An Activity-Based Land Use and Transportation Optimization Model

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Abstract: This paper investigates the joint optimization problem of land use development and transportation network improvement. A novel bi-level model is proposed to solve this problem. The upper level is to determine residential/employment developments and road link capacity expansions with a budget so that the maximal population (residents/ employees) is accommodated while satisfying user utility (in terms of level of service) requirement. The accommodated population distribution by residential/employment locations and user utility are obtained by the lower level. The lower level is the combined network user equilibrium model. It models user location choice, activity pattern choice and path choice behaviors under the determined land use development and transportation network improvement in the upper level. A heuristic solution algorithm is designed for solving the proposed bi-level programming problem. A numerical example is presented to illustrate the applications and merits of the proposed model and solution algorithm.

Key Words**:** *activity-based approach, land use and transportation optimization*

1. INTRODUCTION

The problem of examining the interaction between land use and transport development has attracted the attention of many researchers in the past decades since it has been considered as a key issue for sustainable development in urban areas. A number of studies modeling relationships between land use and transport development can be found in the literature. The most operational spatial interaction model was firstly proposed by Lowry (1964). This Lowry model is considered as a remarkable accomplishment and has been adapted or extended by many other researchers. (Wilson, 1977; Meng et al., 2000; Wong et al, 1999; Wong et al., 2001). Alonso (1964) proposed a bid model in which land is assigned to the highest bidder in terms of a bid-auctioning process. Abraham and Hunt (1997) developed a discrete choice model which uses the random utility theory to describe the interactions between transport and land use so as to investigate the joint choice of residence and workplace. Based on the bid model and discrete choice model, the bid-choice model was developed and extended by Martí nez (2007). Some mathematical programming models integrating a spatial interaction model with a traffic assignment were also proposed (Boyce and Southworth, 1979; Boyce and Mattsson, 1999; Chang and Mackett, 2006).

In the above land use and transport interaction studies, attempts have been made to gain insights into the long-term effects of a change in the urban land use on the transport system and the impact of changes in accessibility in the transportation network on the location of population and employment. However, in most of these studies, the transport model is the trip-based model. The disadvantage and limitations of trip-based model have been discussed by many researchers (Lam and Yin, 2001). It has been recently believed that the activity-based approach provides a better understanding of travel behaviors. The departure time, destination and route choices are directly motivated by the predetermination of activities such as shopping or eating (Hoogendoorn and Bovy, 2004; Lam and Yin, 2001; Lam and Huang, 2003).

In recent years, there is growing number of literature on land use and transport models which studied travel behavior by the activity-based approach instead of the trip-based approach. However, to our knowledge, the activity-based approach has not been used to analyze the location choice behavior in the land use model together with their impacts on the transport model. In this paper, the activity-based approach will be used to model user travel behavior and location choice behavior simultaneously.

Generally, the network design problem has been studied in three manners (Yang and Bell, 1998): a discrete manner which deals with the additions of links or roadway segments to an existing transportation network (Gershwin and Tan, 1979; Friez et al., 1990; Meng et al., (2000); Suh and Kim, (1992), a continuous manner which deals with the optimal capacity expansion of existing links (Bruynooghe, 1972; Chen and Alfa, 1991), and a mixed manner which simultaneously deals with the link addition and capacity improvement problem for the existing transportation network.

The conventional network design problem was usually represented as a leader-follower game where the transportation planning departments are leaders, and the users are the followers who can freely choose the path (Gao et al., 2005). It is usually formulated as bi-level models where the upper level is the objective function of the leaders that may be the minimization of the social travel time and the lower level is the followers that road users' travel behavior model that may be the user equilibrium, stochastic user equilibrium, or stochastic user equilibrium with elastic demand etc. There appeared various bi-level network design models with different upper-level objectives and lower-level transport models (Yang and Bell, 1998; Szeto and Hong, 2005). However, little attention has been paid to the joint land use and network optimization problem, in which link capacity is enhanced or new link is constructed and simultaneously residential and employment accommodation abilities are developed.

As aforementioned, there exist strong interactions between land use and transport development. Through the interactions between land use and transport, it could be concluded that both the location choice and travel choice behaviors are constrained by the network accessibility and capacity. The land use policies affect the travel demand (in terms of origin-destination matrix) while the network design deals with link addition and link capacity enhancement problems with given travel demand matrix. On the basis of the existing transportation network, the government may relieve the traffic congestion by adopting not only appropriate transport policies or but also appreciated land use policies since land use policies affect the travel demand. At the meantime, the transport policies including network expansion projects affect user location choices while the land use development deals with residential/employment allocation according to users' location choice behavior. With the existing land use development, the transportation authority may improve the residential/employment environment through an optimized network design strategy since transport policy would in turn affect the location choices.

The design of transportation network should be consistent with the urban land use allocation strategy. An inappropriate land use development plan or an inefficient network improvement scheme would make the traffic conditions getting worse. Therefore, it is highly desirable to move forward by developing an integrated model for optimization of land use and transportation network plans simultaneously (Yim, 2005).

In this paper, the conventional network design problem will be extended to the joint optimization problem of land use development and network enhancement where the interactions between land use policies and transport policies are considered. Furthermore, in view of the merits of activity-based approach as mentioned above, the travel and location choice behaviors will be captured by an activity-based approach.

An activity-based bi-level model is proposed to solve the integrated optimization problem of simultaneous residential/employment allocation and link capacity expansions. For both a land use development project and a network improvement project, the system of land use and transportation network should be sustainable while the growth of population is expected to continue. Therefore, in the proposed bi-level model, the upper level is to maximize the number of population that could be accommodated. The objective is reached through residential and employment allocation and link capacity expansions with a limited budget but without decreasing users' utility. The lower level is the user behavior model which captures users' location choice, activity pattern choice and path choice behaviors. The users' utility contains the location choice cost, activity performance utility and travel cost. Users' behaviors in the lower level will be affected by and simultaneously influences decisions of residential /employment allocations and link capacity enhancements in the upper level. In the proposed integrated framework, the land use optimization problem and network improvement problem can be considered in a consistent manner to improve the efficiency of available resources of the community such as land and transport facilities, money and time etc.

This paper is organized as follows: In Section 2, the basic components of the proposed model are firstly presented. In Section 3, a bi-level programming model is formulated for the joint optimization problem of land use development and transportation network improvement. In Section 4, a solution algorithm is proposed for solving the bi-level programming problem. Section 5 presents a numerical example for illustrating the applications of the proposed model and solution algorithm. Finally, the conclusions are drawn in Section 6 together with recommendations for further studies.

2. BASIC CONSIDERATION

2.1 Assumptions

To facililate the presentation of the essential ideas, the following basic assumptions are made in this paper.

1. The proposed land use and transport network optimization model is mainly used for strategic planning purpose. Thus, the upper level of the proposed bi-level model is a static model. The lower level of the proposed bi-level model is a time-dependent model to capture the users' behaviors of location choice, activity pattern choice and travel choice in a typical day. It is assumed that both the original situation and optimal solution would satisfy the location and travel choice equilibrium conditions.
2. The residential location choice is a household choice problem, but for simplicity we

assume that households behave as a representative individual. Then, either the residential or employment location choice is an individual choice problem.

3. The population is confined to workers but different classes of workers with different perception for evaluating activity utility, money and travel time are considered. This different perception is represented by parameters α_1^m , α_2^m , α_3^m , α_4^m and α_5^m in the rest of this paper. Users can be classified by many ways such as their income (high income users and low income users) or their spending habits. The classification should be determined according to the study purpose.
4. The studied time horizon T (e.g. one day-24 hours) is divided equally into a number of intervals $K = \{k : k = 1,2,\dots K\}$.
5. Users choose their residential location and employment location to obtain the maximum utility by considering the trade off between their activity utility obtained from completing scheduled activity pattern and location choice cost. The activity pattern is scheduled to maximize their total utility received from the undertaken activities; such activities can be performed at home or elsewhere with time constraint. The shortest path is chosen by users to make their trips to the activity destinations.
6. It is assumed that once an activity is chosen, at least one interval should be occupied by performing this activity. It means that activity durations are calculated with unit of intervals.

2.2 Activity utility component & Activity pattern

An activity utility contains travel time disutility, activity cost disutility, activity performance utility, congestion disutility, and environment items (Yim and Lam 2001). Define $U_{rs,hw}^{j,m}(k)$ as the activity utility obtained by an individual, in class m with residential location h and employment location w , departing at interval k from origin r to destination s , to perform activity j at s for one interval.

$$U_{rs,hw}^{j,m}(k) = -T_{rs}^m(k) - \alpha_2^m P_s^{j,m}(k_s) + \left(\alpha_3^m \mu_{s,hw}^{j,m}(k_s)\right)\left[1 - b_s^j\left(\frac{q_s^j(k_s)}{C_s^j}\right)^n\right] + \alpha_4^m \mu_s + \alpha_5^m \mu_s^j \qquad \forall r \in R, s \in S, j \in J, h \in H, w \in W, k \in K \tag{1}$$

Define: An activity pattern y is the collection of all activity/destination choices undertaken during the studied time horizon and the order of their occurrence. We have $y = \left[\tau_{j,s}^1(k), \tau_{j,s}^2(k), \cdots, \tau_{j,s}^i(k), \cdots\right]$ where i is the order of the activity in the scheduled activity pattern, j, s, k is respectively the activity, activity performance destination, and time interval of starting to perform the correspondingly activity, $\tau_{j,s}^i(k)$ is the activity duration. The utility of an activity pattern, for users in class m with location choice (h , w), can be obtained by $U_{hw}^{m,y} = \sum_{i=1}^{n} U_{rs,hw}^{j,m,i}(k)$, where $U_{rs,hw}^{j,m,i}(k)$ is calculated by Equation (1) and n is the number of activities included in the activity pattern.

Let $\left[\tau_{j,s}^i(k), \tau_{j',s'}^{i+1}(k')\right]$ represent two adjacent activities in an activity pattern, then $k' = k + \tau_{j,s}^i(k) + T_{ss'}\left[k + \tau_{j,s}^i(k)\right]$, where $T_{ss'}\left[k + \tau_{j,s}^i(k)\right]$ is the time-dependent travel time departure from previous activity destination s to the next activity destination s' while departure at the time interval $\left[k+\tau^{i}_{j,s}(k)\right]$. The total travel time for completing an activity pattern is the sum of travel time of destinations for two adjacent activities, $T_y = \sum_{ss'\in y} T_{ss'}\left[k+\tau^{i}_{j,s}(k)\right]$. The sum of total travel time and activity durations equals to the studied time horizon period $T_y + \sum_{i} \tau^{i}_{j,s}(k) = T$.

2.3 Travel time function

Define $t^m_a(k)$ is the time-dependent link travel time.

$$t^m_a(k) = \alpha^m_1 t^0_a \left[1 + 0.15\left(x_a(k)/(C_a + L_a)\right)^4\right] \quad \forall a \in A, k \in K \tag{2}$$

where t^0_a the free-flow travel time of link a , $x_a(k)$ is the link flow at interval k , C_a is the link capacity and L_a is the proposed link capacity expansion.

3. MODEL FORMULATION

In this section, a bi-level programming model is proposed that integrates the residential and employment allocations and link capacity enhancements. The upper-level problem is to maximize the allocated population with the constraints of budget and system performance index requirement that is the user utility should be nondecreased after the population increase to the maximum one. The lower-level problem is the activity-based user equilibrium assignment model for individual location and travel choices.

In the lower level, the location choice depends on where to reside, and where to work people obtain the maximum total utility in a typical day. The total utility is affected by the location cost (housing price, or rent) and the total activity utility of performing activities with the selected residential and employment locations. The location cost is represented as the daily housing price. The total activity utility is determined by the activity pattern choice. The time-dependent activity/destination choices in a whole day constitute as the daily activity pattern. It is assumed that the daily activity pattern choice also follows the maximum utility principle. The path choice also abides by the shortest path principle. It is assumed that location/activity/destination/path choice follows a hierarchical choice structure which implies to firstly make location choices, secondly activity/destination choices and path choices at last.

According to the above statement, we have the following bi-level optimization model.

Upper-level problem: $\max imize\, Q(M, \mathbf{L})$ (3)

subject to

(Budget constraint)

$$\sum_h A_h(h)\cdot M_h + \sum_w A_w(w)\cdot M_w + \sum_a B_a(a)L_a \le B \tag{4}$$

(System performance index constraint)

$$V^{m*} \ge \overline{V}^m \tag{5}$$

(Minimum and maximum development/expansion constraints)

$$0 \le M^{\min}_h \le M_h \le M^{\max}_h \tag{6}$$

$$0 \le M^{\min}_w \le M_w \le M^{\max}_w \tag{7}$$

$$0 \le L_a^{\min} \le L_a \le L_a^{\max} \tag{8}$$

(Population conservation constraints)

$$Q = \overline{Q} + \Delta Q \tag{9}$$

$$q^h = \overline{q}^h + M_h \tag{10}$$

$$q^w = \overline{q}^w + M_w \tag{11}$$

$$\sum_{h \in H} q^h = Q \text{ and } \sum_{w \in W} q^w = Q \tag{12}$$

$$\sum_{h \in H} \overline{q}^h = \overline{Q} \text{ and } \sum_{w \in W} \overline{q}^w = \overline{Q} \tag{13}$$

$$\sum_h M_h = \Delta Q \text{ and } \sum_w M_w = \Delta Q \tag{14}$$

Lower-level problem: (The activity-based network equilibrium problem for residential/employment location choice, activity pattern choice, and travel path choice.)

$$\sum_m \sum_{hw} V^{m*}\left(q_{hw}^m - q_{hw}^{m*}\right) + \sum_m \sum_{hw} \sum_y U_{hw}^{m*}\left(F_{hw}^{m,y} - F_{hw}^{m,y*}\right) - \sum_m \sum_{rs} \sum_p T_{rs}^{m*}(k)\left(f_{rs}^{m,p}(k) - f_{rs}^{m,p*}(k)\right) \le 0 \tag{15}$$

subject to

(Path choice constraints)

$$x_a^m(k) = \sum_r \sum_{s \ne r} \sum_p f_{rs}^{m,p}(k)\delta_{rs}^{ap} \tag{16}$$

$$f_{rs}^{m,p}(k) = \sum_{hw} \sum_y f_{rs,hw}^{m,y,p}(k) \tag{17}$$

$$t_a^m(k) = \alpha_1^m t_a^0\left[1 + 0.15\left(x_a(k)/\left(C_a + L_a\right)\right)^4\right] \quad \forall a \in A, k \in K \tag{18}$$

$$T_{rs}^{m,p}(k) = \sum_a t_a^m(k)\delta_{rs}^{ap} \quad \forall r \in N, s(\ne r) \in N, k \in T, p \in P, a \in A \tag{19}$$

$$T_{rs}^m(k) = \min\left\{T_{rs}^{m,p}(k), \forall p, r, s\right\} \tag{20}$$

(Activity pattern choice constraints)

$$y = \left[\tau_{j,s}^1(k), \tau_{j,s}^2(k), \cdots, \tau_{j,s}^n(k)\right] \tag{21}$$

$$U_{hw}^{m,y} = \sum_1^n U_{rs,hw}^{j,m,i}(k) \tag{22}$$

$$U_{rs,hw}^{j,m,i}(k) = -T_{rs}^m(k) - \alpha_2^m P_s^{j,m}(k_s) + \left(\alpha_3^m \mu_{s,hw}^{j,m}(k_s)\right)\left[1 - b_s^j\left(\frac{q_s^j(k_s)}{C_s^j}\right)^n\right] + \alpha_4^m \mu_s + \alpha_5^m \mu_s^j \tag{23}$$

$$T_y + \sum_{i=1}^n \tau_{j,s}^i(k) = T \tag{24}$$

$$U_{hw}^m = \max\left\{U_{hw}^{m,y}, \forall h, w, m, y\right\} \tag{25}$$

(Location choice constraints)

$$V_{hw}^m(h,w) = -\alpha_2^m P^h\left(q^h\right) + U_{hw}^m(h,w) \quad h \in H, w \in W \tag{27}$$

$$V^m = \max\left\{V_{hw}^m, \forall h, w\right\} \tag{28}$$

(Population conservation constraints)

$$\sum_{w \in W} \sum_{h \in H} q^{hw} = Q \ \forall h \in H, w \in W \tag{29}$$

$$\sum_{w\in W} q^{hw} = q^h \text{ and } \sum_{h\in H} q^{hw} = q^w \ \forall h \in H, w \in W \tag{30}$$

$$\sum_{h\in H} q^h = Q \text{ and } \sum_{w\in W} q^w = Q \ \forall h \in H \tag{31}$$

$$\sum_{m} q_{hw}^m = q^{hw} \ \forall h \in H, w \in W \tag{32}$$

$$q_{hw}^m = \sum_{y\in Y} F_{hw}^{m,y} \qquad \forall h \in H, w \in W, y \in Y \tag{33}$$

$$F_{hw}^{m,y} = \sum_{rs}\sum_{y}\sum_{p} f_{rs,hw}^{m,y,p}(k) \tag{34}$$

4. SOLUTION ALGORITHM

In this section, a heuristic iterative solution algorithm is introduced for solving the proposed bi-level programming problem P (B-L). A Simulated Annealing Approach (SAA) (Friez et al., 2003) is adapted to solve the residential/employment allocation and link capacity expansion problem in the upper level. The general Method of Successive Average (MSA) is applied to solve the combined network user equilibrium model of location choice and travel choice problem in the lower level. The activity pattern choice problem in the lower level is solved by a generated activity-time-space (ATS) network method.

4.1 The Generated Activity-Time-SpaceNetwork (ATS)

In the lower level model, the critical problem is how to solve the activity pattern choice problem. The generated SATN method is introduced to solve that problem. We first need to enumerate all feasible activity/destination choice (j,s) based on the road transportation network. m,h,w is the parameter related to individual. For each group q_{hw}^m, the daily activity pattern can be described as a path following in the ATS network $\Lambda(h,w,m)$ that we construct in what follows:

- Nodes: Each feasible activity/destination choice $(j,s)(j \in J, s \in S)$ performing activity j at destination s is stated as a node $N^{j,s}(j \in J, s \in S)$ and it is expanded to k nodes $N_k^{j,s}, k = 1,2,\ldots,K$ $(j \in J, s \in S)$ of the generated network.
- Links: For each node $N_{k*}^{j*,s*}(j* \in J, s* \in S, k* \in K)$, we construct links:
  - links $(N_{k*}^{j*,s*}, N_k^{j,s})$ $(\forall j \in J, s \in S)$, if $1 \le k* \le K-1$ and $k \le K$, , k is determined by the traffic conditions on the base network, $k = k*+1+INT(C_{s*s}(k*+1)/\Delta + 0.5)$; the utility of this link b is $U_{hw}^{m,b}$. $U_{hw}^{m,b} = U_{s*s,hw}^{j,m}(k*+1)$ obtained by Equation (1).

These links correspond to the process of departure $s*$ at interval $k*+1$ to destination s , and then performing activity j at s at the arrival interval k for one interval. The link utility is the utility gained by experiencing that process.

In order to show how to make activity/destination choice at interval $k=1$, and enter the generated network, one dummy node O_{hw}^m is introduced as the starting node:

- Starting node: For each node $N_{k*}^{j,s}(j \in J, s \in S, k* = 1)$, we construct:
  - One link $(O_{hw}^m, N_{k*}^{j,s})$; the utility of this link b is $U_{hw}^{m,b}$. $U_{hw}^{m,b} = U_{Os,hw}^{j,m}(k*)$ obtained by Equation (1) . $C_{Os}(k*) = 0$.

This links correspond to choose to perform activity j at the destination s at interval $k^*=1$.

We also introduce a dummy node D_{hw}^{m} as the ending node:

- Ending node: For each node $N_{k^*}^{j,s}\left(j\in J, s\in S, k^*\in K\right)$, we construct:
  - One link $\left(N_{k^*}^{j,s}, D_{hw}^{m}\right)$; the utility of this link equals 0.

This links correspond to exit of workers performing activity j at the destination s at interval $k^*=K$.

The ATS network is generated based on the road transportation network and traffic conditions on it. Every path of ATS network represents a feasible activity pattern in the study time horizon. Thus, the activity pattern choice problem with complicated activity chains is transformed to the longest path problem in ATS network. The time constraint, space constraint, the time and space coordinates, activity duration and activity chain are considered.

4.2 Method of Successive Average for solving the lower-level problem:

The general Method of Successive Average (MSA) adapted to solve the combined network user equilibrium problem of location choice and travel choice in the lower level can be summarized as shown in the flowchart of the algorithm given in Figure 1.

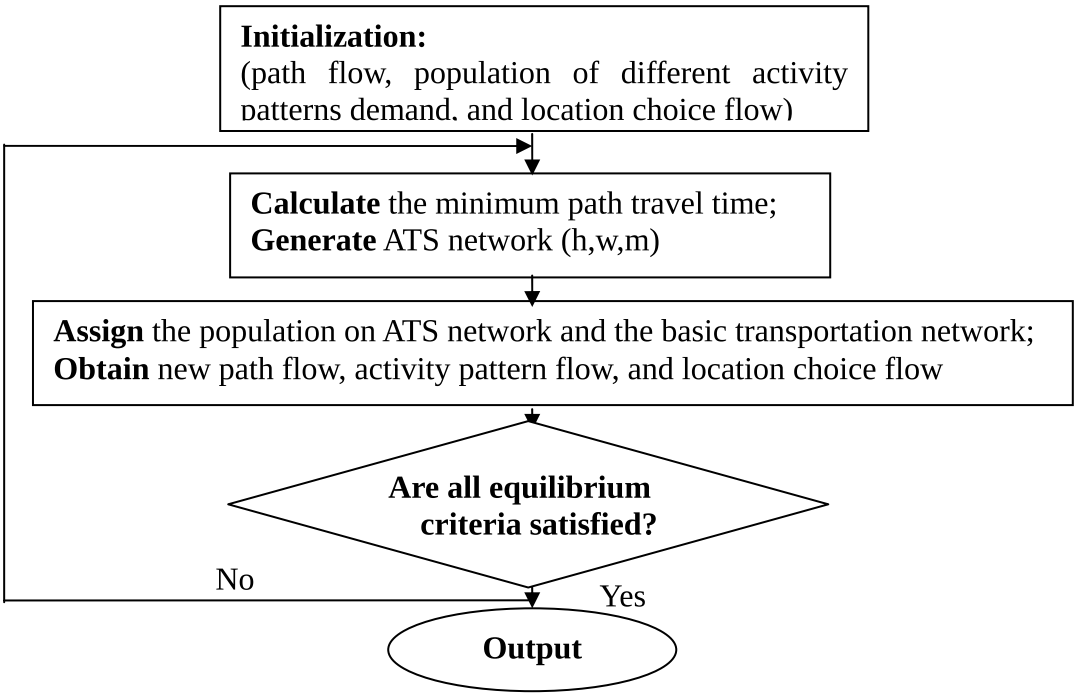

Figure 1 Flowchart of method of successive average for solving the lower-level problem.

4.3 The Simulated Annealing Approach for solving the upper-level problem

Initialization: It is assumed that original resident and employee distribution satisfy the combined location and travel choice network equilibrium conditions. Set the original population as the initial population $\overline{Q}$. Let $n=0$ and $Q^{n}=\overline{Q}$ .Solve the lower-level problem with the initial population to obtain the original user equilibrium utility V^{0*}, and then the required level of network service could be set as $\overline{V}=V^{0*}$. The original residential location choice $\overline{q}^{h}$ and employment location choice $\overline{q}^{w}$ also can be obtained with the results of the lower level assignment. Select an initial value T^0 for the "temperature". Set stop temperature T^s and thermal equilibrium iterations $Mtime$ to guarantee the iterations in each temperature are the same.

Step0: Let $n = n+1$ and determine an appropriate incremental amount ΔQ. Set $Q = \overline{Q} + \Delta Q$.

Step1: Given an initial feasible solution for $(\mathbf{M}^0, \mathbf{L}^0)$, $\sum_h M_h = \Delta Q$, $\sum_w M_w = \Delta Q$, $\sum_h A_h(h) \cdot M_h + \sum_w A_w(w) \cdot M_w + \sum_a B_a(a) L_a \le B$. Calculate objective function value V^0. $(\mathbf{M}, \mathbf{L}) = (\mathbf{M^0}, \mathbf{L^0})$, $T = T^0 * \lambda$.

Step 2: For each temperature stage do the following:

Step 2.1 Generate a feasible new solution $(M^*, \mathbf{L}^*)$ by a small random perturbation from the current solution. $(M^* = M + \beta \Delta M, \mathbf{L}^* = \mathbf{L} + \gamma \Delta \mathbf{L})$. The total development cost of $M^*, \mathbf{L}^*$ satisfies the budget constraint. Conduct the lower-level assignment and get the new equilibrium user utility. Evaluate the difference in equilibrium user utility $-\Delta V$ between the current solution and new solution.

Step 2.2 If $\forall m, -\Delta V^m \ge 0$, the new solution has a larger equilibrium user utility than the current solution. Hence, accept this solution and replace the current solution with the newly accepted solution. $(M, \mathbf{L}) = (M^*, \mathbf{L}^*)$

If $\exists m, -\Delta V^m < 0$, some of the new solutions have worse equilibrium user utility than the current solutions. Accept this solution with a probability given by $P(\Delta V^m) = \exp(\frac{-\Delta V^m}{k_B T})$ and random (0, 1). If for $\forall m, P(-\Delta V^m) > random(0,1)$, accept. Otherwise, reject. Update the current solution if necessary.

Step 2.3 If the "thermal equilibrium" is not reached, go to Step 2.1. Otherwise go to Step 3.

Step3: If the annealing process is incomplete $(T \ge T^s)$, reduce the temperature by $T = T * \lambda$, $0 < \lambda < 1$ and go to Step 1. Otherwise go to step 4.

Step4: If $V^{m*} > \overline{V}^m, \forall m$, go to Step 0. Otherwise stop.

5. NUMERICAL EXAMPLE

The purpose of this numerical example is to show the application of the proposed bi-level model and the proposed algorithm solution for solving the joint optimization problem.

5.1 Problem description and parameter setting

Figure 2 shows an example urban network of 5 nodes and 16 links. The free-flow travel times of the different directed links between two nodes are assumed as the same and they are given in the Figure 2. The capacity of each link is set to 1000 vehicles per hour. It is assumed that Node 1 and 5 are the residential areas where people could do eating activity and other at home activities, Node 2 and 3 are working zones where people could do work activity and eating activity, and Node 4 is a place where workers could do shopping and other activities. The area inside the dummy cycle is central business district (CBD).

The original total population in the urban network is assumed as $Q = 5000$ and it is classified into higher income workers as group 0 and lower income workers as group 1, and $\lambda^0 = \lambda^1 = 0.5$. The daily housing price function in Node 1 and Node 5 is respectively assumed as $P^1(q^1) = 20 \times \left[1 + 0.5 \times \left(q^1/(4000 + M_1)\right)^4\right]$ and $P^5(q^5) = 20 \times \left[1 + 0.8 \times \left(q^5/(3000 + M_5)\right)^4\right]$. Parameters in Equation (1) are given as follows: $\left(P_{rs}^m(k) + P_s^{j,m}(k_s)\right) = 0$ for $\forall m, r, s \neq 4, k, k_s$; $P_4^{j,m}(k_4) = 10\ \forall m, r, s = 4, k_s$; The item $\mu_s = 5$ for $s = 1, and\ s = 5$; $\alpha_1^0 = 2$, $\alpha_1^1 = 1$; $\alpha_2^0 = 0.3$, $\alpha_2^1 = 1$;. $\alpha_3^m = 0.1, \forall m$; $\alpha_4^0 = 4$, $\alpha_4^1 = 1$; $\alpha_5^m = 0, \forall m$. The time-dependent value of $\left(\alpha_3^m \mu_{s,hw}^{j,m}(k_s)\right)$ for $\forall m$ in Equation (1) is displayed in Figure 3. The parameter b_s^j and C_s^j are also given in Figure 2. For instance, in the expression "j=at home, s=Node1 (0.05, 4000)", the number (0.05, 4000) means that $b_s^j = 0.05$, and $C_s^j = 4000$. The activity type is at home activity and destination s is Node 1. To simplify the presentation, the eating activity and other at home activities at the residential nodes are combined into at home activity, and all activities performed at Node 4 are combined into entertainment activity.

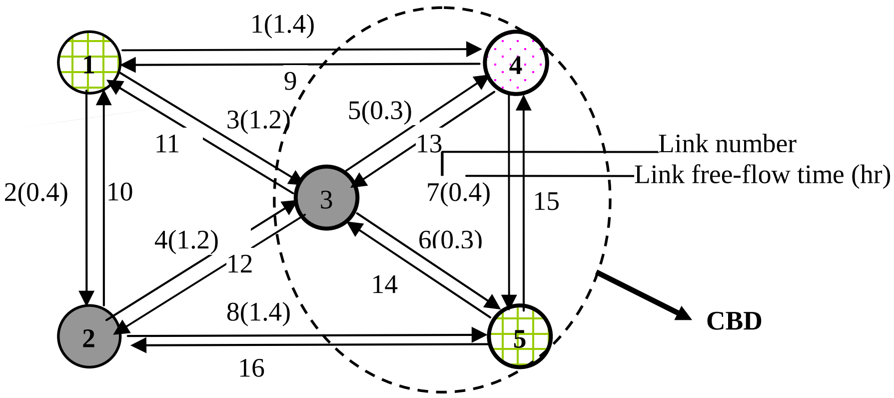

Figure 2 An example urban network.

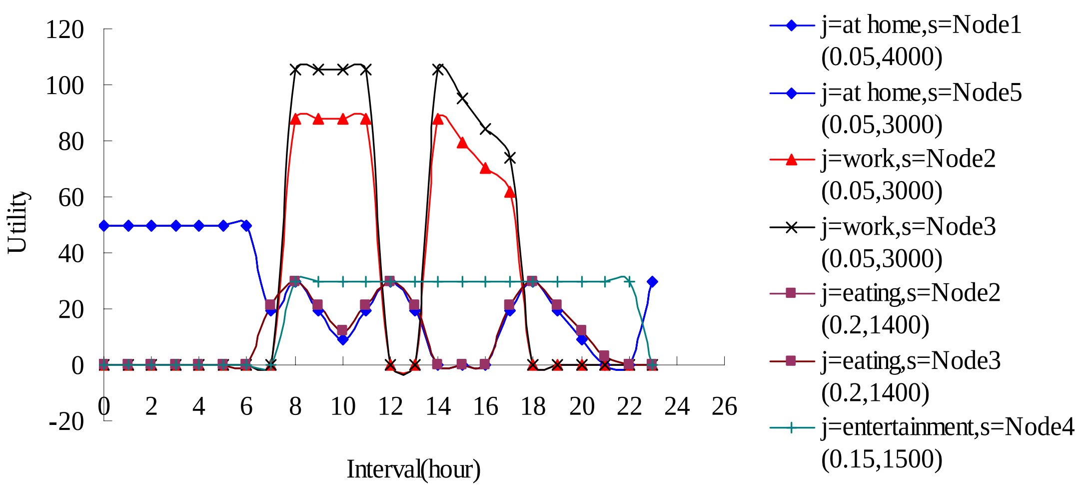

Figure 3 Simplified temporal utility profiles of various activities.

The selected locations for land use development are Node1, Node 2, Node3, Node5 (increasing the residential service capacity at Node1 and Node5, employment service capacity at Node2 and Node3). The selected links for link capacity expansion are Link2, Link3, Link4, Link5, Link6, Link7, Link10, Link11, Link12, Link13, Link14, and Link15. Parameters used in the development constraints and the SAA solution algorithm is given as Table 1.

Table 1 Related parameter setting

Parameters	Value	Parameters	Value
B	100000	T^0	500
B_a, $a = 2,3,4,10,11,12$	15	T^s	50
B_a, $a = 5,6,7,13,14,15$	30	λ	0.8
A_1, A_5, A_2, A_3	30, 50, 40, 60	$Mtime$	300
$(M_h^{\min}, M_h^{\max})$ $h = 1,5$	(0,2000)		
$(M_w^{\min}, M_w^{\max})$ $w = 2,3$	(0,2000)		
$(L_a^{\min}, L_a^{\max})$ $a = \forall selected\ link$	(0,800)		

5.2 Numerical results

The values of the numerical example solution are shown in Table 2. As it shown in Table 2, the main residential development is in Node 1, while the main employment development is in Node 2. This is caused by the lower development cost in these two areas. Differently, although the cost of link capacity of Links in the suburban (Link 3, Link 11, Link4, and Link 12) is lower than that of Links in the Centre Business District (CBD), the main link capacity expansion is conducted in the CBD (Link 5, Link 13, Link 6, Link 14). This is because the high congestion in CBD requires network enhancement urgently.

Table 2 The example solution for the joint optimization model

$\bar{V}^0$		132.625
$\bar{V}^1$		79.882
$\bar{Q}$		5000
ΔQ		1126
B		100000$
Land use expansion		**90920$**
Residential	Node 1	**907**
Allocation	Node 5	219
Employment	Node 2	**740**
Allocation	Node 3	386
Link capacity expansion		19080$
Link 2, Link10		**(82.80, 89.93)**
Link 3, Link11		(60.90, 34.13)
Link 4, Link12		(12.16, 0.00)
Link 5, Link13		**(93.05, 80.42)**
Link 6, Link14		**(99.81, 97.05)**
Link 7, Link15		(60.29, 65.42)

5.3 Sensitivity tests

The original situation is regarded as Case I. The new situation with the maximum population after the joint optimization project is regarded as Case II. The explicit effects on location and travel choice behaviors of the implementation of the above joint optimization project are illustrated in Table 3 and Table 4. From Table 4, the behaviors of moving house and job-hopping could be examined. With the ratios of different classes in a residential/

employment Node, we can also analyze the behaviors of moving house or job-hopping for different class users. Table 4 shows that the time allocation (implicitly, activity pattern) would not change greatly after population increase to the maximum one with the project.

Table 3 Population distribution by location choices

	Case I			Case II		
	Population	Percentage	Ratio Class 0: Class1	Population	Percentage	Ratio Class0: Cass1
Residents at Node 1	2669	53%	40:60	3576	60%	24:76
Residents at Node 5	2331	47%	61:39	2550	42%	87:13
Employees at Node 2	1534	31%	70:30	2274	38%	64:36
Employees at Node 3	3466	69%	41:59	3852	64%	42:58

Table 4 Effects of the joint optimization project on time allocation

	Average travel time (hr/person)	Average duration at home (hr/person)	Average duration at office (hr/person)	Average duration of eating outside (hr/person)	Average duration of entertainment activity (hr/person)
Case I	2.23	10.40	8.00	1.52	1.85
Case II	1.82	10.57	8.23	1.53	1.86
Difference II-I	-0.41	0.17	0.23	0.01	0.01
Percentage of difference	-18%	2%	3%	0%	1%

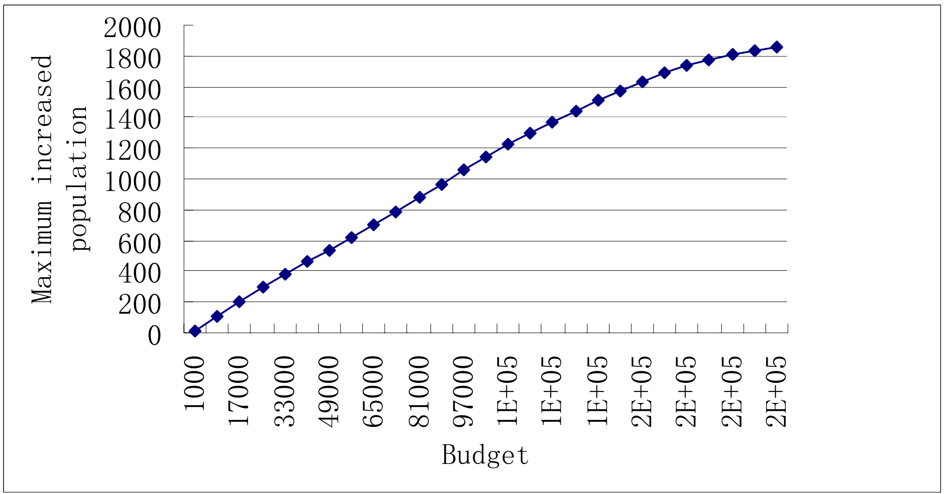

Figure 4 The maximum increased population versus different values of budget level

Figure 4 illustrates that the maximum additional population that are accommodated without decreasing users' utility by the joint optimization project, rises with the budget level increase. However, the slope of the curve in Figure 4 decreases with the increase of the budget level. It means that, the marginal cost for allocating increased population rises with the increase of population. If the marginal cost of allocation increased population rises to greater than the marginal cost by developing new residential/employment zones and adding new links, the government should turn to the late development program instead of only making residential/employment enhancement and link capacity expansions.

6. CONCLUSION

In this paper, we have proposed a bi-level model for solving the residential/employment development and link capacity expansion optimization problems. This joint optimization model could capture the interactions between land use and transportation planning. In this paper the activity-based approach was used to model the residential/employment location and travel choice behaviors. The activity pattern choice problem with complicated activity chains was transformed to the longest path problem in a activity-time-space (ATS) network which was generated on the basis of the study network. A heuristic solution method was adapted and applied to a numerical example for illustration of the merits of the proposed model.

It should be noted that the generated ATS network is self-consistent and can be handled as a usual network. Thus the traditional link-based algorithms for solving static trip-based traffic assignment problem such as MSA(used in this paper) and the Frank-Wolfe algorithm can be applied to solve the lower level problem. However, the Dijkstra's algorithm which is always applied to find the shortest path in each iteration in the traditional link-based algorithms is not suitable for ATS network since Dijikstra's algorithm works only with positive costs. For ATS network with positive (activity utility), negative (travel time), zero link value we can use the Bellman-Ford or Shortest Path Faster Algorithm (SPFA) to find the longest path. Then, as the link-based solution algorithm without path enumeration for solving traditional trip-based traffic assignment problem can be adapted to solve the lower level problem efficiently and SAA is a mature algorithm for solving the upper level problem, the proposed model and solution algorithm in this paper can be applied in large networks. Nevertheless, according to Assumption 6, the activity duration and travel time are measured in the unit of intervals. Thus, the accuracy of the results mainly depends on the length of the interval. The length of interval should be small enough to ensure satisfying the accuracy requirement of activity duration, and travel time. However, too short interval, the increased number of intervals may lead to the burdensome computation. The length of interval should be carefully determined according to the application environment.

Further research work should be carried out in three aspects: (a) the solution property of the proposed model should be further discussed; (b) the lower-level model for modeling users' location choice behavior and travel behavior should be improved by considering the land supply model and household activity pattern; (c) to calibrate and validate the parameters of utility functions with empirical data, etc.

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APPENDIX: NOTATIONS

T , k	studied time horizon; time interval
N	set of zones

R , S	set of origin zones; set of destination zones
r , s	origin zone index; destination zone index
H , W	set of residential locations ; set of employment locations
h , w	residential location index; and employment location index
J , j	set of activities; activity index
m	user class index
a , p	link index; path index
$U_{rs,hw}^{j,m}(k)$	activity utility obtained by an individual, in class m with residential location h and employment location w , departing at interval k from origin r to destination s , to perform activity j at s for one interval
α_1^m, α_2^m	utility parameter of travel time/money, $\alpha_1^m > 0, \alpha_2^m > 0$
$T_{rs}(k)$	actual travel time from r to s departing at interval k
k_s	interval index arriving at destination s , $k_s = k + \overline{T}_{rs}(k)$
$\overline{T}_{rs}(k)$	equivalent number of intervals for the actual travel time from r to s , defined as $INT\left(T_{rs}(k)/\Delta + 0.5\right)$, Δ is the duration of each interval.
$P_s^{j,m}(k_s)$	activity cost/benefit that individuals in class m should pay for or obtain by performing activity j at destination s at the arrival interval k_s .
$\mu_{s,hw}^{j,m}(k_s)$	utility of performing activity j at destination s at the arrival interval k_s for individuals in class m with location choice (h,w)
$\alpha_3^m, \alpha_4^m, \alpha_5^m$	utility parameters for individuals in class m , $\alpha_3^m > 0$, $\alpha_4^m > 0$, $\alpha_5^m > 0$
μ_s	systematic component of utility common to all elements with destination s , just vary across destinations and are independent of activities
μ_s^j	systematic component of utility that vary across both activities and destinations
$-b_s^j\left(\frac{q_s^j(k_s)}{C_s^j}\right)^n$	activity performance congestion disutility
$q_s^j(k_s)$	number of accumulated population at destination s to conduct activity j at interval k_s
C_s^j	capacity of population performing activity j at destination s
b_s^j , n	congestion parameters, vary with the destination s and activity type j
y	activity pattern index
$\tau_{j,s}^i(k)$	activity duration, and i is the order of the activity in the scheduled activity pattern, j,s,k is respectively the activity, activity performance destination, and time interval of starting to perform the correspondingly activity
$U_{hw}^{m,y}$	utility of activity pattern y for individual in class m with location choice (h,w)
T_y	total travel time for completing an activity pattern
$t_a^0 ; C_a$	free-flow travel time of link a ; link capacity
$x_a(k)$	link flow at interval k ; flow on path p from r to s departing at

$f_{rs}^{m,p}(k)$	interval k
$f_{rs}^{m,p*}(k)$	equilibrium value
$f_{rs,hw}^{m,y,p}(k)$	flow on path p from r to s departing at interval k of individual class m with location choice (h,w)
$t_a^m(k)$	link a travel time of individual in class m at interval k
$T_{rs}^{m,p}(k)$	path p travel time of individual in class m from r to s departing at interval k ;
$T_{rs}^m(k)$, $T_{rs}^{m*}(k)$	$T_{rs}^m(k) = \min\{T_{rs}^{m,p}(k), \forall p, r, s\}$; equilibrium value
Q , $\overline{Q}$, ΔQ	total population, original population, increased population
$A_h(h), A_w(w)$, $B_a(a)$	respectively the cost functions for residential allocation, employment allocation, and capacity enhancements
M_h, M_w, L_a	respectively the residential allocation, employment allocation, and link capacity enhancements
B	total budget
V^{m*} , $\overline{V}^m$	equilibrium user utility after the optimization; required user utility
$(M_h^{\min}, M_h^{\max})$	range of feasible residential allocation
$(M_w^{\min}, M_w^{\max})$	range of feasible employment allocation
$(L_a^{\min}, L_a^{\max})$	range of feasible link capacity expansion.
q^h , q^w	population at residential location h ; population at employment location w
$\overline{q}^h$, $\overline{q}^w$	original population at residential location h ; original population at employment location w
q^{hw} , q_{hw}^m , q_{hw}^{m*}	population with location choice (h,w) ; population in class m with location choice (h,w) ; equilibrium value
U_{hw}^m, U_{hw}^{m*}	daily activity utility obtained by individual in class m , with location choice (h,w) , $U_{hw}^m = \max\{U_{hw}^{m,y}, \forall h, w, m, y\}$; equilibrium value
$P^h(q^h)$	daily housing price function of h
V_{hw}^m	location choice utility of individual in class m , with location choice (h,w)
V^m, V^{m*}	$V^m = \max\{V_{hw}^m, \forall h, w\}$; equilibrium value
$F_{hw}^{m,y}, F_{hw}^{m,y*}$	population of individual in class m , with location choice (h,w) with activity pattern y; equilibrium value