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Modeling Departure Time and Route Choice Problems in Stochastic Road Networks for Online ATIS Applications

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Abstract: This paper investigates the departure time and route choice problem (DTARCP) for the development of online routing applications in advanced traveler information system (ATIS). The DTARCP is concerned with the problem of determining the best departure time and reliable shortest path simultaneously in stochastic and time-dependent networks. It is motivated by the fact that travelers make these two choices simultaneously for their travel. A new model is proposed for modeling the DTARCP with particular application in ATIS. An efficient solution algorithm is developed to solve the DTARCP. Numerical results based on real-world ATIS are also presented to demonstrate the features of the proposed model and solution algorithm. The computational results suggest that the proposed algorithm could be applicable to the online ATIS-based routing applications in large-scale road network.

Key Words: *departure time and route choice problems, reliability, stochastic and time-dependent network, advanced traveler information system*

1. INTRODUCTION

With the advances of information and communication technologies, a lot of interests have been given to the development of advanced traveler information system (ATIS) for alleviating traffic congestion. The ATIS includes technological infrastructures for real-time data collection, processes for generating traffic information, services for disseminating the information to users, and applications for route guidance. There are numerous techniques for real-time travel time data collection, including inductive loop detectors, automatic vehicle identification (AVI), global positioning system (GPS) equipments, cellular phone tracking and imaging processing techniques. The collected data are transmitted to a central traffic management center, where they are automatically processed to estimate and/or predict the states of transportation network. The traffic information is then disseminated to travelers using various media, such as websites portals, variable message signs (VMS), or 3-G cellular phones. With updated or predicted traffic information, informed travelers could make a better decision through ATIS-based routing applications. It hopes that collectively better decisions made by travelers would result in a relief from congestion. The benefits of ATIS have been studied by several researches (Chorus *et al.* 2006; Toledo and Beinhaker, 2006). It was found that the ATIS-based routing may lead to travel times saving of up to 14% and a reduction of travel time variability of up to 50% (Toledo and Beinhaker, 2006).

In urban road network, travel times are highly stochastic due to random fluctuations of traffic demand, and interruptions caused by traffic control devices, accidents, road construction and inclement weather (Lee et al., 2009). In this case, the travel time is not constant resulting in uncertainty when attempting to predict it. Under travel time uncertainty, travelers may consider travel time variation as a risk in their route choice decision when they are planning for important events (e.g. a job interview). In this case, travelers may react at least in two ways. They may choose reliable routes; and/or may set up a travel time safety margin by departing from origin earlier. The former captures travelers' behavior in spatial aspect whereas the latter describes the temporal aspect (Siu and Lo, 2009). The spatial and temporal effects of travel time uncertainty on travelers' route choice behaviors have been confirmed in numerous empirical studies (Palma and Rochat, 1999; Lam and Small, 2001; Tam et al., 2008). Therefore, sophisticated ATIS-based routing applications should provide routing functions that are able to determine the optimal departure time and reliable shortest path simultaneously.

The path finding problem in stochastic and time-dependent network (STDSPP) has been addressed by several researches. Hall (1986) pointed out that the STDSPP cannot be solved by traditional shortest path algorithms because of its violation of Bellman's principle of optimality. Fu and Rilett (1998) presented a way to generate the mean and variance of path travel time distribution in stochastic and time-dependent (STD) network using Taylor series expansions. Based on this method, a heuristic solution algorithm was developed to find the least expected time path. Miller-Hooks and Mahmassani (2000) presented an exact solution algorithm for finding the same least expected time path in STD network. Chang et al. (2005) proposed a method to find non-dominant paths for multiple routing objectives in STD network. However, the path finding problem considering travelers' route choice behaviors under travel time uncertainty has received little attention.

This paper proposes a new model and a solution algorithm to solve route searching problem of determining both the departure time choice and route choice for ATIS-based online routing applications. For convenience, the problem is hereafter referred to as departure time and route choice problem (DTARCP). To meet real-time operations of ATIS-based routing applications, the proposed model and solution algorithm of DTARCP fulfill the following requirements: (1) the model should consider the departure time choice and route choice simultaneously; (2) the model should take account travelers' various preferences toward travel time, travel time reliability, travel cost and travel distance in STD networks; (3) the solution algorithm should be computational efficient in order to provide ATIS-based routing services to public.

The rest of paper is organized as follow. Section 2 presents the definition of the DTARCP. Section 3 introduces a solution algorithm for solving the DTARCP. We then conduct a case study based on real-world ATIS to demonstrate the features of proposed model and the performance of solution algorithm in Section 4. Finally, conclusions are given together with recommendations for further study.

2. PROBLEM DEFINITIONS AND MODEL FORMULATIONS

Let G = (N, A) be a stochastic and time-dependent (STD) network, where N = $\{n_1, n_2, ..., n_m\}$ is a set of nodes and A = $\{a_1, a_2, ..., a_n\}$ is a set of links A⊆N×N. Each link a_{ij} connecting two adjacent nodes i and j has an attribute vector ($X_{ij}(t)$, τ_{ij} , d_{ij}), where $X_{ij}(t)$ is the predicted link travel time, τ_{ij} is the toll charge associated with the link, and d_{ij} is the link length. Without loss of generality, the link toll rate and length are assumed to be deterministic and static; while the link travel time, $X_{ij}(t)$, is assumed to be a STD variable whose distribution varies over time.

Let P^{rs} = { p_1, …, p_i} be a set of non-cycle paths from origin r to destination s. Given a particular path p_i from an origin to a destination, assume that a series of journeys are conducted along this path. Each journey $p_i^{rs}(t_r)$ represents a travel departing from origin r at departure time t_r, traveling along the path p_i^{rs}, and arriving destination s at T_s. Note that no waiting at node is allowed in this paper. Let $X_{p_i^{rs}}(t_r)$ denote the path travel time of the journey, we have

$$T_s = t_r + X_{p_i^{rs}}(t_r) \tag{1}$$

Consequently, the arrival time T_s is also a random variable whose distribution is dependent on the link travel time distribution of each link along this path and the departure time at origin node.

Given an origin-destination (O-D) pair and a preferred arrival time $\tilde{t}_s$, the departure time and route choice problem (DTARCP) concerned in this paper is to find the latest departure time t_r and reliable shortest path p^* that required to ensure a certain level of the probability of on time arrival. It can be formulated as follow

$$Min \;\; U(t_r, p) = \tilde{t}_s - t_r + (\tau_p + d_p VOD)/VOT \tag{2}$$

Subject to

$$\Pr(T_s \le \tilde{t}_s) \ge \alpha \tag{3}$$

$$T_s = t_r + X_{p_i^{rs}}(t_r) \tag{4}$$

$$X_p(t_r) = \sum_{(i,j)\in A} X_{a_{ij}}(t)\delta_{ij} \tag{5}$$

$$\tau_p = \sum_{(i,j)\in A} \tau_{ij}\delta_{ij} \tag{6}$$

$$d_p = \sum_{(i,j)\in A} d_{ij}\delta_{ij} \tag{7}$$

$$\sum_{j\in SCS(i)} \delta_{ij} - \sum_{k\in PDS(i)} \delta_{ki} = \begin{cases} 1 & \forall\, i = r \\ 0, & \forall\, i \ne r;\, i \ne s, \\ -1 & \forall\, i = s \end{cases} \tag{8}$$

$$\delta_{ij} \in \{0, 1\}, \quad \forall (i,\, j) \in A \tag{9}$$

Eq. (2) defines the disutility cost of a journey that travelers want to minimize. A value of distance denoted as *VOD* is introduced for converting travel distance d_p into the petrol cost. A value of time *VOT* are defined as weighting factors for converting the toll charge τ_p and petrol cost into the travel time unit. Eq. (3) defines the probabilistic constraint that ensures the probability of on-time arrival is greater or equal to a confidence level α. The confidence level α is a predetermined threshold representing the travelers' attitude towards travel time uncertainty. Eq. (4) defines the arrival time as mentioned in Eq. (1). And Eqs. (5-7) give the travel time, toll charges and distance of the path respectively. Eq. (8) ensures that the links on the reliable shortest path are feasible connecting from origin to destination. Eq. (9) is concerned with the link-path incidence variables which should be binary.

3. SOLUTION ALGORITHM FOR SOLVING DTARCP

Similar to Fu and Rilett (1998), the arrival time of DTARCP can be regards as a continuous-time stochastic process, which indicates that the travel time on a link a_{ij} is the conditional travel time of the enter time at upstream node. As pointed out by Fu and Rilett (1998), identifying arrival time at a number of downstream nodes would become quickly intractable even if simple link travel time distributions are given. Therefore, it is impractical to generate true distributions of arrival time in the real-world ATIS.

To simplify this problem, Chen et al. (2009a) proposed a method to approximate arrival time distribution using discrete probability mass function (PMF). A brief description of the method is given below for the sake of completeness. The cumulative distribution function (CDF) of the link travel time $X_{ij}(t)$ at each time interval is firstly discretized into λ intervals with an equal cumulative probability denoted as ($q\omega$-ω, $q\omega$), q=1,2,…, λ, where ω is the cumulative probability of an interval, $0<\omega<1$ and $\lambda\omega=1$. According to the Mean-Value Theorem, within each interval ($q\omega$-ω, $q\omega$) there is at least one point x_{ijq} satisfies:

$$f(\hat{x}_{ijq}) = \omega / \{\Phi^{-1}_{x_{ij}}(q\omega) - \Phi^{-1}_{x_{ij}}(q\omega - \omega)\} \tag{10}$$

where $f(\hat{x}_{ijq})$ is the probability density function of link travel time at point $\hat{x}_{ijq}$, and $\Phi^{-1}_{x_{ij}}(q\omega-\omega)$ and $\Phi^{-1}_{x_{ij}}(q\omega)$ are inverse of CDF at the boundary points of the interval. Thus, the PMF can be determined as

$$p(\hat{x}_{ijq}) = \omega, \quad \mathrm{q} = 1,2,\ldots,\lambda \tag{11}$$

Similarly, the CDF of arrival times at node *i* can be discretized into η equal cumulative probability intervals denoted as ($q\varepsilon$–ε, $q\varepsilon$), q=1,2,…,η, where ε is the cumulative probability of an interval, $0<\varepsilon<1$ and $\lambda\eta=1$. The PMF of arrival travel time T_i at node *i* is given:

$$P(\hat{t}_{iw}) = \varepsilon, w = 1,2,...,\eta \tag{12}$$

Consequently, the arrival time T_j which depends on arrival time at upstream node *i* can be calculated as:

$$\hat{t}_{j\upsilon} = \hat{t}_{iw} + \hat{x}_{ijq}(y_{iw}), \quad q = 1,2,...,\lambda, \quad w = 1,2,...,\eta, \quad \upsilon = 1,2...,\lambda\eta \tag{13}$$

To prevent the number of elements in the CDF of arrival time from growing exponentially with the number of constituent links, the PMF of arrival time shown in Eq. (13) have to be aggregated:

$$\hat{t}_{jw} = (\sum_{(w-1)\lambda+1}^{w\lambda} y_{jw})/\lambda, \qquad w = 1,2,...,\eta \tag{14}$$

Thus, the PMF of arrival time T_j can be obtained:

$$P(\hat{t}_{jw}) = \varepsilon, \quad w = 1,2,...,\eta \tag{15}$$

In such method, each value of PMF of arrival time ($\hat{x}_{ijq}$) can be treated as an independent routing criterion. As a result, the DTARCP is a multi-criteria shortest path problem (MSPP) in terms of entire PMF of arrival time. Typically, the MSPP relies on the dominance condition to find a set of non-dominated paths, which are defined as those such that it is not possible to find another path with a better value in at least one criterion without worsening the value of at least one other criterion (Soroush, 2008). As mentioned, the dominance condition in the DTARCP should be defined against the entire PMF of arrival time, and this dominance condition is known as first-order stochastic dominance (FSD) condition (Miller-Hooks and Mahmassani, 2003). By recursively applying Eqs. (10-15), one could calculate the arrival time distribution for each departure time and each path, and discard those dominated paths according to the FSD condition. The optimal departure time and reliable shortest path can then be identified by examining all generated non-dominated paths. Clearly, such a naïve approach is computational impractical for the online ATIS-based routing applications in large-scale network.

Using the same method and FSD condition, Chen et al. (2009b) presented a forward-based solution algorithm for finding reliable shortest path in STD network from origin node at given departure time. A backward search from destination node at preferred arrival time was also proposed to find the reliable shortest path by generating departure time distribution. They found that the backward-based searching of reliable shortest path could give a good initial departure time and reliable shortest path for the DTARCP. Based on this idea, a solution algorithm for solving the DTARCP is presented as following.

Inputs: O-D pair, preferred arrival time $\tilde{t}_s$, and confidence level α

Returns: optimal departure time t_r^* and reliable shortest path p^*

Step 1. Find reliable shortest path p_0^{rs} from destination to origin at preferred arrival time $\tilde{t}_s$, using backward-based reliable shortest path algorithm.

Step 2. Based on generated departure time distribution obtained from Step 1, choose the initial departure time t_{r0} according to inverse CDF of departure time at α confidence level.

Step 3. Calculate arrival time distribution T_s by departing origin at t_{r0} and recursively applying Eqs. (10-15) along the p_0^{rs}.

Step 4. Check whether T_s satisfy the probabilistic constraint in Eq. (3). If satisfy, then stop and denote t_{r0} and p_0^{rs} as t_r^*, p^* respectively. Otherwise, update $t_{r0} = t_{r0} + \tilde{t}_s - \Phi_{T_s}^{-1}(\alpha)$, where $\Phi_{T_s}^{-1}(\alpha)$ is the inverse CDF of T_s at confidence level α.

Step 5. Find reliable shortest path p_0^{rs} and arrival time distribution T_s using forward-based searching algorithm from origin at t_{r0}. Go to Step 4.

4. NUMERICAL EXAMPLES

In this section, numerical examples are presented based on a real-world ATIS. The proposed algorithm is coded by C# language for numerical experiments. All experiments are conducted in the computer with a duo-core 1.6GHz CPU and run on Windows XP operation system.

In Hong Kong, real-time traffic information in the whole network of Hong Kong is provided by a Real-time Travel Information System (RTIS) (Tam and Lam, 2008). In the RTIS, area-wide link travel times are generated at 5-min intervals. A GIS-based website portal has been developed for delivering the RTIS results to road users (http://rtis.td.gov.hk/rtis). Based on the RTIS, a solution algorithm for short-term travel time prediction in urban roads of Hong Kong has also been proposed (Tam and Lam, 2009). As shown in Figure 1, a road network of the whole territory of Hong Kong with 1367 nodes and 3655 links is adopted in the RTIS. In this study, three month's RTIS travel time estimates are collected from April to June in 2008. The estimated travel times on thirteen Wednesdays are used to generate the mean travel time and travel time variance for the whole network.

The toll rates of tunnels / bridges in 2008 are also collected from the Transport Department of Hong Kong Government in this study. According to the information from the Transport Department of Hong Kong Government, there are totally 11 toll tunnels/bridges in Hong Kong in 2008, in which three of them are cross-harbor tunnels. These three cross-harbor tunnels are significantly different in toll rates, mean travel times and travel time variances. Cross Harbor Tunnel (CHT) is the cheapest but with the largest travel time variation. Western Harbor Crossing (WHC) is the most reliable one and the expected travel time is also the least. However, the toll charge is the most expensive. The expected travel time for Eastern Harbor Crossing (EHC) is the largest but its travel time variation is similar to that of WHC.

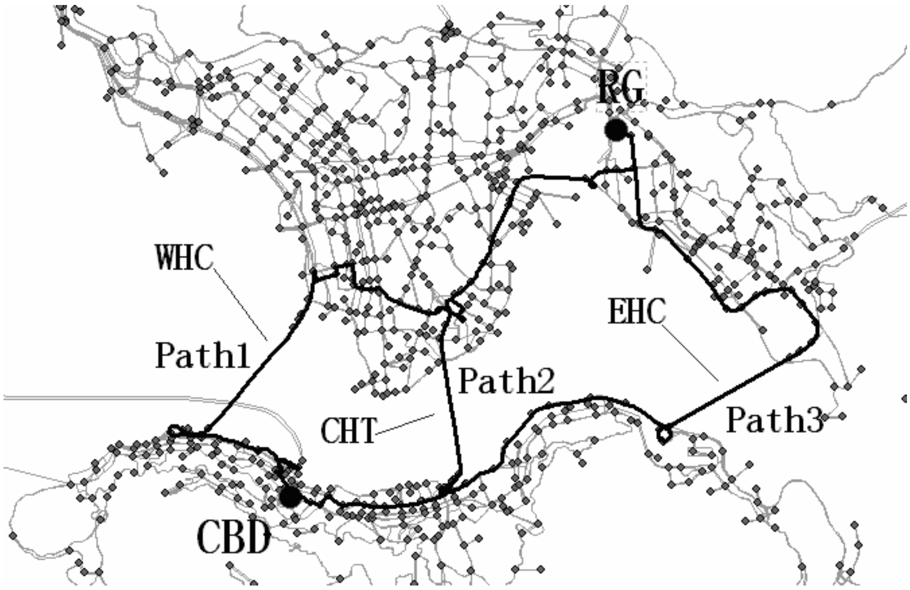

Figure 1 Reliable shortest paths from RG (origin) to CBD (destination)

In the experiments, we set a residential zone Richland Gardens (RG) as origin and Central Business District (CBD) in Central as destination to examine the effect of different confidence level, value of time and value of distance on travelers' route choice. The locations of RG and CBD are illustrated in Figure 1.

Table 1 provides the numerical results under various confidence levels, value of times and value of distance. The reliable shortest paths from RG to CBD are also illustrated in Figure 1. As shown in the figure, three paths are found under various confidence levels and value of times. The Path2 passing through CHT accommodates the requirement of travel cost sensitive travelers, as it is the cheapest path. In contrast, travelers with a high value of time would prefer to Path1 and Path3 which are faster and more reliable.

Table 1 Numerical results from RG (origin) to CBD (destination)

Confidence level	*VOT* (cents / min)	*VOD* (cents / km)	Path	Departure Time	$U(p, t_r)$ (min)	Travel cost (HKD)	Expected travel time (min)	Std. dev. of travel time (min)
0.5	100	10	Path2	**7:30:46**	50.42	21.14	29.28	5.44
0.6	100	10	Path2	**7:29:20**	51.78	21.14	29.28	5.44
0.7	100	10	Path2	**7:27:54**	53.25	21.14	29.28	5.44
0.8	100	10	Path2	**7:26:14**	54.98	21.14	29.28	5.44
0.9	100	10	Path2	**7:23:49**	57.38	21.14	29.28	5.44
0.99	**100**	10	Path2	**7:18:02**	63.09	**21.14**	29.28	5.44
0.99	**300**	10	Path3	7:29:23	39.44	**26.64**	27.62	1.26
0.99	**500**	10	Path3	7:29:23	35.89	**26.64**	27.62	1.26
0.99	**1000**	**10**	Path1	7:31:25	33.20	**46.60**	25.72	1.21
0.99	1000	**20**	Path1	7:31:25	33.36	**48.20**	25.72	1.21
0.99	1000	**30**	Path1	7:31:25	33.52	**49.80**	25.72	1.21
0.99	1000	**50**	Path1	7:31:25	33.84	**53.00**	25.72	1.21

As shown in Table 1, the utility cost is increases with an increase of confidence level. In this case, travelers may become more concern in travel time variability and they would assign a higher travel time budget and depart from origin earlier to ensure on-time arrival. By keeping the confidence level at 0.99, we can observe from Table 1 that with the increase of value of times and value of distance, travelers are more willing to pay a premium to choose the path with less mean travel time and travel time variance to ensure on-time arrival. We can conclude that by varying the values of time, value of distance and levels of confidence, the proposed model is robust to take account traveler' preference towards travel time, travel cost and travel time variability.

The computational time of the proposed algorithm are tested with respect to various network sizes. Three networks with the size of about one-fourth, half and full of the road network in Hong Kong are used. In each road network, reliable shortest paths are computed from all zone centroids to the CBD. In the experiments, CDF of arrival time and link travel time are divided into 20 intervals. During the experiments, we found that the backward-based searching in Step 1 of solution algorithm can give a very good initial departure time and reliable shortest path, and almost the forward-based searching in Step 5 can be avoided. The average computational times for the experiment are shown in Table 2. It shows that computational time required by proposed algorithm increase with the size of the road network. However, the proposed algorithm can find the best departure time and reliable shortest path in the road network of Hong Kong within a satisfied computational time (within 0.5 second). Thus, the proposed algorithm could be applicable for the online ATIS-based routing applications in large-scale road network.

Table 2 Computational time of proposed algorithm

Number of nodes	Number of links	Average computational time (seconds)
300	792	0.067
799	2191	0.107
1367	3655	0.329

5. CONCLUSIONS AND FURTHER RESEARCH

In this paper, a new model was proposed for modeling the departure time and route choice problem (DTARCP). An efficient solution algorithm has been developed to solve the DTARCP based on the proposed model. Numerical results showed that the proposed model was able to determine simultaneously the optimal departure time and route choices. It was robust to take account travelers' various preference towards the travel cost, travel time and travel time variability of their journeys. The computational results indicated that the proposed solution algorithm was applicable for solving the DTARCP efficiently in large-scale road networks. It was suggested that the proposed model and solution algorithm can be used for development of routing applications in large-scale road networks with real-world advanced traveler information system (ATIS). Further study is required to validate the model results with empirical surveys under various congested conditions.

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REFERENCES

Chang, T. S., Nozick, L. K., and Turnquist, M. A. (2005) Multiobjective path finding in stochastic dynamic networks, with application to routing hazardous materials shipments, **Transportation Science, Vol. 39, No. 3**, 383-399.

Chen, B. Y., Lam, W. H. K., Tam, M. L. and Sumalee, A. (2009a). Finding reliable shortest paths in the stochastic time-dependent road networks for online ATIS applications. **Proceeding of the third International Forum on Shipping, Ports and Airports.** Hong Kong, China, pp. 28-36.

Chen, B. Y., Lam, W. H. K. and Shao, H. (2009b). Reliable shortest path problems in stochastic and time-dependent network, Working Paper.

Chorus, C. G., Arentze, T. A., Molin, E. J. E., Timmermans H. J. P. and Wee B. V. (2006) The value of travel information: Decision strategy-specific conceptualizations and numerical examples, **Transportation Research Part B, Vol.** 40, **No. 6,** 504-519.

Fu, L. and Rilett, L. R. (1998) Expected shortest paths in dynamic and stochastic traffic networks, **Transportation Research Part B, Vol. 32, No. 7**, 499-516.

Hall, R. W. (1986) The fastest path through a network with random time-dependent travel-times, **Transportation Science, Vol. 20, No. 3**, 182-188.

Lam, T. C. and Small, K. A. (2001) The value of time and reliability: measurement from a value pricing experiment, **Transportation Research Part E**, **Vol. 37, No. 2-3**, 231-251.

Lee, W. H., Tseng, S. S. and Tsai, S. H. (2009). A knowledge based real-time travel time prediction system for urban network. **Expert Systems with Applications**, **Vol. 36, No. 3**, 4239-4247.

Miller-Hooks, E. D. and Mahmassani, H. S. (2000) Least expected time paths in stochastic, time-varying transportation networks, **Transportation Science, Vol. 34, No. 2**, 198-215.

Miller-Hooks, E. D and Mahmassani, H. S. (2003). Path comparisons for a priori and time-adaptive decisions in stochastic, time-varying networks. **European Journal of Operational Research, Vol. 146, No. 1**, 67-82.

Palma, A. D. and Rochat, D. (1999) Understanding individual travel decisions: results from a commuters survey in Geneva, **Transportation**, **Vol. 26, No. 3**, 263-281.

Siu B. and Lo, H. (2009) Equilibrium trip scheduling in congested traffic under uncertainty. **Proceedings of 18th International Symposium on Transportation and Traffic Theory**, Hong Kong, China, pp. 147-167.

Soroush, H. M. (2008). Optimal paths in bi-attribute networks with fractional cost functions. **European Journal of Operational Research, Vol. 190, No. 3**, 633-658.

Tam, M. L. and Lam, W. H. K. (2008) Using automatic vehicle identification data for travel time estimation in Hong Kong, **Transportmetrica, Vol. 4, No. 3**, 179-194.

Tam, M. L., Lam, W. H. K. and Lo, H. P. (2008) Modeling air passenger travel behavior on airport ground access mode choices, **Transportmetrica, Vol. 4, No. 2**, 135-153.

Tam, M. L. and Lam, W. H. K. (2009) Short-term Travel Time Prediction for Congested Urban Road Networks. **Proceeding of 88th Transportation Research Broad Annual Meeting (CD-ROM)**, Washington, D.C.

Toledo, T. and Beinhaker, R. (2006) Evaluation of the potential benefits of advanced traveler information systems, **Journal of Intelligent Transportation Systems, Vol. 10, No. 4**, 173-183.