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Optimization of Vehicle Routing and Scheduling Problem with Time Window Constraints in Hazardous Material Transportation

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Abstract: Unlike widely available literatures in hazardous material (HAZMAT) transportation that basically aim at finding non dominated paths for a given origin-destination pair, our main focus in this study is on vehicle routing problem with time window (VRPTW) aspect of HAZMAT transportation problem that has received very less attention in literatures. We present a new multi-objective optimization model and its meta-heuristic solution technique using Ant Colony System for HAZMAT routing. In contrast to existing local routing models, we consider minimization of risk and transportation cost in both route choice and routing phases of transportation process. Moreover, route choice and routing have been carried out as a single step process. Lastly, the proposed algorithm has been tested for normal VRPTW by testing on Solomon benchmark instances and the results obtained show that the proposed algorithm outperforms while maintaining realistic computation time.

Key Words: *Multi-objective optimization, Hazardous Material, VRPTW, Meta-heuristic*

1. INTRODUCTION

A wide class of hazardous materials is produced, transported and used to meet the daily requirements of industrial activities of a country. Due to nature of these materials, every production, storage and transportation activity related to their use inherits many risks for both society and environment. Despite the continuous effort to mitigate the adverse effects of HAZMAT, accidents do happen "Erkut *et al.* (1995)" and though these accidents are very small in numbers the consequences in most cases are undesirable. This is the reason that these accidents are commonly perceived as low probability high consequences (LPHC) events. Data on HAZMAT shipments in most cases are not readily available and are often difficult to access but the sizeable shipments of these materials and their potential adverse conditions are the reason that HAZMAT transportation is a growing issue in logistical decision making.

While risk is the primary ingredient that separates HAZMAT transportation problems from other transportation problems "Erkut *et al.* (2007)", HAZMAT transportation is a multi-objective issue that involves a number of parties with often conflicting priorities and viewpoints during decision making process. The transporting company for example has priority to minimize the transportation cost while the local government intends to minimize the risk associated with the transportation process. One of the possible methods to resolve this issue is application of a proper routing technique that enables decision maker to come up with safe and economical HAZMAT transport routes, and it is this aspect of HAZMAT transportation that is focused in this study.

A large number of single objective and multi-objective models for finding paths for HAZMAT shipment for a given origin and destination pair are available in literatures. However, in practical situation, HAZMAT shipments specially those based on truck mode like ordinary Vehicle Routing Problems with Time Windows (VRPTW) calls for determination of a set of routes to be used by a fleet of vehicles serving a fixed number of customers. In this study, we present a new multi-objective optimization model for routing and scheduling of a fleet of vehicles carrying HAZMAT from a single common depot and satisfying the demand and time window constraints of a fixed number of customers. Owing to the complexity raised in this study due to our attempt to consider route choice and routing in single step, the dynamic nature of the problem will not be considered in this study. A new heuristic approach using Ant Colony System (ACS) has been developed in order to solve this model. Selection of a path from depot to customer or customer to customer or customer to depot, also known as route choice and finding order of customers to be visited by vehicles for optimal routing called routing are the two terminologies of routing and scheduling of any VRPTW. Unlike in previous studies where route choice is carried out beforehand determining a single path between each customer pair which is then used for routing process to determine order of customers to be visited for optimal routing, both route choice and routing process in this study have been carried out in multi-objective aspects and proceeded as a single step process.

The rest of this paper is organized into five sections. Section 2 provides a thorough review of past researches in HAZMAT routing. A brief description of HAZMAT transportation problem considered in this study, its formulation and the method of assessment of risk involved has been presented in Section 3. Section 4 presents a new heuristic solution technique developed to solve this HAZMAT routing problem. The proposed algorithm has been tested for normal VRPTW in Solomon's benchmark problem and the numerical results obtained are presented in Section 5. Finally, Section 6 provides concluding remarks of the present study and possible future research plans.

2. LITERATURE REVIEW

HAZMAT transportation has been a very active area of interest for a large numbers of researchers since last few decades. Erkut *et al.* (2007) have presented an extensive bibliography of the researches on HAZMAT logistics classifying all of them into four different classes - risk assessment, routing, combined facility location and routing and network design. A similar attempt by List *et al.* (1991) resulted in a documentation of research studies since 1980. While a large number risk related researches are available in literatures, our study being focused on multi-objective routing and scheduling of HAZMAT transportation, we would limit our search to the literatures on HAZMAT routing and scheduling only.

A number of both single objective and multi-objective routing models have been presented in the past for HAZMAT routing. All these researches in general are related with two different variants of HAZMAT routing problems (Meng *et al.* 2005). A large number of researches relating to first category of HAZMAT routing are available in literatures that employ a single or multi-objective shortest path algorithms for finding non dominated paths minimizing risks or other attributes in transportation process for a given origin-destination pair. In contrast researches related to second category aiming for optimal routing and scheduling of a fleet of vehicles to distribute HAZMAT from a depot point to a fixed number of customers satisfying their demand and time window requirements are very limited in literatures. Our problem in this study is basically related to second category and here we present a list of available literatures for this category of HAZMAT vehicle routing and scheduling problem.

Cox and Turnquist (1986) were the first to consider scheduling problems of HAZMAT vehicles. They developed a dynamic algorithm considering the effects of presence of locally imposed curfews for identifying an efficient frontier of a bi-objective routing and scheduling problem of HAZMAT truck shipments in graph. However, the problem they considered is more related to first category of Hazmat routing problems. So far known, Tarantilis and Kiranoudis (2001) and Zografos and Androutsopoulos (2004) are the only two studies that explicitly considered the VRPTW prospective of HAZMAT transportation problem. Both of these studies used a bi-objective VRPTW model and applied heuristic techniques to find solutions for routing vehicles carrying HAZMAT. However, the later study ultimately used a single objective model obtained by transforming the proposed bi-objective model using weighing approach and solved it using insertion based heuristic for routing and used Dijkstra shortest path algorithm for route choice between customers. While the former maintained bi-objective model for routing using a List Based Threshold Accepting (LBTA) meta-heuristic algorithm, they used a risk based single objective approach for route selection using the same Dijkstra algorithm. The model and the heuristic algorithm developed in the later study were extended in Zografos and Androutsopoulos (2008) for developing a GIS based decision support system for integrated hazardous materials routing and emergency response decisions. So far known none of the studies till now are able to represent the multi-objective nature of these problems in both route choice and routing processes. The consideration of multi-objective nature for both route choice and routing processes calls for use of non-dominated paths in both steps of HAZMAT transportation. This emphasizes the requirement of proceeding both these processes in single step to obtain the truly non dominated pareto-optimal paths for routing vehicles carrying HAZMAT.

3. HAZMAT TRANSPORTATION PROBLEM

The core concept of HAZMAT transportation problem is similar to that of a Capacitated Vehicle Routing Problem with Time Window (C-VRPTW). C-VRPTW is a variant of VRPTW in which a fleet of delivery vehicles with uniform capacity must service fixed customers demand within pre-defined time windows for a single commodity from a single common depot. VRPTW are topics of a great deal of ongoing research in the operations research community. The details on the topic including its variants, formulations and solution techniques can be referred from Desrosiers *et al.* (1995) and Taniguchi *et al.* (2001). To facilitate prospective readers, attempts have been made during formulation of HAZMAT vehicle routing problem with time window presented in this study to use standard notations used by the later. However, at places where it becomes inconvenient due to introduction of

new terms or possibility of creating confusion with repetition of the notation, efforts have been made to provide more detail information.

3.1 Problem Definition

Route choice and routing are the two major processes of VRPTW. While solving VRPTW in general, the two processes are completed in two separate steps. A single best path for moving vehicle from a customer to another is first determined in route choice process using some shortest path algorithms for customer-customer or customer-depot node pairs. These pre-defined paths are then used in routing process in which the order of customers to be visited for optimal case is determined. In reality, route choice considering multiple numbers of objectives results into several non dominated paths for each customer node pair including the depot node. Using a pre-defined single route for proceeding routing process would cancel the possibility of a number of non dominated paths to participate in routing process and this at times may hinder decision maker to reach the actual optimal solution. Keeping this in mind, we attempt to process route choice and routing as a single step process thus providing a chance to all non dominated paths of route choice to be part of optimal routing process.

Accordingly, HAZMAT vehicle routing problem here can be defined as a problem of determining a set of pareto-optimal routes for a fleet of vehicles carrying HAZMAT in order to serve a given set of customers satisfying following conditions:

a. Both route choice and routing must be based on multi-objective requirements described below in Section 3.2 and should be carried out as single step process.
b. All vehicles must start and end their routes at depot node.
c. Demand of each customer must be serviced within pre-defined time windows.
d. Waiting at points of early arrival is possible while late arrival is not allowed at all. The final set of routes are however expected to minimize waiting since minimizing total scheduling time that includes waiting at customer nodes is one of the requirement in routing process.

3.2 Objectives

As previously mentioned, a key reason behind ongoing popularity of HAZMAT transportation study is the risk term associated with the transportation process. While minimizing risk is the primary objective in all HAZMAT transportation problems, HAZMAT shipments are subjected to another important objective of minimizing transportation cost from shipper's point of view. The objectives of HAZMAT routing problem considered in this study are to minimize both cost and risk associated with transportation process with equal consideration without singling out any preference.

Like normal VRPTW, minimizing transportation cost calls for minimizing fixed cost and operating cost in transportation. Taking this into account, two objectives of minimizing total number of vehicles in use and minimizing total scheduling time including travel time, waiting and service time have been introduced; the first objective being related with former component of transportation cost and the second one being related with the later component.

Risk assessment has been an active area of research study since long and a number of qualitative and quantitative risk modeling are available in literatures. The present study adopts the widely used risk model referred as traditional risk model by Erkut and Ingolfsson (2004) for risk calculation. According to the model, risk associated with a path R_{path} can be presented

by equation (1).

$$R_{path} = \sum_{link \in path} [(HAZMAT\ accident\ \ probability)_{link}\ .\ (consequence\ of\ \ the\ accident)_{link}] \qquad (1)$$

Though a number of consequences in relation to a HAZMAT accident are possible, safety for human life counts for top priority. Therefore in our study, the risk $R^{p(i)}_{n(i)n(i+1)}$ associated with transportation of HAZMAT from customer node $n(i)$ to $n(i+1)$ using path $p(i)$ which consists of numbers of links joining nodes $v(j)$ to $v(j+1)$ is modeled as presented in equation (2).

$$R^{p(i)}_{n(i),n(i+1)} = \sum_{v(j)v(j+1) \in p(i)} AR_{v(j)v(j+1)}.EP_{v(j)v(j+1)} \qquad (2)$$

Here $AR_{v(j)v(j+1)}$ is the probability of HAZMAT accident for link connecting node $v(j)$ to $v(j+1)$ and $EP_{v(j)v(j+1)}$ is the exposure population for the same link that is the number of people lying within λ distance from the link segment. The distance λ is dependent upon the HAZMAT class being transported and has been defined with the assumption that all persons within this distance from the accident spot are subjected to the same consequence of life loss while the consequences outside this distance have been ignored. Detail on this threshold distance λ is available in Batta and Chiu (1988).

3.3 Problem Formulation

A multi-objective VRPTW model for HAZMAT transportation has been formulated based on details provided on Section 3.1 and Section 3.2. The objective function Z which is a multi-objective three dimensional vector for minimizing total number of vehicle in use (Z_1), the total scheduling time (Z_2) and the total risk exposure associated with the transportation process (Z_3), here is dependent upon two decision variables *X and Y*. $X = \{x_l \mid l = 1, m\}$ is the traditional decision variable of order of visiting customer nodes for all vehicles and $Y = \{y_l \mid l = 1, m\}$ is a new decision variable of order of paths to be visited by all vehicles, introduced for proceeding route choice step. The detail formulation is presented here from Equation (3) to (6):

$$Min \quad Z(X,Y) = \begin{bmatrix} Z_1(X,Y) & Z_2(X,Y) & Z_3(X,Y) \end{bmatrix}^T \qquad (3)$$

$$Here,\ Z_1(X,Y) = \sum_{l=1}^{m} \delta(x_l, y_l) \qquad (4)$$

$$where \quad \delta(x_l, y_l) = \begin{cases} 1 & if\ vehicle\ l\ is\ used \\ 0 & otherwise \end{cases}$$

$$Z_2(X,Y) = \sum_{l=1}^{m} Z_{tl}(x_l, y_l)$$

$$= \sum_{l=1}^{m} \sum_{i=0}^{\hat{N}_l} (\overline{T}^{p(i)}_{n(i),n(i+1)} + t_{c,n(i+1)} + t_{w,n(i+1)}) \qquad (5)$$

$$where \quad t_{w,n(i)} = \begin{cases} (e_{n(i)} - t_{l,n(i)}) & if\ t_{l,n(i)} < e_{n(i)} \\ 0 & otherwise \end{cases}$$

$$Z_3(X,Y) = \sum_{l=1}^{m} Z_{rl}(x_l, y_l) = \sum_{l=1}^{m}\sum_{i=0}^{\hat{N}_l} R^{p(i)}_{n(i),n(i+1)} \quad (6)$$

The problem has been defined in a network of nodes and arcs (V, A), where $V = \{v_1, v_2, v_3, ..., v_k\}$ is a finite set of vertices and $A = \{a_1, a_2, a_3, ..., a_k\}$, a finite set of arcs that includes all possible connections between vertices in V. The set of customer nodes to be visited which is subset of V can generally be represented by set $N = \{n_1, n_2, n_3, ..., n_{\hat{N}}\}$. Specifically for this study, x_l which is the order of customers to be visited by vehicle l is represented as $x_l = \{n(i) \mid i = 0, \hat{N}_l\}$. $n(i)$ here is the customer to be visited by vehicle l, m being the maximum number of vehicles in use. $\hat{N}_l$ is the total number of customers to be visited by vehicle l, $\hat{N}_l + 1$ being zero. Since each vehicle l in use has to start from depot node, it is considered as a temporary customer node $n(0)$ for all vehicles. $[e_{n(i)}, f_{n(i)}]$ is the time window representing earliest and latest possible service time at node $n(i)$. The decision variable $y_l = \{p(i) \mid i = 0, \hat{N}_l\}$ is the order of paths to be used by vehicle l while visiting its customer nodes where path $p(i) \in P$, P being set of all non-dominated paths between customer-customer and customer-depot node pairs. $p(0)$ is the path to be followed from depot node to customer node $n(1)$ and $p(N_l)$ is the path to be used while visiting from customer node $n(N_l)$ back to depot node.

Equation (5) shows detail calculation of total scheduling time where Z_{tl} is total scheduling time associated with vehicle l for hard time window condition and is dependent on the average travel time value from $n(i)$ to $n(i+1)$ that is $\overline{T}^{p(i)}_{n(i)n(i+1)}$ which ultimately relies both on the order of customers and the path used while moving from one customer node to another. Terms $t_{c,n(i)}$, $t_{l,n(i)}$ are the service time and service start time respectively of vehicle l at node $n(i)$. In calculation of risk objective presented in Equation (6), Z_{rl} is the risk value associated with vehicle l which is dependent on $R^{p(i)}_{n(i),n(i+1)}$ that is the risk value associated with each path followed while moving from customer $n(i)$ to $n(i+1)$ of that vehicle, the detail calculation of it being presented in Section 3.2.

The whole model is subjected to time window, demand-capacity and customer number constraints as in traditional VRPTW. Early time window constraints have been considered during total scheduling time calculation in Equation (5). Equation (7) is the late time window constraint for the model. Mathematical expressions for demand-capacity and customer number constraints are given below in Equation (8) and (9) respectively. $D_{n(i)}$, $W_l(x_l)$ and $W_{c,l}$ here are demand at node $n(i)$, total weight carried by the vehicle l and capacity of vehicle l in use respectively. It should be noted that all these constraints hold true only during routing process and selection of nodes within path $p(i)$ is not subjected to these constraints.

$$t_{l,n(i)} \leq f_{n(i)} \quad (7)$$

$$\sum_{n(i)\in x_l} D(n(i)) \quad = \quad W_l(x_l) \quad \leq \quad W_{c,l} \tag{8}$$

$$\sum_{l=1}^{m} \hat{N}_l \quad = \quad \hat{N} \tag{9}$$

4. ANT COLONY SYSTEM FOR HAZMAT ROUTING

VRPTW are related to a class of NP-hard (Non-deterministic Polynomial) combinatorial optimization problems and heuristic or meta-heuristic algorithms must be used to solve problems of larger instances and for timely computation of the solution. A large number of different meta-heuristic approaches have been proposed in recent years for solving different variants of VRPTW. A comprehensive survey on the available approaches can be referred in Toth and Vigo (2002). In this study, we present a new Ant Colony System (ACS) based meta-heuristic solution technique in order to solve the HAZMAT transportation problem presented in Section 3.

Ant Colony Optimization is a meta-heuristic approach inspired by the foraging behavior of real ant colonies. Complete explanation on this meta-heuristic is available in Dorigo and Stutzle (2004). Ant System (AS), a variant of Ant Colony Optimization was first used by Bullnheimer *et al.* (1998) for solving vehicle routing problems. Number of researches focused on improvement and use of the approach for solving a range of problems. Gambardella *et al.* (1999) presented a multi-objective ant colony system MACS-VRPTW algorithm for solving multi-objective vehicle routing problems with time window constraints. The concept was based on Ant Colony System (ACS) and two ant colonies were used, each ant colony being specialized for a particular objective. The algorithm presented however provided precedence for the first objective over the second one. The proposed ACS for this study is adopted version of the MOACS-VRPTW approach presented by Baran *et al.* (2003). The reason behind this particular selection is that the mentioned study has been found to be able to deal with multiple numbers of objectives using a single ant colony system and each objective has been given equal consideration. Moreover, the approach provides a good convergence to all the pareto front surface which is one of the desirability of this study for coming out with number of solutions to decision making process, the alternative choices of which can lead to equitable distribution of the risk value.

Figure 1 shows the flowchart of proposed ACS for HAZMAT VRPTW. The proposed ACS mainly differs from the MOACS-VRPTW in terms of solution construction process and the local search method. Details on these processes have been provided in Section 4.1 and Section 4.2 respectively. As shown in the flowchart, the first step of initialization of trail pheromone value and setting of pareto-optimal set S is based on a routing solution obtained using nearest neighborhood (nn) heuristic. The solution obtained itself represents the first member of pareto-optimal set S. Equation (10) is the expression for evaluating the initial trail pheromone value τ_0 which is reciprocal of the total scheduling time, total risk value and the average number of nodes ($|N|$) that includes total number of customers and the average number of vehicles for nearest neighborhood solution. The initial pheromone values for subsequent generations are calculated based on average objective values for pareto-optimal set of the previous iteration similar as in MOACS-VRPTW.

$$\tau_0 = \frac{1}{(|N|)_{nn} * (Z_2)_{nn} * (Z_3)_{nn}} \tag{10}$$

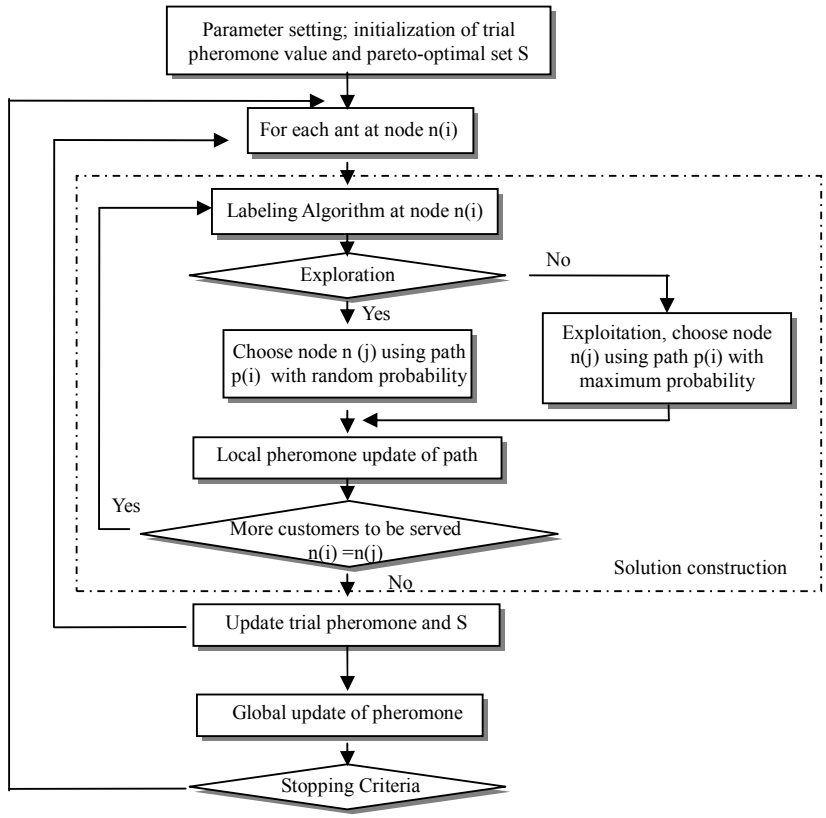

Figure 1 Flowchart of ant colony system for HAZMAT VRPTW

4.1 Solution Construction

Unlike in MOACS-VRPTW, each of m numbers of ants in the proposed ACS employ Labeling Algorithm before proceeding to customer node insertion to find out all non dominated paths from the present node of ant's stay to all feasible customer nodes. Customer node $n(j)$ among feasible set of customer nodes N' for ant at $n(i)$ is then inserted in routing order corresponding to path $p(i)$. This path $p(i)$ is decided based on pseudo-random proportional rule and is added to form an order of routing paths. Choice of path $p(i)$ and thereby customer node $n(j)$ is made based on expression in Equation (11) if $q \le q_0$. The choice otherwise is made randomly based on a probability value presented in Equation (12). q is a random number such that $0 \le q \le 1$ and q_0 is a parameter that defines relative importance of exploration and exploitation.

$$\max_{p(i)\in P'} \left(\tau_{p(i)} . [\eta_{p(i)}]^{\theta^{\eta}\beta} . [\nu_{p(i)}]^{\theta^{\nu}\mu}\right) \tag{11}$$

$$\Pr(p(i)) = \frac{\tau_{p(i)} . [\eta_{p(i)}]^{\theta^{\eta}\beta} . [\nu_{p(i)}]^{\theta^{\nu}\mu}}{\sum_{p(i)\in P'} \tau_{p(i)} . [\eta_{p(i)}]^{\theta^{\eta}\beta} . [\nu_{p(i)}]^{\theta^{\nu}\mu}} \tag{12}$$

Here $\tau_{p(i)}, \eta_{p(i)}$, $\nu_{p(i)}$ are the pheromone value and heuristic values relating to the scheduling time and risk value of path $p(i)$, respectively. β and μ are the parameters that define relative influence of the time and risk objectives. θ^{η} and θ^{ν} are the ant specific weights/preferences for normalizing time and risk objectives, respectively. $P' \in P$ is a set of all non-dominated paths from customer node $n(i)$ to each customer nodes in set N'. $\Pr(p(i))$ is the probability of path $p(i)$ to be chosen. Calculation of time related heuristic value is similar to the one for delivery time in MOACS-VRPTW that depends on the waiting time and time window at customer node. The risk related heuristic value depends on the risk value associated with path $p(i)$ that connects customer node $n(i)$ to $n(j)$ and is evaluated as given in equation (13).

$$\nu_{p(i)} = \frac{1}{R^{p(i)}_{n(i),n(j)}} \tag{13}$$

Basic knowledge on labeling algorithm can be referred from Ahuja *et al.* (1993). The labeling algorithm used in this study is based on the template labeling algorithm proposed by Irnish and Villenuve (2003) for shortest path problem with resource constraint. The concept is to make use of information that the labels created at vertex carry to determine all non-dominated paths in the presence of multiple resource constraints. The label at a vertex in general carries information about a path leading to it by maintaining a linkage with other label at the predecessor vertex. The label in resource constrained shortest path problems are made capable of describing state of the resources at the given node as well. Since labeling algorithm here is basically used for proceeding route choice step, travel time values and the risk values associated with each link of movement along the path constitute the resource constraints of the problem.

4.2 Local Search

The proposed ACS for HAZMAT VRPTW implements insertion local search procedure to improve quality of the feasible solutions. However being a time consuming process, we apply local search to only those solutions belonging to pareto-optimal set *S* which is updated at each of the iterations unlike in traditional ACS where local search is carried out to each solution. Update of pareto-optimal set *S* for each iteration is carried out based on a dominance rule in which all the solutions that are dominated in terms of all objectives are discarded. For example, if ψ_1 and ψ_2 are two solutions belonging to pareto-optimal set *S* with objective values of $Z_1(1)$, $Z_2(1)$, $Z_3(1)$ and $Z_1(2)$, $Z_2(2)$, $Z_3(2)$ respectively, ψ_1 is said to be dominated by ψ_2 and discarded from the set *S* if Equation (14) to (16) are satisfied except when $Z_1(1) = Z_1(2)$ & $Z_2(1) = Z_2(2)$ & $Z_3(1) = Z_3(2)$.

$$Z_1(1) \leq Z_1(2) \tag{14}$$
$$Z_2(1) \leq Z_2(2) \tag{15}$$
$$Z_3(1) \leq Z_3(2) \tag{16}$$

Hoos and Stutzle (2004) provide a wide view on local search procedures including their developments, analysis and application. The insertion local search used in this study utilizes insertion neighborhoods of a typical solution. All the nodes of a previously obtained feasible

solution are given chances to be inserted to the same vehicle route or to the route of other vehicles without violating feasibility requirements and the newly obtained solutions are checked for improved objective values. Figure 2 shows two typical insertion neighbors, IN-1 and IN-2 obtained inserting a node of solution ψ to position in the same vehicle route and in another vehicle route respectively.

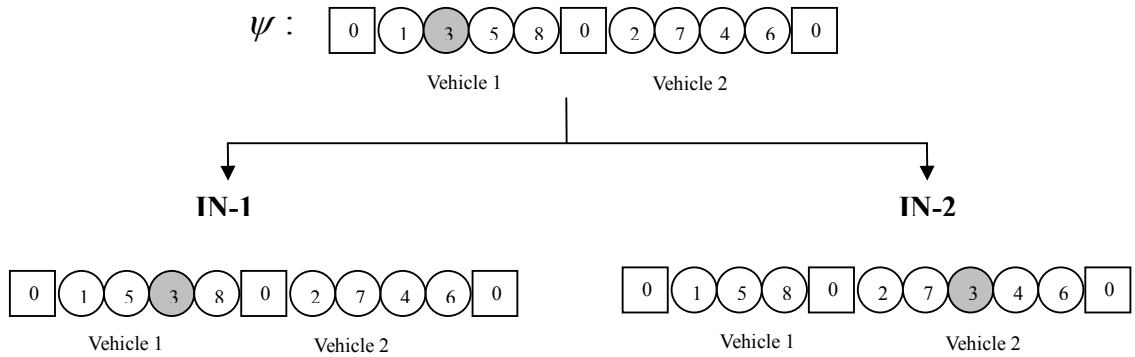

Figure 2 Insertion neighbors

4.3 Pheromone Update

The present ACS solution algorithm for HAZMAT VRPTW utilizes local and global update of pheromone procedures similar to the one used in MOACS-VRPTW method. Each path used by ant for constructing solution is subjected to local pheromone update as given in Equation (17) where ρ is the evaporation coefficient which powers exploration process by evaporating trail pheromone values for these used paths.

$$\tau_{p(i)}^{new} = (1-\rho)\tau_{p(i)}^{old} + \rho\tau_0 \tag{17}$$

In each iteration, pareto-optimal set S is updated after local search process and to each path belonging to pareto-optimal solution $\psi \in S$, global update of pheromone is carried out based on Equation (18) below.

$$\tau_{p(i)}^{new} = (1-\rho)\tau_{p(i)}^{old} + \rho/[(Z_2)_\psi .(Z_3)_\psi] \qquad where \;\; p(i) \in \psi \tag{18}$$

5. NUMERICAL ANALYSIS

Before proceeding to implementation of proposed model and the developed solution algorithm for a particular case of HAZMAT routing, it is essential to compare their performance for a standard case of routing based on earlier available results. However computational tests on HAZMAT transportation problem become unattainable due to lack of availability of standard datasets for the case of HAZMAT transportation. Therefore attempts have been made here to test their performance in standard VRPTW datasets making appropriate modifications for instance removing risk terms, replacing total scheduling time to distance objective etc. The details of the problems that have been considered for testing and the results obtained have been presented in Section 5.1 and Section 5.2 respectively.

5.1 Test Instances

Solomon (1987) provides a platform for testing efforts in VRPTW by presenting variety of

problems for normal vehicle routing problems with time windows. Computational experiments have been conducted for Solomon's benchmark instances for 100 customer problems of R, C and RC classes. All instances provided have a central depot, capacity constraint and time window constraints. While the customers in C type of problems are clustered, those in R type of problems are randomly distributed and those in RC type are of mixed type where customers are both clustered and randomly distributed. For this computational test, random selection of a problem from each problem type has been made.

5.2 Results and Discussions

In order to adjust proposed model for solving the problem instances, appropriate modifications have been made in the proposed algorithm by removing risk term and changing the second objective of reducing total scheduling time to reducing overall distance. Thus the problem here reduces to bi-objective minimization of number of vehicles and the total distance traveled by these vehicles. The proposed algorithm has been coded using Borland C and is executed in Core 2 Duo desktop PC of 2.67GHz with 2GB RAM. The parameters used are $m=10, \theta^{\eta}=1, \beta=1, \rho=0.1, q_0=0.9$, which are the same as those used in MOACS-VRPTW and MACS-VRPTW. Each problem has been run for 10000 number of iterations and the best solution obtained among 20 test trials has been presented.

Table 1 Comparison of results obtained for test instances using various approaches

	Number of Vehicles	Distance	Computation time (secs)
Problem C107-100			
Nearest Neighborhood+ Insertion	10	1386.5	
Proposed ACS	10	828.94	472
KDMSS (2-path cuts)	10	827.3	
Deviation from exact solution = 0.2%(Distance)			
Problem R110-100			
Neighborhood+ Insertion	14	1263.08	
Proposed ACS	12	1145.35	484
CR (A parallel cutting-plane algorithm)	12	1068	
Deviation from exact solution =7.24%(Distance)			
Problem RC102-100			
Neighborhood+ Insertion	15	1628.33	
Proposed ACS	14	1565.45	476
CR (A parallel cutting-plane algorithm)	14	1457.4	
Deviation from exact solution =7.41% (Distance)			

Table 1 present results obtained for the test instances C107, R110 and RC102 for 100 customer case. The results from proposed ACS presented here are the average objective values of the final pareto-optimal set *S*. It has been found that some test instances for example C101 for 100 customer case are so organized that its optimal solutions is obtainable just by using insertion heuristic along with the initial solution created using nearest neighborhood search. However, the results obtained using this approach for other problems is very poor even for other problems in C type. To check improvement caught with implementation of ACS technique along with nearest neighborhood initialization and insertion heuristic, the results obtained based on only nearest neighborhood and insertion heuristic approach has been also

included in the table. It has been observed that the results obtained with the use of proposed ACS technique with nearest neighborhood-based solution initialization and insertion-based local search are comparatively better than the one without ant approach in terms of both the reduction in number of vehicles and distance traveled. A comparison of the results with the best known solutions by the exact approaches (referred as KDMSS in C107 problem (Kohl *et al.* (1999)) and as CR in R110 and RC102 problems (Cook and Rich (1999)) in the table) has been also presented. It should be clear to readers that the exact approach referred here is a single objective approach with the main objective of minimizing total distance traveled and its results has been used just as a reference for comparison. It has been observed that the results obtained from the proposed algorithm are quite satisfactory, showing almost negligible deviation from the best solution using the exact approaches for the C type problems, while maintaining an acceptable limit of deviation of around 7.5% for R and RC type of problems. Moreover, the computation time for each case is within 500 seconds which is practical for the case of 100 customers.

For a clearer concept on performance of the approach along various generations, a plot on distance improvement along various generations for the case with number of vehicles being 10 has been created as shown in Figure 3 for C107 problem. Distance improvements with more number of vehicles have also been observed in some generations. However to maintain consistency of the graph, those results have been excluded for this particular plotting. As is clearly observable from the plot, the effect of the approach is found to be quite fast for the first 100 generations, however gradual improvements in the results has been observed even up to 3000th generation. These gradual improvements in later generation can in fact be due to typical features of ACS of exploration in addition to exploitation of the previously obtained best solutions.

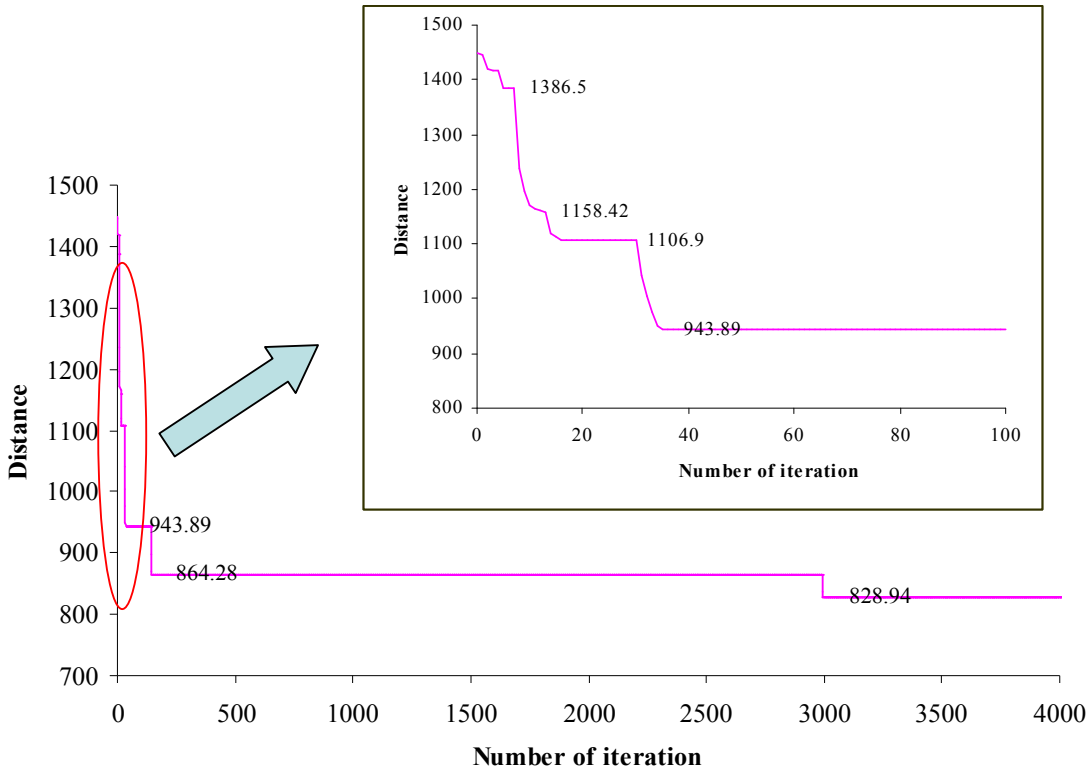

Figure 3 Distance trend for C107

6. CONCLUSION AND FUTURE WORK

Proper routing being an important aspect for safe and economic logistics of HAZMAT, the main focus in our study is to build an appropriate HAZMAT VRPTW model and develop a suitable algorithm to solve this model, which in fact in literatures of HAZMAT has received very less attention. A model illustrating the need of multi-objective consideration both for route choice and routing phase of HAZMAT transportation has been presented. A new ACS-based meta-heuristic algorithm has been proposed in order to solve this model for optimal case, and the numerical experiment performed shows its competency in all types of normal VRPTW problems, maintaining a good convergence to pareto-front which is one of the main requirements for HAZMAT application. Moreover, the performance for C type of problem has been found to be almost comparable with the best known solutions obtained by exact approach. Although normal VRPTW problems are very different from HAZMAT problems, both of these problems belong to combinatorial optimization problems. With regard to the excellent performance of the algorithm for VRPTW problems in general, similar results can also be expected for HAZMAT case. Besides, the pareto-optimal set consideration in the proposed approach provides numbers of alternative non dominated routing choices which will be more prominent in case of HAZMAT transportation with addition of risk objective. Alternative use of such paths in decision making process assists in maintaining risk equity. Based on its performance on test instances, the next aim of our study is to test its applicability for some test problems of HAZMAT routing. Nevertheless, testing different types of local search methods to make the algorithm more efficient, applying it to some real world problems and including its dynamic nature are other important aspects of the study.

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