Network Analysis Method in Seeking Optimal Longitudinal Section of Railway

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Abstract: Network Analysis Method – PERT/CPM based on oriented graph theory. The method is widely applied to determine the longest path called the critical path from the start to the finish of a project. This article presents methodological basis of the method and applying the method in solving problem on seeking the optimal longitudinal section of railway, one of the important requirements of railway designing work in the field of transport construction.

Keywords: Network Analysis Method; Optimal longitudinal section of railway; The Critical Path; PERT/CPM.

1. INTRODUCTION

A project is often implemented by a set of tasks, also known as activities associated with each other sequence from the start activity to the finish one. There is need time requirements and specific resources for implementation of the activities.

Traditionally, people use Gantt Charts to present progress of activities, in which the activities are listed from top to bottom, and the time is shown by the horizontal lines from left to right.

Disadvantages of Gantt Charts is not enabling to determine the relationship between the activities, so it shall not be applied for big projects or large complex jobs. These projects require to establish planning, operating and checking schedule systematically and efficiently - often to optimize the efficiency of time and resources. Based on the demands above circa 1956 - 1958 Network Analysis Method was developed.

This article presents methodological basis of the Network Analysis Method – PERT/CPM and applying the method in solving problem on seeking the optimal longitudinal section of railway from huge feasibility alternatives, one of the important requirements of railway designing work in the field of transport construction.

2. NETWORK ANALYSIS METHOD

CPM (Critical Path Method) was the discovery of M.R.Walker of E.I.Du Pont de Nemours & Co. and J.E.Kelly of Remington Rand, circa 1957. In March 1959, the method was applied to a maintenance shut-down at the Du Pont works in Louisville, Kentucky.

PERT (Program Evaluation Review Technique) was devised in 1958 for the POLARIS missile program by the Program Evaluation Branch of the Special Projects office of the U.S.Navy, helped by the Lockheed Missile Systems division and the Consultant firm of Booz-Allen & Hamilton, see Vanhoucke, M. (2013).

The two methods above were formed independently, but very similar. The difference between the two methods is the estimated time for activities is deterministic in the CPM but can be random in the PERT. Nowadays these methods are developed widely and regarded as one method under one common named Method for operating of projects PERT/CPM or Network Analysis Method (NAM).

Network Analysis Method including 3 main stages as planning, operating, checking and adjustment. Because the purpose of applying, this article presents the first two stages and the estimated time for these activities is deterministic.

2.1. The first Stage

The first stage of NAM is planning embodied in a network diagram that is represented as a oriented graph as follows:

Define the Project and all of it's significant activities or tasks.

Determine the relationships among the activities. Decide which activities must precede and which must follow others, the first activities, the end activities.

Draw the "Network" connecting all the activities. Each Activity should have unique event numbers. Dummy arrows are used where required to avoid giving the same numbering to two activities.

Assign time and/or cost estimates to each activity

The longest time/or lowest cost path pass the network that is called the critical path then shall be tracked.

2.2. Operating Stage

This stage is responsible for analyzing the target time and give the necessary data on the network map.

Figure 1 below presenting activities sequence shall be implemented to complete the project. It could be used as an example for explanation of NAM.

- ***Earliest time (ET)*** *of an event* E_i

  Node 1 (Start): $E_1 = 0$

  Node 2: $E_2 = E_1 + t_{12}$

  ....

  Node j:

$$E_j = E_i + t_{ij} \quad (1)$$

Where:

Node i is immediate predecessor of node j.

E_i: Earliest time of event i;

E_j: Earliest time of event j;

t_{ij}: Duration of activity (i-j). See Oberlender, G.D. (2000).

In the case of multiple arcs enter a node, it means that multiple activities end at the node, then Ej is time that all activities end at that node has just finished, which should get the maximum of totals. For example, diagram on Figure 1:

$E_7 = max \{E_4 + T_{47} , E_5 + t_{57}\} = max \{16 + 7, 20 + 5\} = 25$

$E_8 = max \{E_5 + T_{58} , E_6 + t_{68}\} = max \{20 + 0, 22 + 7\} = 29$

Generalize for all cases:

$E_j = max\ \{E_i + tij\}$ (2)

E_i showed on Figure 1 is the first number in parentheses at each node.

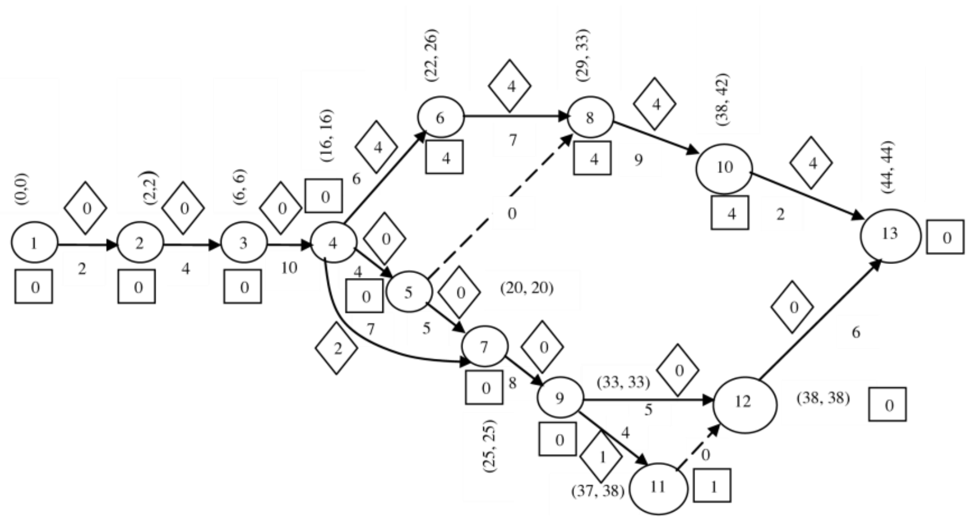

Figure 1: Diagram illustrating for Network Analysis Method

Latest time *for an event* L_j is the latest time of any activity enter node j that should be completed without altering the end of the project as soon as possible, symbol: *Lj*. In contrast to E_j, L_j could be calculated backward pass. It means that the calculation will be started from the ending node. By definition, at the ending node E_n equals to L_n.

In the case of only one arc goes out from a node, i.e. one activity is started, the latest time as follows:

$$L_i = L_j - t_{ij} \quad (3)$$

Where:

L_i: Latest time of event i;

L_j: Latest time of event j;

t_{ij}: Duration of activity (i-j).

The nodes 12, 11, 10, 8, 7, 6, 3, 2, 1 in Figure 1 belongs to the case. In the case of multiple arcs go out of a node, then by definition we have:

$$L_i = min\ \{L_j - t_{ij}\} \quad (4)$$

The nodes 9, 5, 4 in Figure 1 belongs to the case, such as:

$L_9 = min\ \{L_{11} - t_{911}, L_{12} - t_{912}\} = min\ \{38 - 4, 38 - 5\} = 33$

• ***The Slack*** *for an event is the difference between its latest time and its earliest time, symbol:*

$$d_i = L_i - E_i \quad (5)$$

There are two types of the slack for an activity (i,j): the total slack and the independent slack.

The total slack *TFij* means the time for an activity (i,j) can be delayed but without affecting the end of the project. It equals to difference of longest time for an actyvity (*Lj – Ei*) and it's duration (tij) as follows:

$$TF_{ij} = L_j - E_i - t_{ij} \quad (6)$$

The independent slack means the difference of time for an activity (*Ej – Ei*) and its duration (tij), assume that all activities could be started as soon as posible, as follows:

$$FF_{ij} = E_j - E_i - t_{ij} \quad (7)$$

On the network diagram, d_i is the difference of two numbers in parentheses of node i that is usually written in the square next to the node. The total slack of the activity *TFij* written in the lozenge next to the arc. The independent slack FF_{ij} less important so in general it could not be written.

The Critical Path is the path pass the start node to the finish node and on it all activities has the total slack TF_{ij} of zero. These activities are called the critical activities. Event i with d_i of zero that is called the critical event. In Firgue 1, the critical path pass nodes: 1 - 2 - 3 - 4 - 5 - 7 - 9 - 12 - 13.

2.3. Key Characters of the Critical Path

- A project has at least one critical path;

- All activities on the critical path has TF_{ij} of zero;

- All events with di of zero, i.e. all critical events shall be on the critical path;

- A path pass the start node to the finish node with all critical events on it that could not be the critical path because there may not be critical activities.

- The critical path is the longest path among paths pass the start node to the finish node.

3. APPLICATION OF NETWORK ANALYSIS METHOD ON SEEKING THE OPTIMAL LONGITUDINAL SECTION OF URBAN RAILWAY

To solve the optimal problem there is need to establish constraints and the objective function. There are key constraints for the problem of seeking the optimal longitudinal section of urban railway as follows:

- The allowable longitudinal gradient (ALG): The equivalent gradient value of any section must also not be allowed greater than ALG;
- Length of a gradient section must be greater than or equal to the allowable length of a section;
- The change of gradient of two adjacent sections must be less than allowable gradient change;
- Vertical curve shall be installed wherever a gradient change greater than allowable gradient change so that it does not interfere with the safe car operation, taking operating speed and the structure of rolling stock into consideration. The radius of a vertical curve shall not be less than the allowable values. Vertical curve shall comply with the following criteria: the distance from the ends of the curve to the ends of a switch not less than 5m. For ballasted track, vertical curve shall not be coincide with transition curve of horizontal alignment. If there is no transition curves then vertical curve shall not be coincide with cant transition section.
- In case of an urban railway pass over another one, the allowable clearance of the urban railway shall be followed;
- In case of an urban railway pass over an ordinary railway, the allowable clearance of the ordinary railway shall be followed;
- In case of an ordinary railway pass over an urban railway, the allowable clearance of the urban railway shall be followed;

- In case of an urban railway pass over a highway, the allowable clearance of the highway shall be followed;
- In case of an urban railway is passed over by a highway, the allowable clearance of the urban railway shall be followed;
- In case of an urban railway pass over a waterway, the allowable clearance of the waterway shall be followed;
- Level of stations: Stations of an elevated urban railway often have two to three floors, depending on the level of adjacent railway sections and the height of the station structure as well as the minimum height, the maximum height of one station's floor. These limited heights also depend on type and scale of the station.
- Other constraints: depending on location of the horizontal alignment and level of the urban railway that based on existing conditions of the city, urban planning, at grade structures, underground structures, requirement on preservation of ancient structures, requirement on landscape architecture and so on.

The objective function of the optimal longitudinal section problem is total cost of an alternative. Total cost of an alternative is different to another, including construction cost and operation cost.

The construction cost involves cost for railway and cost for structures, cost for facilities such as formation, slab, rails, fastening, viaduct, tunnel, drainage system, stations, rolling stock, power system, etc.

The operation cost involves cost for train running and maintenance such as power supply, administration, repair and maintenance for rolling stock and infrastructure and so on.

In the example shown below the constraints are deemed to have been satisfied in the process of designing. Key task is reflected by the objective function is to find out the alternative with the lowest cost among possible alternatives.

Example for seeking the optimal longitudinal section of urban railway

The task is designing longitudinal section of an urban railway track with start point 0 and finish point 7. The line pass intermediate points as 1, 2, 3, 4, 5, and 6 and is divided into 7 sections as 0-1, 1-2, 2-3, 3-4, 4-5, 5-6, 6-7. Assuming that level of the start point and the finish point are determined and then level of the intermediate points shall be defined.

For each intermediate gradient change point, there are a lot of reliable level options that meet requirement of constraints. In the example, an intermediate point has three options (note that no limitation on options for each intermediate point) as follows:

For point 1, including options: 1.1, 1.2, 1.3;
For point 2, including options: 2.1, 2.2, 2.3;
For point 3, including options: 3.1, 3.2, 3.3;
For point 4, including options: 4.1, 4.2, 4.3;
For point 5, including options: 5.1, 5.2, 5.3;
For point 6, including options: 6.1, 6.2, 6.3;

(See figure 2 below)

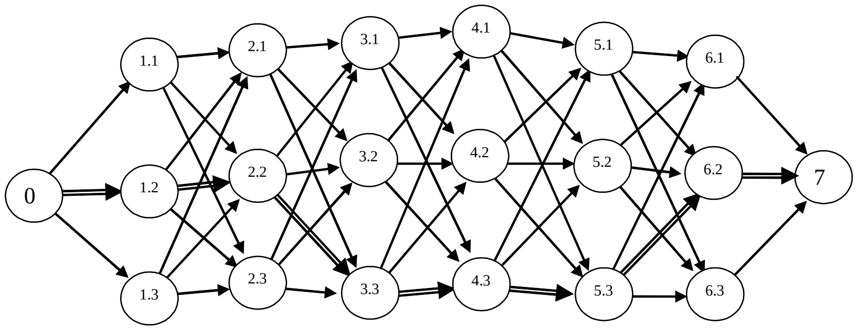

Figure 2: Network diagram for seeking the optimal longitudinal section of urban railway

Cost for each alternative of each section (called activity cost) shown in table 1 below. There are many longitudinal section alternatives that pass from the start point to the finish point. Which is lowest total cost alternative? It shall be found out.

According to Network Analysis Method, the critical path is longest path. So to apply the method for seeking the optimal longitudinal section of urban railway there is need to transform the problem of max seeking into min seeking as follows.

In table 1, We recognize that all activity cost is less than 200 currency units. We replace an activity cost with a new variable as follows:

$$b_{ij} = 200 - c_{ij}$$

Where:

b_{ij} and c_{ij} no less than zero;

c_{ij}: activity cost (i,j). It is easy to recognize that if total c_{ij} reaches min value i.e. total b_{ij} reaches max value.

We call point 1 to point 6 as node 1 to node 6, point 1.1 to point 6.3 as node 1.1 to node 6.3 respectively. So cost for each activity on the traditional Network Analysis Diagram is time unit then the cost here is currency unit.

Application of (1) and (2) to compute:

Node 0: $E_0 = 0$

Node 1 : $E_{1.1} = E_0 + b_{0\text{-}1.1}$, $E_{1.2} = E_0 + b_{0\text{-}1.2}$, $E_{1.3} = E_0 + b_{0\text{-}1.3}$,

$E_{1.1} = 0+80 = 80$, $E_{1.2} = 0+70 = 70$, $E_{1.3} = 0+55 = 55$

Node 2: $E_{2.1} = \max\{E_{1.1} + b_{1.1\text{-}2.1}, E_{1.2} + b_{1.2\text{-}2.1}, E_{1.3} + b_{1.3\text{-}2.1}\}$,

$= \max\{80+70, 70+74, 55+70\} = \mathbf{150}$

$E_{2.2} = \max\{E_{1.1} + b_{1.1\text{-}2.2}, E_{1.2} + b_{1.2\text{-}2.2}, E_{1.3} + b_{1.3\text{-}2.2}\}$,

$= \max\{80+55, 70+100, 55+75\} = \mathbf{170}$

$E_{2.3} = \max\{E_{1.1} + b_{1.1\text{-}2.3}, E_{1.2} + b_{1.2\text{-}2.3}, E_{1.3} + b_{1.3\text{-}2.3}\}$,

$= \max\{80+70, 70+55, 55+65\} = \mathbf{150}$

Table 1: Activity cost (currency unit)

Section1			Section2			Section3			Section4			Section5			Section6			Section7		
activ	c_{ij}	b_{ij}	**activ**	c_{ij}	b_{ij}	**activ**	c_{ij}	b_{ij}	**activ**	c_{ij}	b_{ij}	**activ**	c_{ij}	b_{ij}	**activ**	c_{ij}	b_{ij}	**activ**	c_{ij}	b_{ij}
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*
0-1.3	145	55	**1.3-2.3**	135	65	**2.3-3.3**	140	60	**3.3-4.3**	120	80	**4.3-5.3**	130	70	**5.3-6.3**	165	35			
			1.2-2.3	145	55	**2.2-3.3**	145	55	**3.2-4.3**	132	68	**4.2-5.3**	140	60	**5.2-6.3**	175	25			
			1.1-2.3	130	70	**2.1-3.3**	155	45	**3.1-4.3**	140	60	**4.1-5.3**	145	55	**5.1-6.3**	185	15			
0-1.2	130	70	**1.3-2.2**	125	75	**2.3-3.2**	115	85	**3.3-4.2**	115	85	**4.3-5.2**	126	74	**5.3-6.2**	160	40			
			1.2-2.2	100	100	**2.2-3.2**	144	56	**3.2-4.2**	140	60	**4.2-5.2**	136	64	**5.2-6.2**	165	35			
			1.1-2.2	145	55	**2.1-3.2**	160	40	**3.1-4.2**	130	70	**4.1-5.2**	140	60	**5.1-6.2**	180	20			
0-1.1	120	80	**1.3-2.1**	130	70	**2.3-3.1**	155	45	**3.3-4.1**	126	74	**4.3-5.1**	115	85	**5.3-6.1**	155	45	**6.3-7**	98	102
			1.2-2.1	126	74	**2.2-3.1**	128	72	**3.2-4.1**	135	65	**4.2-5.1**	120	80	**5.2-6.1**	160	40	**6.2-7**	80	120
			1.1-2.1	130	70	**2.1-3.1**	155	45	**3.1-4.1**	130	70	**4.1-5.1**	135	65	**5.1-6.1**	175	25	**6.1-7**	155	45

Node 3 : $E_{3.1} = max\{ E_{2.1} + b_{2.1\text{-}3.1}, E_{2.2} + b_{2.2\text{-}3.1}, E_{2.3} + b_{2.3\text{-}3.1}\}$

$= max\{150+45, 170+72, 150+45\}=\mathbf{242}$

$E_{3.2} = max\{ E_{2.1} + b_{2.1\text{-}3.2}, E_{2.2} + b_{2.2\text{-}3.2}, E_{2.3} + b_{2.3\text{-}3.2}\}$

$= max\{150+40, 170+56, 150+85\}=\mathbf{235}$

$E_{3.3} = max\{ E_{2.1} + b_{2.1\text{-}3.3}, E_{2.2} + b_{2.2\text{-}3.3}, E_{2.3} + b_{2.3\text{-}3.3}\}$

$= max\{150+45, 170+55, 150+60\}=\mathbf{225}$

Similar results:

Node 4 : $E_{4.1} = max\{ E_{3.1} + b_{3.1\text{-}4.1}, E_{3.2} + b_{3.2\text{-}4.1}, E_{3.3} + b_{3.3\text{-}4.1}\}=312$

$E_{4.2} = max\{ E_{3.1} + b_{3.1\text{-}4.2}, E_{3.2} + b_{3.2\text{-}4.2}, E_{3.3} + b_{3.3\text{-}4.2}\}=312$

$E_{4.3} = max\{ E_{3.1} + b_{3.1\text{-}4.3}, E_{3.2} + b_{3.2\text{-}4.3}, E_{3.3} + b_{3.3\text{-}4.3}\}=305$

Node 5 : $E_{5.1} = max\{ E_{4.1} + b_{4.1\text{-}5.1}, E_{4.2} + b_{4.2\text{-}5.1}, E_{4.3} + b_{4.3\text{-}5.1}\}=392$

$E_{5.2} = max\{ E_{4.1} + b_{4.1\text{-}5.2}, E_{4.2} + b_{4.2\text{-}5.2}, E_{4.3} + b_{4.3\text{-}5.2}\}=379$

$E_{5.3} = max\{ E_{4.1} + b_{4.1\text{-}5.3}, E_{4.2} + b_{4.2\text{-}5.3}, E_{4.3} + b_{4.3\text{-}5.3}\}=375$

Node 6 : $E_{6.1} = max\{ E_{5.1} + b_{5.1\text{-}6.1}, E_{5.2} + b_{5.2\text{-}6.1}, E_{5.3} + b_{5.3\text{-}6.1}\}=420$

$E_{6.2} = max\{ E_{5.1} + b_{5.1\text{-}6.2}, E_{5.2} + b_{5.2\text{-}6.2}, E_{5.3} + b_{5.3\text{-}6.2}\}=415$

$E_{6.3} = max\{ E_{5.1} + b_{5.1\text{-}6.3}, E_{5.2} + b_{5.2\text{-}6.3}, E_{5.3} + b_{5.3\text{-}6.3}\}=410$

Node 7 : $E_7 = max\{ E_{6.1} + b_{6.1\text{-}7}, E_{6.2} + b_{6.2\text{-}7}, E_{6.3} + b_{6.3\text{-}7}\}=535$

So, the alternative reaches max value is:

0-1.2-2.2-3.3-4.3-5.3-6.2-7, i.e. the alternative reaches min cost value of **865** (=7x200-535).

Activity costs in the alternative 0-1.2-2.2-3.3-4.3-5.3-6.2-7 is:

130+100+145+120+130+160+80 = **865.**

The result of seeking optimal longitudinal section is presented in Table 2.

Table 2: Result of Seeking Optimal Longitudinal Section of Railway

Section 1		Section 2			Section 3			Section 4			Section 5			Section 6			Section 7		
Activ	C_{ij}	Activ	C_{ij}	Min value	Activ	C_{ij}	Min value	Activ	C_{ij}	Min value	Activ	C_{ij}	Min value	Activ	C_{ij}	Min value	Activ	C_{ij}	Min value
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0-1.3	145	1.3 - 2.3	135		2.3 - 3.3	140		3.3 - 4.3	120	495	4.3 - 5.3	130	625	5.3 - 6.3	165	790			
		1.2 - 2.3	145		2.2 - 3.3	145	375	3.2 - 4.3	132		4.2 - 5.3	140		5.2 - 6.3	175				
		1.1 - 2.3	130	250	2.1 - 3.3	155		3.1 - 4.3	140		4.1 - 5.3	145		5.1 - 6.3	185				
0-1.2	130	1.3 - 2.2	125		2.3 - 3.2	115	365	3.3 - 4.2	115		4.3 - 5.2	126	621	5.3 - 6.2	160	785			
		1.2 - 2.2	100	230	2.2 - 3.2	144		3.2 - 4.2	140		4.2 - 5.2	136		5.2 - 6.2	165				
		1.1 - 2.2	145		2.1 - 3.2	160		3.1 - 4.2	130	488	4.1 - 5.2	140		5.1 - 6.2	180				
0-1.1	120	1.3 - 2.1	130		2.3 - 3.1	155		3.3 - 4.1	126		4.3 - 5.1	115		5.3 - 6.1	155	780	6.3 - 7	98	
		1.2 - 2.1	126		2.2 - 3.1	128	358	3.2 - 4.1	135		4.2 - 5.1	120	608	5.2 - 6.1	160		6.2 - 7	80	865
		1.1 - 2.1	130	250	2.1 - 3.1	155		3.1 - 4.1	130	488	4.1 - 5.1	135		5.1 - 6.1	175		6.1 - 7	155	

CONCLUSION

By this paper, the author presents a summary of the methodological basic of the Network Analysis Method and applying the method in solving problem on seeking the optimal longitudinal section of railway in general and urban railway in particular. This is a fundamental task in railway design to find the lowest cost option from huge possible alternatives that can go from the beginning to the end of a route that satisfies the constraints given by the designer. The problem becomes even more significant if the length of the route and the number of feasible options increase. By applying the algorithm of Network Diagram Method, the volume of calculation is greatly reduced, helping the designer to quickly find the optimal solution.

Among modern and popular seeking optimal methods as dynamic programming method, genetic algorithm and so on, Network Analysis Method is a method widely applied in project management, scheduling construction, production and installation of complex construction works in the military as well as in construction sector, social-economic sectors.

This method is based on strong theoretical basis but the algorithm is quite simple and visual. Based on principle of Network Analysis Method, with initial application mainly in the time management of project implementation. We now can develop the application for other problems with different expense resources such as time, money, facilities, supplies, personnel, etc, in which seeking longitudinal section with minimal investment costs is a highly effective application.

With algorithm based on the methodology of Network Analysis Method, computerisation with the common software such as Excel, Microsoft Project, etc. becomes quite simple. The application of NAM to assist designers easily finding the exact solution for the problem on seeking optimal longitudinal section, contributing positively in investment savings during carrying out construction design on urban railway and transportation works in general.

REFERENCES

Ha, L. H., Ky, P.V. (2010) *Survey and Design of Railway*, Transport Publishing House, Hanoi.

Kiem, H., Thai, L.H. (2000) *Genetic algorithms*, Education Publishing House, Hanoi.

Ky, P.V., Ha, L.H., Thien, N.H., Hai, D.V. (2016) *Modern Methods for Railway Design*, Science and Technics Publishing House, Hanoi.

Nagarajan, K. (2005) *Project Management*, New Age International Publishers, New Delhi.

Oberlender, G.D. (2000) *Project Management for Engineering and Construction*, McGraw – Hill, New York.
Phan, V.N. (2005) *Optimization*, Post office Publishing House, Hanoi.
Punmia, B.C., Khandelwan, K.K. (2002) *Project Planning and Control with PERT and CPM*, Laxmi Publications (P) LTD, New Delhi.
Rory, B. (2003) *Project Management Planning and Control Techniques*, Wiley, London.
Sharma, S.C. (2006) *Operation Research PERT, CPM & Cost Analysis*, Discovery Publishing House, New Delhi.
Tri, B.M. (1995) *Optimization*, Hanoi University of Science and Technology, Hanoi.
Thomas Friedlob, G. (2003) *Essentials of Financial Analysis*, John Wiley& Sons, Hoboken, New Jersey.
Vanhoucke, M. (2013) *Project Management with Dynamic Scheduling*, Springer, Berlin.