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The Rotated Hexagonal Lattice Model for Pedestrian Flows

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Abstract: Various researches have been studied on the topic of pedestrian traffic flow. At the beginning of pedestrian researches, the modeling and simulation method for the vehicular traffic flow was simply applied to pedestrian traffic flow. Recently, CA based simulation models are frequently applied to pedestrian flow analysis. Initially, the square lattice model (SLM) is a base model for applying to pedestrians of counterflow and then hexagonal lattice model (HLM) improves its network as a hexagonal cell for more realistic movement of the avoidance of pedestrian conflicts. In this paper, we suggested the rotated hexagonal lattice model (RHLM) to take the advantages of SLM and HLM for more realistic pedestrian motion. Two simulation models such as SLM and RHLM are compared by the speed, density and flow. The comparison shows that RHLM can simulate pedestrian flow more realistically from various perspectives.

Key Words: *square lattice, hexagonal lattice, rotated hexagonal lattice, CA model, counterflow, pedestrian.*

1. INTRODUCTION

Recently, a great deal of attention has been directed to the topic of pedestrian traffic flow because walking mode is very attractive from energy and environmental perspectives. At the beginning of pedestrian research, pedestrian traffic flow is treated as a special case of vehicular traffic flow so that fundamental theories of vehicular flow are readily applied to pedestrian flow (Fruin, 1971a; Fruin, 1971b). However, the differences between pedestrian and vehicular flow are substantial. First, pedestrian walkways are not strictly regulated as roadways so that pedestrian flows are mostly not channeled (Blue and Adler, 1999). Second, unlike roadways separated by direction, pedestrian moves multi-directionally (Hoogendoorn, 2003)). Third, pedestrians can change their walking directions, speeds, and routes more freely and often than vehicles (Hao *et al*, 2007). Hence, more proper and advanced analysis methods are required to take those distinct characteristics of pedestrian flow into account.

Due to the complex characteristics of pedestrian flow, the simulation model is more frequently adopted than the mathematical model as an analysis tool (Helbing *et al.*, 1994; Kerridge *et al.*, 2000). Simulation models for pedestrian flow can be categorized into macroscopic and microscopic simulation model. Because of the microscopic nature of pedestrian movement, the microscopic simulation model is the most prevailing approach for pedestrian analysis. Furthermore, the microscopic simulation model can also be divided into three types such as cellular-based, physical force-base and queuing network model (Teknomo, 2002; Teknomo, 2006). Among the three types of simulation models, CA model is very popular because of its simplicity and low computational cost (Blue and Adler, 1999; Blue and Adler, 2000a; Blue and Adler, 2000b).

CA models generally use a square shape of cell as an analysis unit. (Blue and Adler, 1999; Blue and Adler, 2000a; Keβel *et al.*, 2001; Zhang *et al.*, 2003) From similar perspectives, the square lattice model (SLM) adopts a square lattice instead of a square cell (Muramatsu *et al.*, 1999). Both models allow pedestrians to move four distinct directions such as forward, backward, right-turn and left-turn. When a person meet other persons come from the opposite direction, pedestrians must change their directions by unrealistic angle 90 degrees because they can move to only 4 directions. SLM can be easily ameliorated if diagonal directions are allowed (Hao *et al*, 2007; Narimatsu *et al*., 2004; Lee *et al*., 2004). Despite of increment of number of movement directions, a pedestrian who decide to move to a diagonal direction must travels $\sqrt{2}$ times longer during unit simulation time. It represents that a pedestrian moves with different speeds by directions, which is not a realistic assumption.
Maniccam (2002) has proposed the hexagonal lattice model (HLM) which is an upgrade version of SLM. HLM uses a hexagonal lattice instead of a square lattice so that it has two more directions than SLM. Although HLM provides realistic diagonal movements, the pedestrian simulated by HLM cannot move straight.

In this paper, we propose the rotated hexagonal lattice model (RHLM) which can tackle problems embedded in SLM and HLM. In order to test efficacy of RHLM, a computer simulation models are developed and simulation results are compared with SLM.

2. LATTICE MODELS

2.1 Square Lattice Model (SLM)

Muramatsu *et al.* (1999) proposed the square lattice model based on the gas lattice model, which is applied to pedestrian flow. A gas particle in the gas lattice model is equivalent to a pedestrian in the square lattice model, who is treated as a biased random walker. The biased random walker can move to any preferential directions except backward direction. Figure 1 shows the possible moving directions in SLM and a black dot in the middle of arrows represents a place occupied by a pedestrian in the lattice. The pedestrians in SLM can move in accordance with the following rules. (i) Pedestrians cannot move to the backward direction. Pedestrians coming from left hand side and going to right-hand side in Figure 1 can move to the one of the following directions: forward (go to number 2 direction), left (go to number 1 direction) and right (to number 3 direction); (ii) Locations of pedestrians in lattice are updated once during the unit-time by the random sequential order. It means that pedestrians can move at most one grid during the unit-time; (iii) Pedestrians are inhibited from overlapping on same spot; (iv) Transition probability decides pedestrians' next position, which depend on the drift strength D of pedestrians where $0 \le D \le 1$. For instance, when a pedestrian going to number 2 direction in Figure 1, transition probabilities of directions to number 1, 2, 3 and 4 are

$P_1 = (1-D)/3$, $P_2 = D + (1-D)/3$, $P_3 = (1-D)/3$, $P_4 = 0$, respectively.

When a pedestrian conflicts with other pedestrians from the opposite direction, the pedestrian should divert her/his route to avoid collisions. The pedestrian will take either direction 1 or 3 to avoid crash shown in Figure 1. However, as shown in Figure 2, the SLM do not simulate pedestrian movement properly because the model does not reflect the characteristics of the pedestrian like pedestrians tend to take the shortest path.

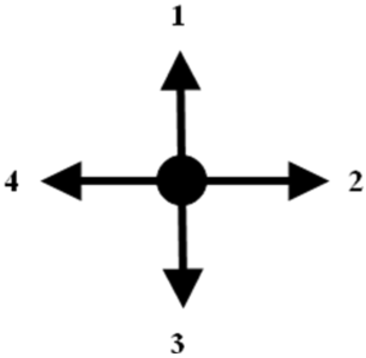


Figure 1 Possible moving direction on the square lattice.

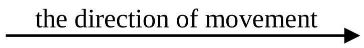


Figure 2 Pedestrian movements on the square lattice in conflict situation.

2.2 Hexagonal Lattice Model (HLM)

Manniccam (2002) proposed the hexagonal lattice model (HLM) as an improvement of SLM because it has more directional movements as shown in Figure 3. HLM simulates pedestrian flow by using the hexagonal lattice instead of the square lattice. The pedestrian movements of HLM also follow similar rules to SLM. Put in another way, pedestrians are treated as biased random walkers and transition probabilities governed by drift strength *d* are also used. However, simulated results are quite different in terms of routes pedestrians take because number of movements and their directions of HLM are different from those of SLM. Since HLM allows pedestrians diagonal movements (i.e., acute angle movement), which is able to represent more realistic pedestrian movement than SLM (i.e., 90 degree angle movement). However, even in the non-conflict conditions, the pedestrians of HLM move in a zigzag path toward the destination, which is a critical drawback because the model cannot express a straight-line movement as well as follow the shortest-path.

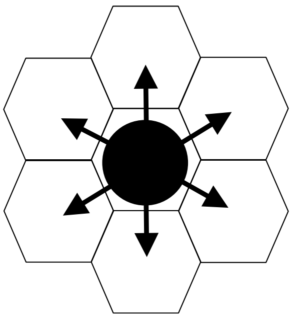


Figure 3 Possible moving directions on the hexagonal lattice.

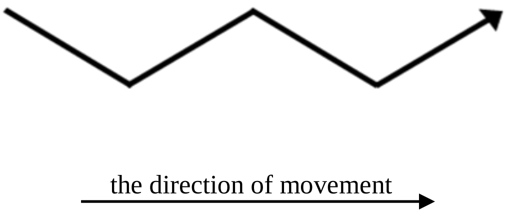
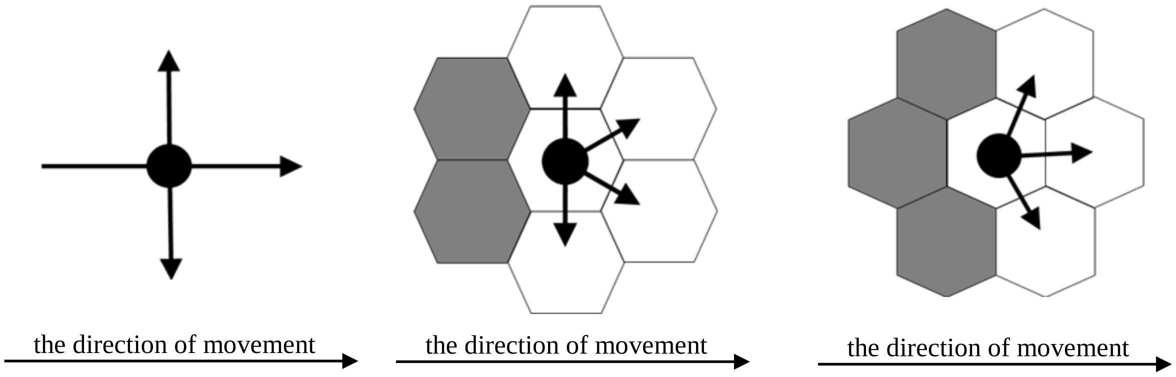


Figure 4 Pedestrian movements on the hexagonal lattice in non-conflict situation.

3. ROTATED HEXAGONAL LATTICE MODEL

3.1 The Concept of Rotated Hexagonal Lattice Model

A new lattice model, namely the rotated hexagonal lattice model (RHLM), is proposed in this study to improve the existing lattice models such as square and hexagonal lattice model. Although the new lattice model simply rotates a hexagonal lattice by 90 degree, the concomitant results from the model is substantial. Figure 5 compares the possible directions of pedestrian movements of three lattice models. SLM has four directions, three of which can be used for pedestrian movements in Figure 5(a). HLM has six directions of movements, four of which are possible movements in Figure 5(b).



(a) SLM (b) HLM (c) RHLM

Figure 5 Possible moving directions of pedestrian going to the right side
(a) SLM, (b) HLM, (c) RHLM.

Like HLM, RHLM has also six directions of movements, but only three directions are available for pedestrian movements in Figure 5(c). While RHLM has same number of directions of movements to SLM and smaller than HLM, RHLM has huge advantages because it can simulate the shortest-path movement and realistic avoidance motion.

When conflicts occur among pedestrians, the pedestrians with lower drift force change their direction to 60 degrees in Figure 6. In addition, when there is a non-conflict, it is possible to express the straight-line movement for their destinations. From these perspectives, RHLM is more reliable and realistic models than any other existing models.

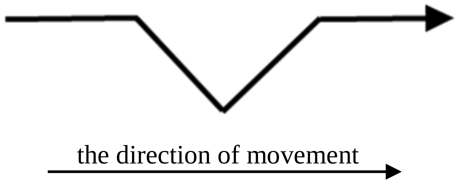


Figure 6 Pedestrian movements on the rotated hexagonal lattice in conflict situation.

3.2 Simulation Algorithm for RHLM

The pedestrians in RHLM are also treated as biased random walkers like SLM and HLM. Basic rules for pedestrian walking behavior in RHLM are same to those in SLM and HLM. In our simulations, pedestrians can ingress from and egress to right and left ends. Pedestrians are randomly generated from the Poisson distribution with the mean of λ. Another important variable is the ratio, definition of which is a proportion of the number of pedestrians from right to left and from left to right. When the ratio is 0, all the pedestrians only move into one direction: left or right. When the ratio is 0.5, equal number of pedestrians ingress from both sides. Hence, the ratio above 0.5 is the meaningless numbers. In addition, transition probability controls pedestrian's location in the next simulation period, which depends on the drift strength D (Muramatsu *et al.*, 1999; Maniccam, 2002). For instance, pedestrians can have at most eight choice sets of possible movement configuration depicted in Figure 7.

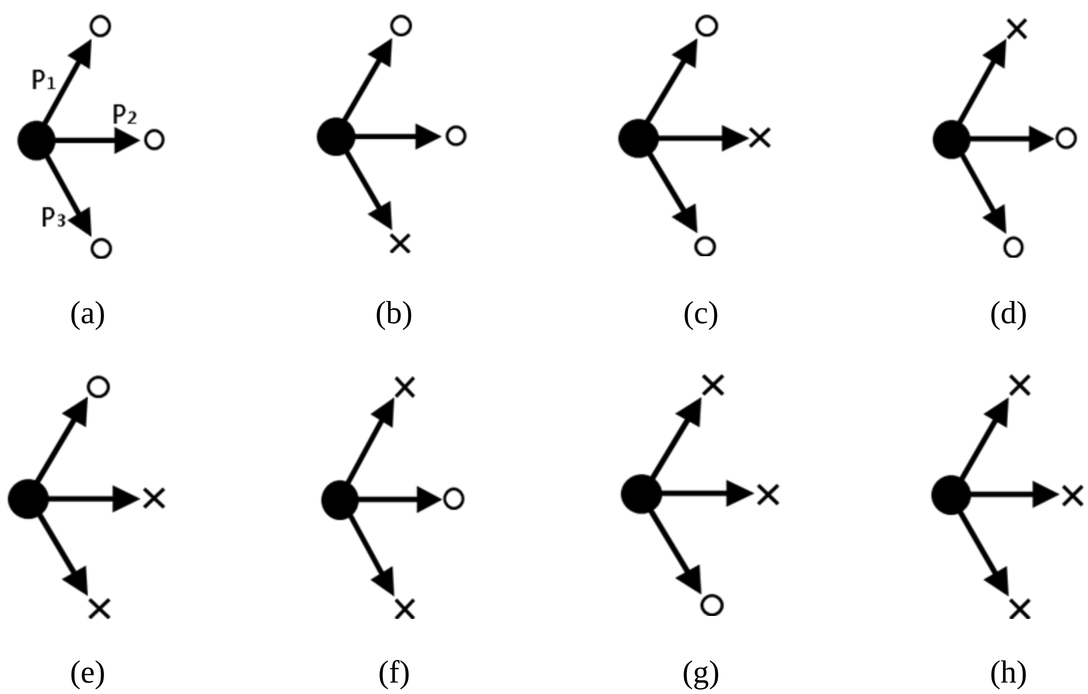


Figure 7 Eight possible configurations of the pedestrian going to the right side in RHLM.
[O]: possible directions to move, [X]: impossible directions to move

The transition probabilities P_1, P_2 and P_3 for each configurations are calculated as follows:

(a) $P_1 = (1-D)/3$, $P_2 = D + (1-D)/3$, $P_3 = (1-D)/3$

(b) $P_1 = (1-D)/2$, $P_2 = D + (1-D)/2$, $P_3 = 0$

(c) $P_1 = 1/2$, $P_2 = 0$, $P_3 = 1/2$

(d) $P_1 = 0$, $P_2 = D + (1-D)/2$, $P_3 = (1-D)/2$

(e) $P_1 = 1$, $P_2 = 0$, $P_3 = 0$

(f) $P_1 = 0$, $P_2 = 1$, $P_3 = 0$

(g) $P_1 = 0$, $P_2 = 0$, $P_3 = 1$

(h) $P_1 = 0$, $P_2 = 0$, $P_3 = 0$

4. SIMULATION RESULTS

A case consisting of 20×20 cells is simulated during 100 unit-times. Parameters for simulation are described in Table 1. The ratio value varies from 0.05 to 0.50 and the drift strength changes from 0.1 to 1.0. For instance, the drift strength ‘1.0’ means that although the pedestrians can go to three directions, they always choose the straight direction and ‘0.0’ means that they do not have any preferential directions (left, right or straight direction). The lambda (λ) which is the mean value of the Poisson distribution indicates the number of generated pedestrians during unit-time and its range is from 2 to 20.

Table 1 Input variables in Simulation

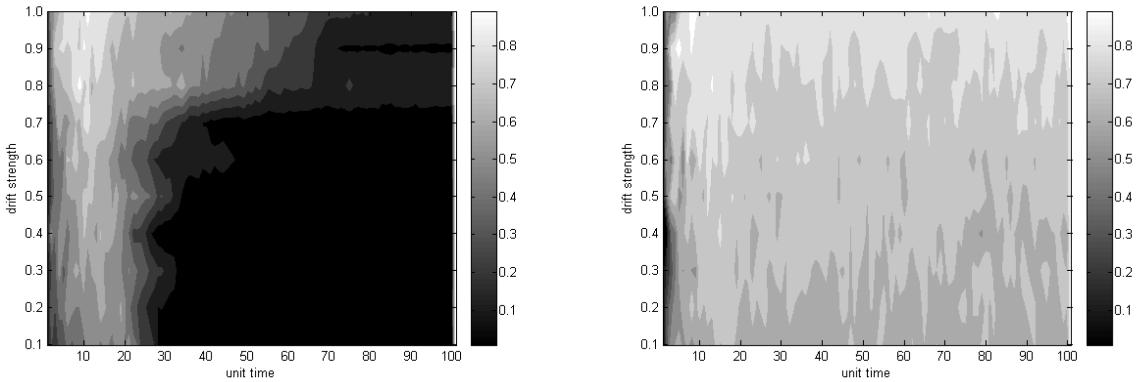
Variable	**Min**	**MAX**	**Increment**
Drift Strength	0.1	1.0	0.1
Lambda(pedestrian/unit-time)	2	20	2
Ratio	0.05	0.50	0.05
Network Width (Cell)	20		
Network Length (Cell)	20		
Simulation Time (unit-time)	100 (unit-time)		

Blue (2001) has applied the speed-volume-density relationships and the fundamental diagrams for the pedestrian network to examine the characteristics of place exchange using the CA model. Same approach as in Blue (2001) is adopted to test efficacy of RHLM compared to SLM.

4.1 Average Speed

The effect of the drift strength on average speed is examined and depicted in Figure 8 (a) and (b). Different the drift strengths from 0.1 to 1.0 with the increment of 0.1 are provided in the simulation while the lambda (λ) and the ratio are constant as 5 and 0.25, respectively. In Figure 8, the various colors represent the different average speed, from the darkest 0.0 cell/unit-time to the lightest 1.0cell/unit-time. While the average speed of the SLM (Figure 8 (a)) tends to decrease rapidly along time axis and for lower drift strength, the average speed of the RHLM (Figure 8 (b)) does not change along the time axis and slightly decreases at lower drift strength. When the drift strength increases, speed increases at the beginning stage. However, as time goes by, lower drift strength impedes the movements of pedestrians so that those with lower drift strength cannot advance to their destinations in the network. The phenomenon is easily observed in SLM rather than RHLM because two models have quite different avoidance motions illustrated in Figure 2 and Figure 6.

More realistic movements of pedestrians in RHLM produce more reasonable average speed with the various drift strength levels. We can also find the speed of 0 cell/unit-time in the SLM given condition under 0.7 drift strength. The unstable condition happens as the number of pedestrians entering to the network increase. As the unstable condition persists, the more pedestrians are in the network, the more avoidance motions occur and at last all pedestrians cannot move anymore. However, RHLM does not produce any the speed 0 cell/unit-time at any situations, but RHLM could create a speed of 0 cell/unit-time when excessive number of pedestrians enter to the system. However, the effect of the drift strength on the average speed is still smaller than SLM.

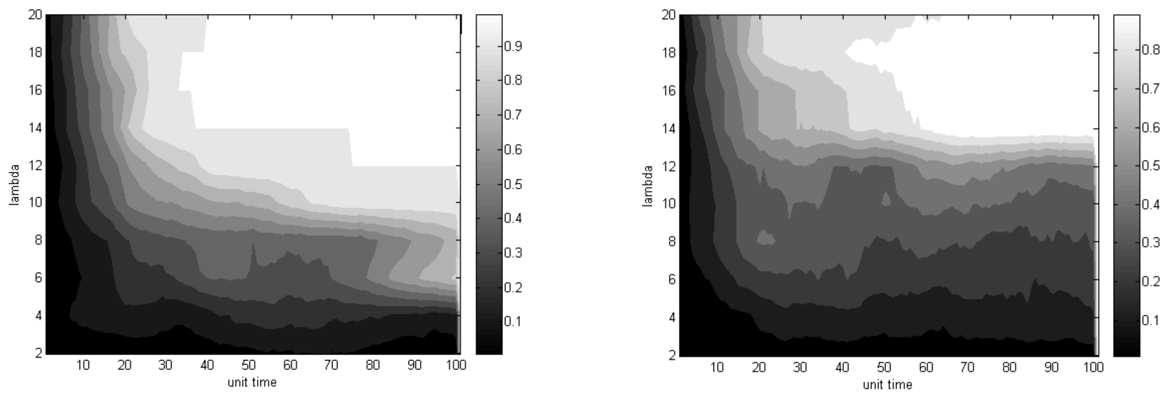


(a) SLM (b) RHLM

Figure 8 Speed on various drift strength. (Drift strength=0.1~1.0, Lambda=5, Ratio=0.25)

4.2 Density

Figure 9 (a) and (b) illustrate effects of Lambda on density. The simulation is conducted for various Lambda 2 (pedestrian/unit-time) ~ 20 (pedestrian/unit-time), the constant drift strength of 0.9 and the ratio of 0.25. As values of the lambda become higher, the density tends to increase rapidly in both cases. However, the density of SLM shown in Figure 9 (b) increases more quickly than RHLM in Figure 9 (a) and in the case of the same lambda the speed of increasing the density is also faster in SLM. The jam densities are observed over lambda 10 (pedestrian/unit-time) in SLM and over lambda 14 (pedestrian/unit-time) in RHLM, respectively. This is because the avoidance motion of RHLM is more realistic than the other. A density of 1.0 (pedestrian/cell) represents that 20×20 cells in the network reaches to the capacity of flow. Put in another way, all cells in the network are occupied by pedestrians so that queues are propagated in the network. During simulation periods, the pedestrians are continuously generated by the lambda (λ) even though density reaches to 1.0 (pedestrian/cell).



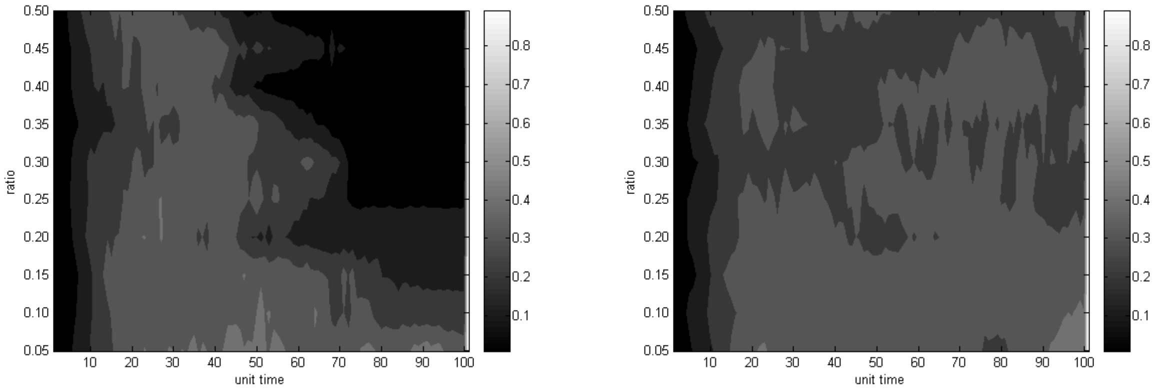
(a) SLM (b) RHLM

Figure 9 Density on various lambda. (Drift strength=0.9, Lambda=2~20, Ratio=0.25)

Hence, any simulation models should reach to jam density condition with higher lambda value. However, time duration reached to a density of 1.0 (pedestrian/cell) varies depending on pedestrians avoidance motion in simulation models. RHLM can produce more reasonable result due to the better avoidance motion.

4.3 Flow

The ratio from 0.05 to 0.50 is applied to investigate the effect of ratio on flow. As shown in Figure 10 (a) and (b), higher ratio tends to produce lower flow because the ratio expresses the balance of number of ingress pedestrians from opposite directions. The ratio of 0.5 represents the perfect balance situation like equivalent number of pedestrians enters to the system from both sides. At the ratio of 0.50, the lowest flow is observed and at the ratio of 0.05, the highest flow is obtained as we expected. Under the ratio of 0.10 representing almost one-way roads, the flow patterns simulated by two models are not much different during 30 unit-time simulation periods. However, beyond that ratio and that time, the flows of SLM decrease considerably because SLM is more susceptible than RHLM for pedestrian counterflow. It means that pedestrians divert their directions either left or right cell to avoid a collision in SLM, which required longer distance than in RHLM.



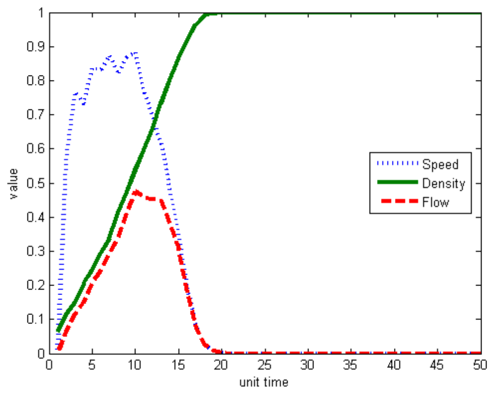
(a) SLM (b) RHLM

Figure 10 Flow on various ratio. (Drift strength=0.9, Lambda=5, Ratio=0.05~0.50)

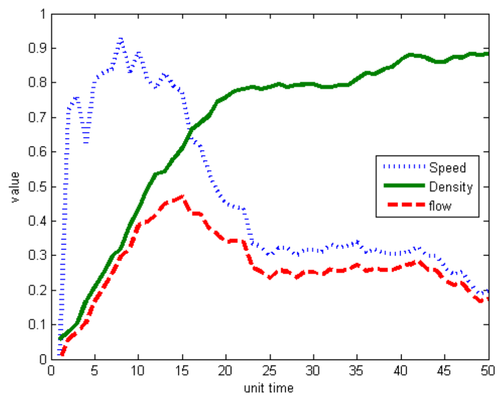
4.4 Flow- density diagram

In Figure 11 (a) and (b), flow, density and speed are plotted along time and all parameters are fixed as the drift strength of 1.0, the lambda (λ) of 5 and the ratio of 0.50. The most different feature between two Figures can be found in the density curve. While density becomes 1.0 (pedestrian/cell) before 20 unit-times in SLM and do not reach to that value in RHLM. Similar patterns are observed in average speed as well. While the maximum speed and flow appear at 10 unit-times in SLM (Figure 11 (a)), the maximum speed reveals before 10 unit-times and the flow does after 15 unit-times (Figure 11 (b)). These results mean that more pedestrians are in the network and the congestion take place later in RHLM. After reaching to the peak of the speed, the speed in SLM decreases drastically and becomes 0.0 (cell/unit-time) quickly. Speed in RHLM reduces gradually because the avoidance motion in RHLM is more efficient for counterflow.

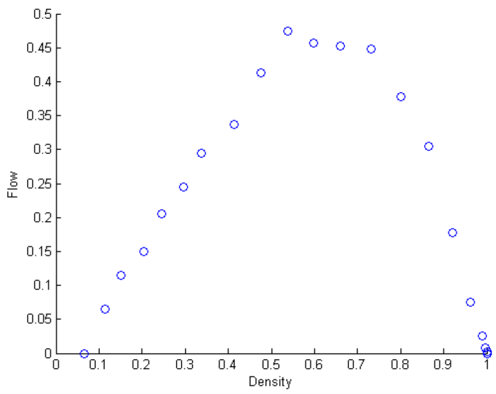
In addition, two flow-density diagrams are shown in Figure 11(c) and (d). The capacity of two models is similar under 0.48 (pedestrian/unit-time/width), at which density is somewhat different. When two models come up to the capacity, they have different pattern of decreasing flow. While SLM keeps flows on density 0.5~0.7 (pedestrian/cell), RHLM decreases them on density 0.6 (pedestrian/cell). And over the capacity the speed of SLM decreases faster than that of RHLM. It is because former model piles up the pedestrian in the network for its unrealistic avoidance motions but later model can make pedestrians to pass immediately. It also implies that the time to jam density of SLM is earlier than RHLM and the speed of RHLM is hard to zero (cell/unit-time).



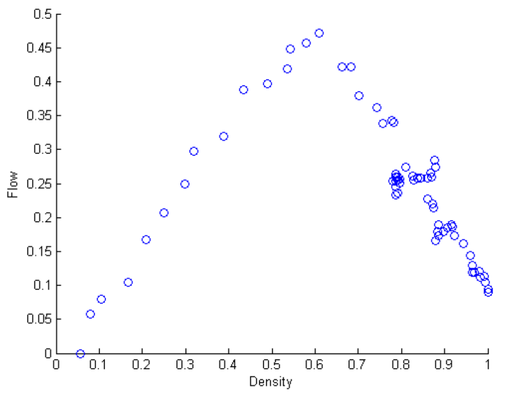
(a) Speed, Density, Flow of SLM



(b) Speed, Density, Flow of RHLM



(c) Flow Density Diagram of SLM



(d) Flow Density Diagram of RHLM

Figure 11 Simulation result of two models

5. CONCLUSION

In this paper, the rotated hexagonal lattice model (RHLM) was proposed, which is applied to pedestrian flow. RHLM is an upgrade version of the square lattice model and hexagonal lattice model. The model was tested using CA-base simulation model and the results are compared with the square lattice model. In order to tackle the pedestrian counterflow problem, the simulation conducts a scanning process to search their available cells. The simulation was performed at the hexagonal lattice 20x20 and evaluated by different speed, density and flow conditions. Simulation results shows that RHLM can replicate the characteristics of pedestrian traffic more effectively and reliably than any other existing models from several perspectives. First, RHLM can simulate the shortest-path movement. If they cannot move straight direction, they can move shorter distance from previous position to destination. Second, RHLM expresses the realistic avoidance motion. When conflicts occur among pedestrians, they can choose their direction to 60 degree. Further improvements to RHLM may include individual pedestrian's characteristics for movement and also the effect of obstacles which affect to pedestrian behaviors.

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