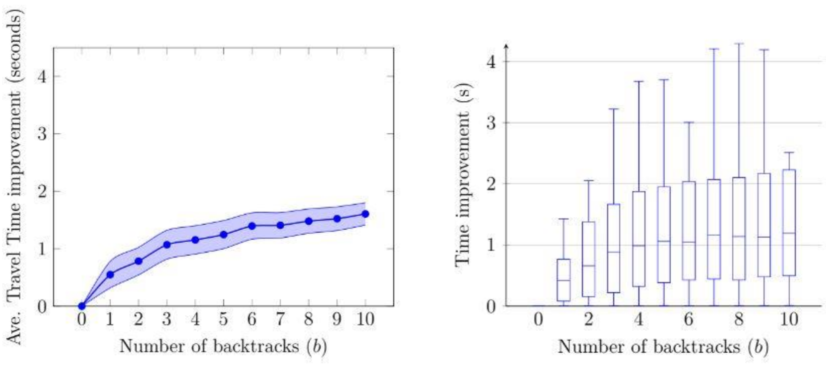
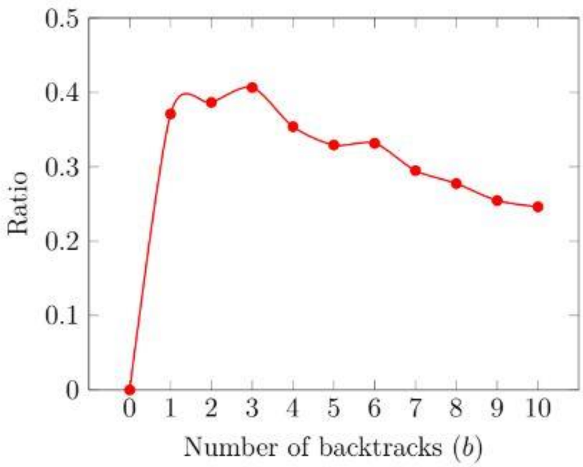


Figure 5.5 shows the ratio between the travel time improvement and normalized running time. From the graph, it can be seen that the ratio peaked at $b = 3$. Hence 3 is the number of backtracks that balances the increase in running time and the travel time improvement from the greedy algorithm.

Figure 5.5 Travel Time Improvement to Normalized Running Time Ratio



5.3 Multiple O-D Pairs

One of the limitations of this study is just examining the algorithm under a single O-D pair. It is possible to do the same setup under multiple O-D pairs, albeit with a different methodological set-up. An important aspect of this study is that entry time of the travelers are controlled and is consistent within the same replications under the same background traffic. In this case, the methodology can be extended. However, a lot of initial conditions might be needed since it's possible for more types of initial conditions due to the possible interaction among the vehicles in more than one O-D pairs.

5.4 Recommendations

Further extensions can be done for game-theoretic treatment for traffic assignment. Listed below are some of the recommendations that could be done for further research:

1. Make the entry of vehicles follow poisson distribution instead of constant. A parameter μ can be fixed for the poisson distribution.
2. See how the optimal number of backtracks changes as the number of optimized vehicles (b) changes.
3. Look at different conflict area models.
4. Perform sensitivity analysis for parameters used in the study like m, n, n_{window}.
5. Parallelizing of processed to reduce running time. Parallelization can be done on the backtrack step by distributing the b backtracks into several processes.

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