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Traffic Incident Management Location Model under Degradable Stochastic Network

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Abstract: This paper applies the concept of price of anarchy to define a new index, namely, traffic incident management ratio (TIMR) for identifying appropriate roads to be installed with traffic incident management program (TIMP). TIMR is defined as the ratio between the expected total travel costs of the degradable transport networks without and with TIMP. The paper assumes user equilibrium (UE) represents the degradable network without TIMP, whereas the social optimum (SO) characterizes the degradable network with TIMP. The traffic incident management location (TIML) model is then proposed for assessing the critical value of link capacity degradation on each link (i.e. road segment) and identifying the critical links for an installation of TIMP. The critical links for the TIMP are ranked by the critical values of capacity degradations, which maximize the TIMR. The proposed model and solution algorithm were tested using two networks to illustrate the application of the proposed model.

Keywords: *Traffic Incident Management Location, Critical Link Identification, Stochastic Road Network*

1. INTRODUCTION

Non-recurrent congestion caused by traffic incidents, which accounts for 60% of congestion delay, is a major concern for transport agencies (Pal and Sinha, 2000). A traffic incident management program (TIMP), which is a planned and coordinated program process, has been implemented extensively in many metropolitan areas to detect, respond to, remove traffic incidents, and restore traffic capacity as safely and quickly as possible (FHWA, 2000). An efficient design of the TIMP is necessary to allocate limited resources to critical and important roads whose degradation (by traffic incidents) may seriously affect network performance. From the literature, several works have been conducted on the optimal design for TIMP; for example, freeway incident response system (Pal and Sinha, 2000), traffic signal control (Shed *et al*., 2003), and incident-responsive local ramp control (Sheu, 2007), among others. Most of these studies focus principally on the operational plan of the TIMP. On the other hand, it is also important to identify critical links for long-term resource allocation of the TIMP. This paper aims to propose a model for identifying the critical links which should be installed with the TIMP. The proposed model is based on the equilibrium network modeling framework.

In this paper, the framework of stochastic road network is proposed to represent a random process of the flows (i.e. demand, path, and link) and the travel time (and cost) in the network. The proposed framework also allows for network degradation caused by traffic incidents. In general, a traffic assignment model considers the behavior of travelers in making route choices within a transport network. Several routing models have been studied extensively in the literature, e.g., deterministic user equilibrium (UE), stochastic user equilibrium (SUE), and social (or system) optimum (SO). However, this paper focuses mainly on two basic routing rationales, namely, the UE and SO models, which correspond to Wardrop's first and second equilibrium principles (Wardrop, 1952). The former describes the phenomenon in the unregulated road network that independently allows self-interested travelers to pursue their personally optimal route choice strategies. On the other hand, the latter aims to minimize the total travel time in the controlled road network.

To simplify the proposed model after traffic incidents occur, this paper assumes the UE to represent the *unregulated network* without TIMP implementation, whereas the SO would characterize the *controlled network* under TIMP application. The UE flow pattern does not always minimize the total travel cost incurred by all network users compared to the SO flow pattern (Roughgarden, 2003). The gap between the total travel costs under the UE and SO conditions represents the potential improvement of network performance from the network control system. Papadimitriou (2001) defined a price of anarchy (POA) as the worst-possible ratio of the total travel cost of the UE to the total travel cost of the SO. This ratio indicates an inefficiency of the transport network from the lack of coordination among its members. Several studies have been conducted to investigate the POA, see e.g., Roughgarden (2003); Haviv and Roughgarden (2007); and Youn *et al*. (2008).

This paper applies the concept of POA to define a new index (i.e. traffic incident management ratio or TIMR) to evaluate an inefficiency of degradable stochastic road network. The inefficiency of stochastic road network in this paper is assessed in terms of the expected total travel cost incurred by uncoordinated drivers in the network without TIMP and that by coordinated drivers in the network with TIMP. TIMR is thus defined as the ratio between the expected total travel times (or costs) of the degradable stochastic road networks without and with TIMP. A traffic incident management location (TIML) model is then proposed to the critical values of link (i.e. road section) capacity degradations which yield the maximum value of TIMR. The critical links are then identified by ranking the links in accordance to the critical values of capacity degradations. The links with lower value of critical capacity degradations are more critical. Adequate preparation for long-term resource allocation of TIMP (e.g. CCTV monitoring and traffic re-routing process) on these critical links may reduce the impacts from traffic incidents. The remainder of this paper is organized as follows. Section 2 describes some basic notations and degradable stochastic road network representation. Section 3 explains two route choice models under degradable stochastic network. Section 4 defines the TIMR and TIIF. In addition, the TIML model and solution algorithm are formulated and developed, respectively, in this section. Section 5 presents numerical examples of the proposed model. Finally, Section 6 concludes the paper and discusses future research issues.

2. NOTATION AND DEGRADABLE STOCHASIC ROAD NETWORK

2.1 Notation

The road network is represented by a directed graph $G = (N, A)$ with a set of nodes N and a set of directed links A. Let O and D be the sets of origin and destination nodes, respectively. The set of all origin-destination (OD) movements is defined as W. Let K_w denote the set of all possible paths connecting OD movement $w \in W$. The total number of paths is $K = \sum_{w \in W} K_w$. The italic uppercase letters are used to denote random variables, whereas lowercase letters represent deterministic variables. The following notations are used throughout the paper.

Q_w travel demand for OD movement $w \in W$;
q_w mean travel demand for OD movement $w \in W$;
σ_w^q standard deviation (SD) of travel demand for OD movement $w \in W$;
$F_{k,w}$ traffic flow on path $k \in K_w$;
$f_{k,w}$ mean traffic flow on path $k \in K_w$;
$\sigma_{k,w}^f$ SD of traffic flow on path $k \in K_w$;
$\mathbf{f}$ column vector of mean path flows, $\mathbf{f} = \left[f_{1,1}, \ldots, f_{K_1,1}, f_{1,2}, \ldots, f_{k,w}, \ldots, f_{K_w,w} \right]^T$;
$\mathbf{\Sigma^f}$ variance-covariance matrix of path flows;
V_a traffic flow on link $a \in A$;
v_a mean traffic flow on link $a \in A$;
σ_a^v SD of traffic flow on link $a \in A$;
$\mathbf{v}$ column vector of mean link flows, $\mathbf{v} = \left[v_1, v_2, \ldots, v_a \right]^T$;
$\mathbf{\Sigma^v}$ variance-covariance matrix of link flows;
$p_{k,w}$ flow proportion on path $k \in K_w$, $p_{k,w} = f_{k,w} / q_w$;
$\mathbf{p}_w$ column vector of path choice proportions for OD movement $w \in W$, $\mathbf{p}_w = \left[p_{1,w}, p_{2,w}, \ldots, p_{k,w} \right]^T$;
$\mathbf{\Delta}$ link-path incidence matrix, $\mathbf{\Delta} = \left[\mathbf{\Delta}_1 \; \mathbf{\Delta}_2 \; \ldots \; \mathbf{\Delta}_w \right]$;
$\mathbf{\Delta}_w$ link-path incident matrix associated with OD movement $w \in W$, $\mathbf{\Delta}_w$ has elements $\delta_{a,k,w}$ with $\delta_{a,k,w} = 1$ denoting link a is a part of path k and $\delta_{a,k,w} = 0$ otherwise;
T_a travel time on link $a \in A$;
t_a mean travel time on link $a \in A$;
σ_a^t SD of travel time on link $a \in A$;
$\mathbf{t}$ column vector of mean link travel times, $\mathbf{t} = \left[t_1, t_2, \ldots, t_a \right]^T$;
$C_{k,w}$ travel cost on path $k \in K_w$;
$c_{k,w}$ mean travel cost on path $k \in K_w$;
$\mathbf{c}$ column vector of mean path costs, $\mathbf{c} = \left[c_{1,1}, \ldots, c_{K_1,1}, c_{1,2}, \ldots, c_{k,w}, \ldots, c_{K_w,w} \right]^T$;
TT total travel time in the stochastic network, $TT = \sum_{a \in A} V_a T_a$.

2.2 Traffic Flow Relationships

Due to the existence of variability and uncertainty in travel demands from day to day, the OD travel demand Q_w is assumed to follow a normal distribution (Asakura and Kashiwadani, 1991; Chen et al., 2003; Lam *et al*., 2008). Note that other types of probability distribution function (pdf) of travel OD demand have been adopted in the literature; for example, binomial distribution (Nakayama and Takayama, 2003), poison distribution (Clark and Watling, 2005), or lognormal distribution (Sumalee *et al*., 2009a). The proposed framework and solution algorithm in this paper can also be generalized and further applied to other pdfs.

Given q_w and σ_w^q, the OD travel demand can be expressed as $Q_w \sim N\left(q_w, \left(\sigma_w^q \right)^2 \right)$. Since the OD demand is a normally distributed random variable, the path flow, which is the product of path choice proportion and stochastic OD demand, i.e. $F_{k,w} = p_{k,w} Q_w$, also follows a normal distribution, i.e. $F_{k,w} \sim N\left(f_{k,w}, \left(\sigma_{k,w}^f \right)^2 \right)$. The mean and variance of stochastic path flow are

$$f_{k,w} = \mathrm{E}\left[p_{k,w} Q_w \right] = p_{k,w} q_w, \; \forall k \in K_w, \; w \in W , \tag{1}$$

$$\left(\sigma_{k,w}^{f}\right)^2 = \mathrm{Var}\left(p_{k,w}Q_w\right) = \left(p_{k,w}\sigma_w^q\right)^2,\ \forall k \in K_w,\ w \in W , \tag{2}$$

where $\mathrm{E}[.]$ and $\mathrm{Var}(.)$ denote expectation and variance operators, respectively.

The covariance between two arbitrary path flows (say $F_{k,w}$ and $F_{j,w}$) joining the same OD movement *w* is formulated, following Lam *et al*. (2008), as

$$Cov\left(F_{k,w}, F_{j,w}\right) = \left(\sigma_w^q\right)^2 p_{k,w}p_{j,w},\ k \neq j,\ \forall k, j \in K_w,\ w \in W , \tag{3}$$

while the covariance of flows on paths connecting different OD pairs is zero.

Let **V** and **F** be the vectors of stochastic link and path flows, respectively. Since the stochastic flow on each link is a sum of stochastic path flows, i.e. $\mathbf{V} = \Delta \cdot \mathbf{F}$. **V** thus follows a multivariate normal distribution, i.e. $\mathbf{V} \sim N\left(\mathbf{v}, \Sigma^{\mathbf{v}}\right)$, with *a*-dimensional mean vector

$$\mathbf{v} = \Delta \cdot \mathbf{f} = \sum_{w \in W} \Delta_w \cdot q_w \cdot \mathbf{p}_w , \tag{4}$$

and $a \times a$ variance-covariance matrix

$$\mathbf{\Sigma}^{\mathbf{v}} = \mathbf{\Delta} \cdot \mathbf{\Sigma}^{\mathbf{f}} \cdot \mathbf{\Delta}^T . \tag{5}$$

Note that the link flows are not mutually independent due to the link-path relationship. However, this paper only uses the mean and variance of traffic flow on each link to express the expected travel time on that link. The details are explained in the next section.

2.3 Link and Path Travel Times

The link travel time function is assumed to follow a standard Bureau of Public Roads (BPR) function, i.e. $T_a\left(V_a\right) = t_a^0 + b_a\left\{V_a/\left(c_a - s_a\right)\right\}^{n_a}$ where t_a^0 is the free-flow travel time; b_a and n_a are parameters of the BPR function; c_a is the average link capacity; and s_a is the link capacity degradation, incurred by traffic incidents, to be determined. This paper also assumes that the stochastic flow (i.e. V_a) and the effective capacity (i.e. c_a-s_a) on each link are mutually independent. Since link flows are stochastic, the link travel times and the path travel costs are random variables. This paper uses the expected link travel times and path travel costs to capture the user travel cost in the route choice models. The paper assumes $n_a = 4$ for all links. By using the moment generating function (MGF) method, the mean link travel time can be expressed in terms of the mean and SD of stochastic link flow (Sumalee *et al*., 2009b), i.e.

$$t_a = \mathrm{E}\left[T_a\left(V_a\right)\right] = t_a^0 + \frac{b_a}{\left(c_a - s_a\right)^4}\left\{v_a^4 + 6v_a^2\left(\sigma_a^v\right)^2 + 3\left(\sigma_a^v\right)^4\right\},\ \forall a \in A . \tag{6}$$

The mean link travel time is formulated as a function of v_a and σ_a^v on each link, without the covariance of flows between two links. This is because the link travel time is a function of the flow on that link only. The mean path travel cost vector is then calculated from $\mathbf{c} = \Delta^T \cdot \mathbf{t}$.

3 ROUTE CHOICE MODELS UNDER DEGRADABLE STOCHASTIC NETWORK

This section explains two basic rationales (i.e. UE and SO) of drivers for making their route choice strategies under degradable stochastic road network (SN). In this paper, the SN-UE represents the network condition without TIMP, whereas the SN-SO describes the coordinated traffic pattern controlled by the TIMP. The formulations for both models are described below.

3.1 SN-UE Formulation

In general, a UE condition under deterministic road network can be formulated as the variational inequality (VI) problem (Nagurney, 1999). The VI problem for the UE condition is to find $\mathbf{v}^* \in \Omega_v$ such that

$$\mathbf{t}\left(\mathbf{v}^*\right)\cdot\left(\mathbf{v}-\mathbf{v}^*\right)^T \geq 0\,,\ \forall \mathbf{v}\in\Omega_v, \tag{7}$$

where $\mathbf{v}^* = \left[v_1^*, v_2^*, \ldots, v_a^*\right]^T$ is the column vector of UE mean link flows, and the feasible region of deterministic link flows is defined as $\Omega_v = \left\{\mathbf{v}\middle|\mathbf{v}=\Delta\cdot\mathbf{f}, \mathbf{f}\geq\mathbf{0}, \sum_{w\in W}\sum_{k\in K_w} f_{k,w} = q_w\right\}$.

Since both link travel time and path travel cost in this paper are treated as random variables, the equilibrium rule for the SN-UE then changes to "*no traveler can reduce his or her* ***expected*** *travel time by unilaterally changing his/her route*" (Sumalee and Xu, 2008). Let $\Omega_V = \left\{\mathbf{V}\middle|\mathbf{V}=\Delta\cdot\mathbf{F}, \mathbf{F}\geq\mathbf{0}, \sum_{w\in W}\sum_{k\in K_w} F_{k,w} = Q_w\right\}$ be the feasible region of stochastic link flows. Thus, the VI problem for the SN-UE is to find $\mathbf{V}^{UE} \in \Omega_V$ such that

$$\mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right]\cdot\left(\mathbf{v}-\mathbf{v}^{UE}\right)^T \geq 0\,,\ \forall \mathbf{v}\in\Omega_v, \tag{8}$$

where $\mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right] = \left[\mathrm{E}\left[T_1\left(V_1^{UE}\right)\right], \mathrm{E}\left[T_2\left(V_2^{UE}\right)\right], \ldots, \mathrm{E}\left[T_a\left(V_a^{UE}\right)\right]\right]^T$ is the column vector of the expected link travel times, and $\mathbf{v}^{UE} = \left[v_1^{UE}, v_2^{UE}, \ldots, v_a^{UE}\right]^T$ is the column vector of SN-UE mean link flows. Note that each element of $\mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right]$ can be calculated by using (6).

The VI problem in (8) may requires heavy computational effort because $\mathbf{v}^{UE}$ is evaluated against the vector of all feasible mean link flows (i.e. $\mathbf{v}$). However, the feasible region Ω_v consists of a set of linear equality constraints, thus constitutes a polyhedron. Let *S* be the set of extreme points (corner points) of the polyhedron Ω_v, then any element of $\mathbf{v}$ in Ω_v can be expressed as a convex combination of the extreme points, i.e. $v_a = \sum_{s=1}^{S}\lambda_s v_a^s$ where v_a^s denotes the s^{th} extreme point of the polyhedron Ω_v, and λ_s is the coefficient ranging between 0 and 1 ($0\leq\lambda_s\leq 1$) and $\sum_{s=1}^{S}\lambda_s = 1$. Let $\mathbf{v}^s$ be the column vector of extreme points. The VI problem of SN-UE condition can be reformulated as

$$\mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right]\cdot\left(\mathbf{v}^s-\mathbf{v}^{UE}\right)^T \geq 0\,,\ \forall s\in 1...S\,. \tag{9}$$

3.2 SN-SO Formulation

In general, a SO aims to assign the flows so as to minimize the total travel time in the deterministic flow network. The VI problem for the SO condition can be formulated by replacing the unit link cost-flow functions *t(v)* in the UE model with the marginal social cost-flow functions *m(v)*. For the network with stochastic demand, the rule of SN-SO changes to minimize the ***expected*** *total travel cost* in the network (Sumalee and Xu, 2008). Similar to the SN-UE condition, the VI problem for the SN-SO condition is to find $\mathbf{V}^{SO} \in \Omega_V$ such that

$$\nabla_{\mathbf{v}} \mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right] \cdot \left(\mathbf{v}^{s} - \mathbf{v}^{SO}\right)^{T} \geq 0, \ \forall s \in 1...S, \tag{10}$$

where $\nabla_{\mathbf{v}} \mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right] = \left[\partial \mathrm{E}\left[\sum_{a \in A} V_a^{SO} T_a^{SO}\right] \big/ \partial v_1^{SO}, \ldots, \partial \mathrm{E}\left[\sum_{a \in A} V_a^{SO} T_a^{SO}\right] \big/ \partial v_a^{SO}\right]$ is the column vector of the expected marginal costs. Each of them is the partial derivative of the expected total travel cost in the network with respect to the mean of SN-SO flow on link *a*, $\forall a \in A$. In order to express $\nabla_{\mathbf{v}} \mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]$ in (10) as a function of the mean and SD of SN-SO link flows (i.e. v_a^{SO} and $\sigma_a^{v^{SO}}$, respectively), the expected total travel time of the network can be derived by using the MGF method (with $n_a = 4$ for all links), i.e.

$$\mathrm{E}\left[TT\left(\mathbf{V}\right)\right] = \mathrm{E}\left[\sum_{a \in A} V_a T_a\right] = \sum_{a \in A} \left\{ v_a t_a^0 + \frac{b_a}{\left(c_a - s_a\right)^4} \left(v_a^5 + 10 v_a^3 \left(\sigma_a^v\right)^2 + 15 v_a \left(\sigma_a^v\right)^4 \right) \right\}. \tag{11}$$

Each element of $\nabla_{\mathbf{v}} \mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]$ can be determined by differentiating (11) with respect to the mean flow and evaluating it at $\mathbf{V}^{SO}$, i.e.

$$\frac{\partial \mathrm{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]}{\partial v_a} = t_a^0 + \frac{b_a}{\left(c_a - s_a\right)^4} \left\{ 5\left(v_a^{SO}\right)^4 + 30\left(v_a^{SO}\right)^2 \left(\sigma_a^{v^{SO}}\right)^2 + 15\left(\sigma_a^{v^{SO}}\right)^2 \right\}, \ \forall a \in A. \tag{12}$$

4. PROBLEM FORMULATION AND SOLUTION ALGORITHM

4.1 Traffic Incident Management Ratio

TIMR is proposed as a new index for evaluating the direct benefit of installing the TIMP on critical links. Following the definition of Price of Anarchy or POA (Papadimitriou, 2001), the TIMR is defined as "*the ratio between the **expected** total travel cost of the **degradable** stochastic road network without and with TIMP*". Note that the TIMR can also be interpreted as an inefficient state of the degradable network due to the lack of coordination when the TIMP is not implemented. The TIMR formulation under degradable stochastic road network is

$$\mathrm{TIMR} = \frac{\mathrm{E}\left[TT\left(\mathbf{V}^{UE}\right)\right]}{\mathrm{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]}, \tag{13}$$

where $\mathbf{V}^{UE}$ and $\mathbf{V}^{SO}$ are the vectors of stochastic link flows at SN-UE and SN-SO conditions, respectively; and $\mathrm{E}\left[TT(.)\right]$ is the expected total travel time of the network which can be calculated by using (11). Note that a high value of TIMR implies that the travel condition of the degradable network without TIMP may be worsened significantly as a result of incidents. This value also suggests a potential benefit of the TIMP implementation in reducing the total travel time in the network once an incident occurs.

4.2 Traffic Incident Management Location Model

A traffic incident management location (TIML) model is proposed to quantify the critical value of link capacity degradations after traffic incidents occur. TIML model is evaluated at the maximum value (worst-possible ratio) of TIMR, as defined in (13). This worst-possible TIMR indicates the greatest gap between the expected total travel costs of the degradable stochastic road networks without and with TIMP. The proposed model can also be used to identify the critical links which should be installed with the TIMP.

TIML model is to find link capacity degradations (denoted by **s**) which maximize the TIMR subject to parameter constraints. The TIML model is formulated as a mathematical program with equilibrium constraints (MPEC):

$$\max_{\mathbf{s}} \mathrm{TIMR} = \frac{\mathrm{E}\left[TT\left(\mathbf{V}^{UE}\right)\right]}{\mathrm{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]} \tag{14}$$

subject to:

$$\mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right]\cdot\left(\mathbf{v}^{s}-\mathbf{v}^{UE}\right)^{T} \geq 0,\ \forall s \in 1 \ldots S, \tag{15}$$

$$\nabla_{\mathbf{v}}\mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]\cdot\left(\mathbf{v}^{s}-\mathbf{v}^{SO}\right)^{T} \geq 0,\ \forall s \in 1 \ldots S, \tag{16}$$

$$\Omega_{V} = \left\{\mathbf{V}\left|\mathbf{V} = \Delta\cdot\mathbf{F}, \mathbf{F} \geq \mathbf{0}, \sum_{w\in W}\sum_{k\in K_w} F_{k,w} = Q_w\right.\right\} \tag{17}$$

$$\Omega_{v} = \left\{\mathbf{v}\left|\mathbf{v} = \Delta\cdot\mathbf{f}, \mathbf{f} \geq \mathbf{0}, \sum_{w\in W}\sum_{k\in K_w} f_{k,w} = q_w\right.\right\}, \tag{18}$$

$$\mathbf{0} \leq \mathbf{s} \leq \mathbf{s}^{\max} \tag{19}$$

where $\mathbf{s}^{\max} = \left[s_1^{\max}, s_2^{\max}, \ldots, s_a^{\max}\right]^T$ is the column vector of the maximum values of link capacity degradations.

The objective function (14) is, therefore, to find the critical levels of link capacity degradations (i.e. $\mathbf{s}$) that maximize the TIMR. Constraints (15) and (16) are the VI problems for SN-UE and SN-SO conditions explained in Section 3.1 and 3.2, respectively. The feasible regions of stochastic link flows and mean link flows are expressed in (17) and (18), respectively. The upper and lower bounds of $\mathbf{s}$ are expressed in (19).

4.3 Solution Algorithm

A solution algorithm for solving the proposed model is based on the sequential quadratic programming (SQP). The SQP solves a series of sub-problems designed to minimize a second-order (quadratic) approximation of the objective function subject to linearized constraints. We also adopt the cutting constraint algorithm (CCA) proposed by Lawphongpanich and Hearn (2004) to avoid enumerating all extreme points in (15) and (16) at once. The CCA will generate a set of necessary extreme points at each iteration. Then the generated extreme points will be used to define a new set of VI constraints (15) and (16). At each iteration, the method of successive average (MSA) (Sheffi, 1985) is used for solving the SN-UE and SN-SO. The overall procedure of the solution algorithm is shown in Figure 1.

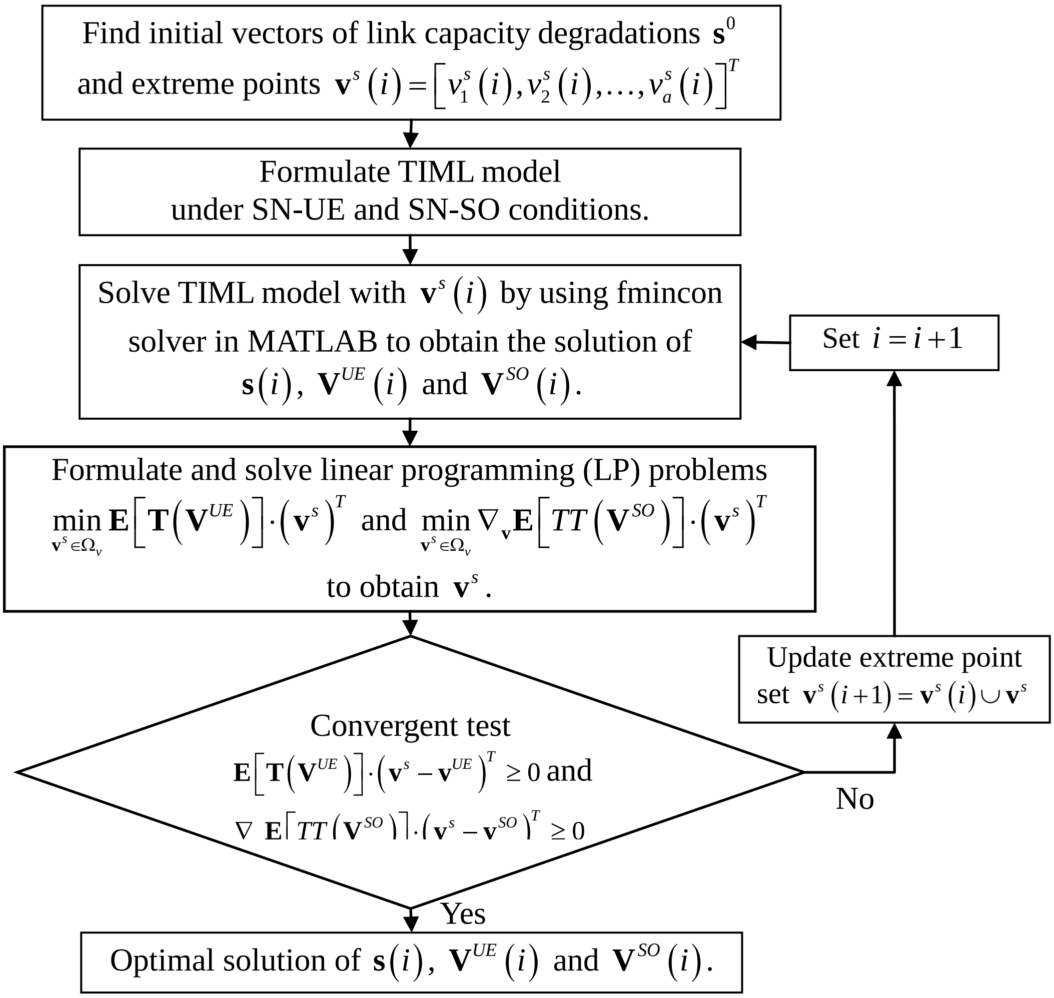


Figure 1. Overall procedure of the solution algorithm

The details of solution algorithm can be described as follows:

Step 0: *Initialization.* Set iteration i = 0; find initial vectors of link capacity degradations $\mathbf{s}^0$ and extreme points $\mathbf{v}^s(i) = \left[v_1^s(i), v_2^s(i), \ldots, v_a^s(i)\right]^T$ and formulate the TIML model with $\mathbf{v}^s(i)$ under SN-UE and SN-SO conditions.

Step 1: *TIML optimization.* Solve the TIML model with $\mathbf{v}^s(i)$ to obtain the current solution of $\mathbf{s}(i)$, $\mathbf{V}^{UE}(i)$ and $\mathbf{V}^{SO}(i)$.

Step 2: *Finding the shortest path.* Formulate and solve linear programming (LP) problems, i.e. $\min_{\mathbf{v}^s \in \Omega_v} \mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right] \cdot \left(\mathbf{v}^s\right)^T$ and $\min_{\mathbf{v}^s \in \Omega_v} \nabla_{\mathbf{v}} \mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right] \cdot \left(\mathbf{v}^s\right)^T$, with the solution of $\mathbf{s}(i)$, $\mathbf{V}^{UE}(i)$ and $\mathbf{V}^{SO}(i)$ as the inputs to obtain the optimal solution (extreme point) $\mathbf{v}^s$. Note that solving these LP problems is equivalent to finding the shortest path for each OD in the network.

Step 3: *Convergence test.* Terminate the algorithm if $\mathbf{E}\left[\mathbf{T}\left(\mathbf{V}^{UE}\right)\right]\cdot\left(\mathbf{v}^{s}-\mathbf{v}^{UE}\right)^{T}\geq 0$ and $\nabla_{\mathbf{v}}\mathbf{E}\left[TT\left(\mathbf{V}^{SO}\right)\right]\cdot\left(\mathbf{v}^{s}-\mathbf{v}^{SO}\right)^{T}\geq 0$. $\mathbf{s}(i)$, $\mathbf{V}^{UE}(i)$ and $\mathbf{V}^{SO}(i)$ is thus the optimal solution of the TIML model. Otherwise, go to Step 4.

Step 4: *Extreme point set updating.* Include the new set of extreme points $\mathbf{v}^{s}$ into the set $\mathbf{v}^{s}(i+1)=\mathbf{v}^{s}(i)\cup\mathbf{v}^{s}$, set $i = i +1$ and go to Step 1

5. NUMERICAL EXAMPLES

Two test networks are used to illustrate the application of the proposed model and solution algorithm. The first network from Sumalee *et al.* (2006) consists of 18 directed links, 6 OD movements, and 30 paths. The network is shown in Figure 2. The mean and coefficient of variation (CV), SD over mean ratio, of OD demands (CV in brackets) are listed in Table 1. The larger the CV is, the less reliable the OD demand is. The cost-link function is $T_a(V_a)=t_a^0+b_a\left\{V_a/(c_a-s_a)\right\}^4$, $\forall a\in A$. The other parameters of the cost-link function are given in Table 2.

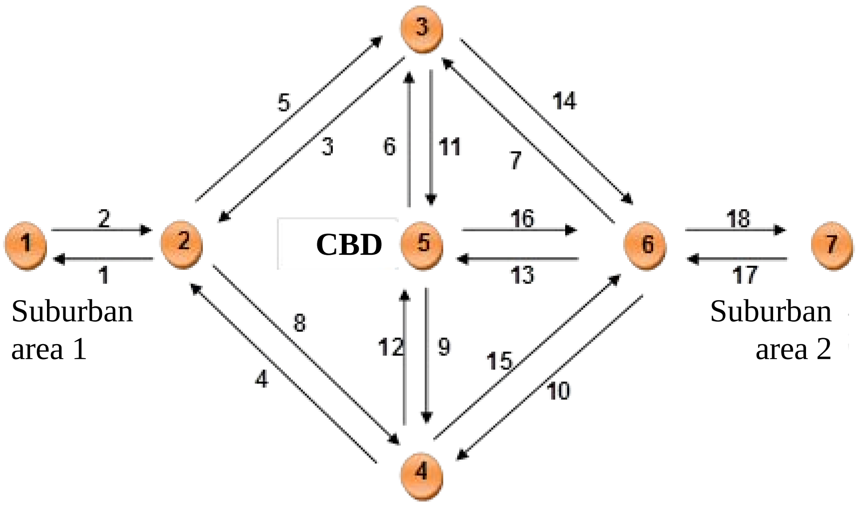


Figure 2. Hypothetical test network

Table 1. Mean and CV of OD normally distributed demands

Origin	Destination		
	1	*5*	*7*
1	-	600 (0.25)	400 (0.20)
5	500 (0.50)	-	600 (0.60)
7	375 (0.20)	800 (0.20)	-

For SN-UE and SN-SO conditions, the convergence criteria and the maximum number iterations in MSA are set as $\varepsilon=1\times10^{-6}$ and i_{max} = 500. Thirty (30) possible paths are defined as shown in the third column of Table 3. Table 3 also presents the assignment results under normal network conditions (i.e. no traffic incident). It shows that the expected travel costs at SN-UE (or the expected marginal social costs at SN-SO) of all used paths between each OD pair are almost equivalent (only different in the last decimal point). In addition, Table 3 shows the mean and SD of path flows. The different flow patterns (mean and SD) at SN-UE and SN-SO can be found from OD pairs (7,1) and (1,7). For each OD pair, only two paths are used at SN-UE, whereas four paths are employed at SN-SO. This result is reasonable since the SN-SO approach aims to utilize a greater number of paths than that under SN-UE in order to minimize the expected total travel time, i.e. E[TT], of the network. From the equilibrium conditions, the E[TT]s are found to be 37,136.07 and 35,141.71 at SN-UE and SN-SO, respectively. These results give the price of anarchy (POA) to be 1.057. Note that the POA is used here because the network is evaluated under normal conditions.

Table 2. Link cost parameters and lengths

Link	t_a^0	b_a	c_a
1	1.2500	0.0253	1800
2	1.2500	0.0253	1800
3	9.1667	6.2610	1100
4	9.1667	6.2610	1100
5	9.1667	6.2610	1100
6	2.5000	1.7075	1100
7	7.5000	1.0866	1100
8	9.1667	6.2610	1100
9	2.5000	1.7075	1100
10	7.5000	1.0866	1100
11	2.5000	1.7075	1100
12	2.5000	1.7075	1100
13	2.0000	1.3660	1100
14	7.5000	1.0866	1100
15	7.5000	1.0866	1100
16	2.0000	1.3660	1100
17	1.2500	0.0253	1800
18	1.2500	0.0253	1800

From SN-UE and SN-SO solutions, we also calculated the marginal cost toll to the users on each link by calculating the difference between (12) and (6). It was found that Links 13 and 16 are the first two congested links since these two links have the greatest marginal cost tolls compared to other links. From the results, Links 13 and 16 may be considered as the important links for the TIMP among the CBD-connected links (i.e. Links 6, 9, 11, 12, 13 and 16).

It was interesting to investigate the performance (i.e. TIMR) of degradable network if traffic incidents occur on these two links. The test assumed three scenarios according to the traffic incident location: 1) only on link 13; 2) only on link 16; and 3) on both links. The TIMRs for scenarios 1) and 2) are shown in Figure 3, whereas the TIMRs for scenario 3) are shown in Figure 4.

Figure 3 shows that the TIMRs can increase up to 1.135 and 1.191 at the critical capacity degradations of 559 and 572 on Links 13 and 16, respectively. Note that the maximum TIMRs (i.e. 1.135 and 1.191) under the degradable network conditions are greater than the maximum POA (1.057) under the normal network condition as shown in the previous test.

Figure 4 illustrates that the TIMR was greater when Links 13 and 16 were degraded. The maximum TIMR was found to be 1.252 at the degradations of 557 and 560 on Links 13 and 16, respectively. The TIMR under scenario 3) was the highest among three scenarios. We can conclude that a higher numbers of degraded links create the worse impact to the network.

Table 3. Equilibrium path flows under SN-UE and SN-SO conditions

OD movement	OD pair	Path	Link sequence	SN-UE			SN-SO		
				Expected travel cost	Flow		Expected marginal social cost	Flow	
					Expected	SD		Expected	SD
1	(5,1)	1	1-3-6	13.230	250	125	14.226	250	125
		2	1-4-9	13.229	250	125	14.225	250	125
		3	1-3-7-16	-	-	-	-	-	-
		4	1-4-10-16	-	-	-	-	-	-
2	(7,1)	5	1-4-10-17	-	-	-	20.427	164	33
		6	1-3-7-17	-	-	-	20.427	164	33
		7	1-4-9-13-17	18.505	187	37	20.427	24	5
		8	1-3-6-13-17	18.506	187	37	20.428	24	5
		9	1-4-7-9-11-17	-	-	-	-	-	-
		10	1-3-6-10-12-17	-	-	-	-	-	-
3	(1,5)	11	2-5-11	13.320	300	75	14.604	300	75
		12	2-8-12	13.319	300	75	14.603	300	75
		13	2-5-13-14	-	-	-	-	-	-
		14	2-8-13-15	-	-	-	-	-	-
4	(7,5)	15	10-12-17	-	-	-	-	-	-
		16	7-11-17	-	-	-	-	-	-
		17	13-17	5.276	800	160	6.202	800	160
		18	4-5-10-11-17	-	-	-	-	-	-
		19	3-7-8-12-17	-	-	-	-	-	-
5	(1,7)	20	2-5-14-18	-	-	-	20.778	160	32
		21	2-5-11-16-18	18.319	200	40	20.778	40	8
		22	2-5-9-11-15-18	-	-	-	-	-	-
		23	2-8-15-18	-	-	-	20.778	161	32
		24	2-8-12-16-18	18.319	200	40	20.778	40	8
		25	2-6-8-12-14-18	-	-	-	-	-	-
6	(5,7)	26	6-14-18	-	-	-	-	-	-
		27	16-18	4.999	600	360	6.175	600	360
		28	15-18	-	-	-	-	-	-
		29	4-5-9-14-18	-	-	-	-	-	-
		30	3-6-8-15-18	-	-	-	-	-	-

After incidents occur, traffic agencies may consider different patterns of efficient TIMP to mitigate adverse impacts from traffic congestion. The next test is to investigate the traffic diversion (re-routing) patterns without and with TIMP under the critical degradations on Links 13 and 16. It is important to understand these characteristics since we can further find an optimal re-routing pattern to put more effort on the paths (or links) with increasing traffics to avoid consequent congestions and crashes.

Figure 5 presents the mean path flow changes under the normal and degradable network conditions. Figure 5 also shows that without TIMP all mean flows on Paths 7 and 8 diverges to Paths 5 and 6, while all mean flows on Paths 21 and 24 shifts to Paths 20 and 23. The re-routing pattern seems to be in an efficient way under TIMP. Although, all mean flows on Paths 7, 8, 21 and 24 moved to Paths 5, 6, 20, and 23 in the similar manner as described for the case without TIMP. With TIMP the traffic flows were distributed to more paths in order to minimize total travel time in the network. It was found that the mean flow on Path 17 moved to Paths 15 and 16 whereas the flow on Path 27 changed to Paths 26 and 28. For the link flow pattern, we found that with TIMP the mean flows on Links 6, 7, 9, 10, 11, 12, 14 and 15 increased. This result emphasizes that transport agencies should pay attention not only to removing traffic incidents from Links 13 and 16 for capacity recovery process, but also to preparing the technology required for carrying out such a re-routing strategy to deal with additional traffic flows, especially on the CBD-connected links (i.e. Links 6, 9, 11 and 12).

The next test was to determine the critical links connecting the CBD of the test network by using the TIML model (14)-(19). The maximum value of capacity degradation on each link (i.e. $s_a^{\max}$) was allowed to be half of the existing link capacity (i.e. c_a). Table 4 shows the critical values of link capacity degradations on the CBD-connected links. Links 13 and 16 are the first inbound and outbound links with the lowest critical capacity degradations (i.e. 158 and 175, respectively). These two critical links also serve high volumes of traffic under the normal network (from SN-UE and SN-SO flow results).

Finally, we tested the TIML model with the Sioux Falls network. The network as shown in Figure 6 consists of 24 nodes, 76 road links and 528 OD movements. The OD demands and link cost parameters are same as defined in Suwansirikul *et al*. (1987). This data is omitted here for brevity. From the network, both directions of ten selected candidate links, connecting node pairs (6,8), (7,8), (9,10), (10,16) and (13,24), are considered for the test. The maximum capacity degradation (i.e. $s_a^{\max}$) is allowed to reach the average capacity (i.e. c_a) for all candidate links.

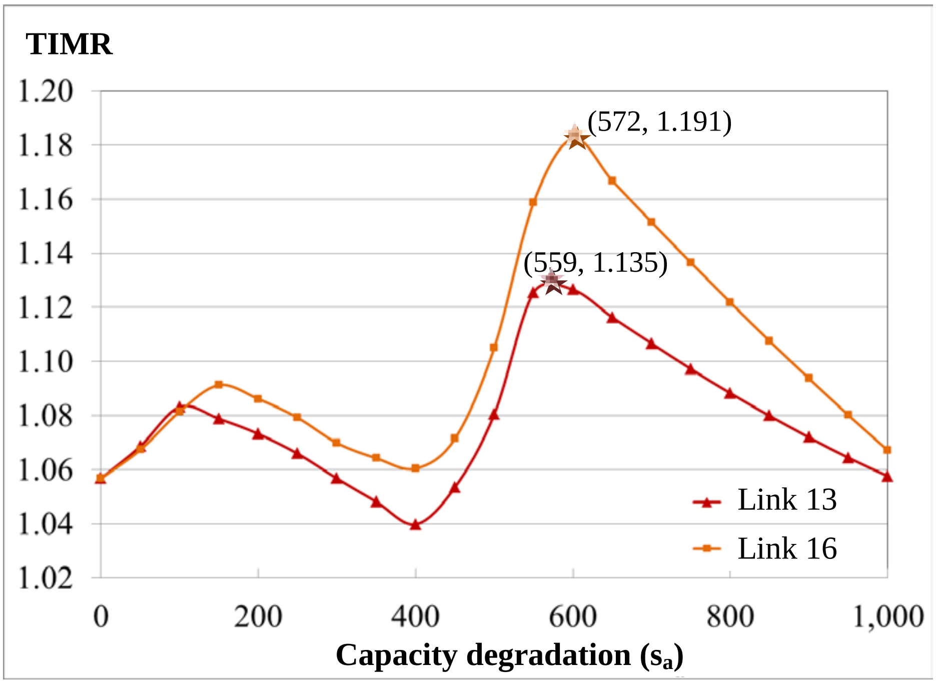


Figure 3. TIMR under the capacity degradation on either Links 13 or 16

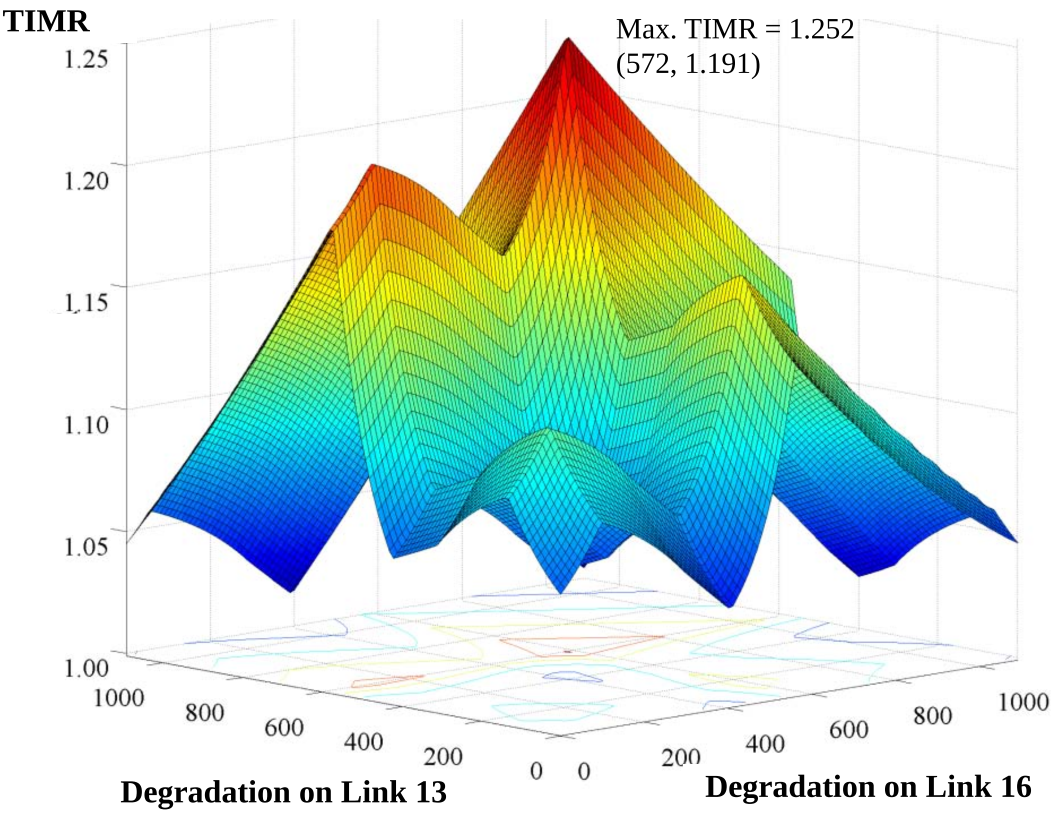


Figure 4. TIMR under the capacity degradations on Links 13 and 16

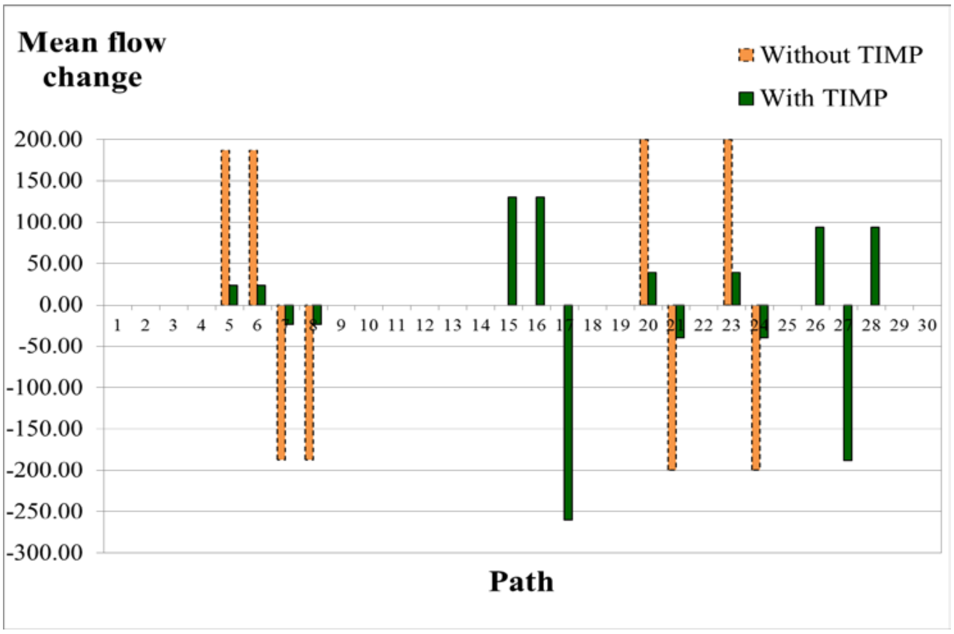


Figure 5. Mean path flow changes under critical degradations on Links 13 and 16

Table 4. Critical values of link capacity degradations obtained from TIML model

Link no.	Critical capacity degradation (s_a)
6	238
9	229
11	285
12	299
13	158
16	175
E[TT] under SN-UE	*49,065*
E[TT] under SN-SO	*40,348*
TIMR	*1.216*

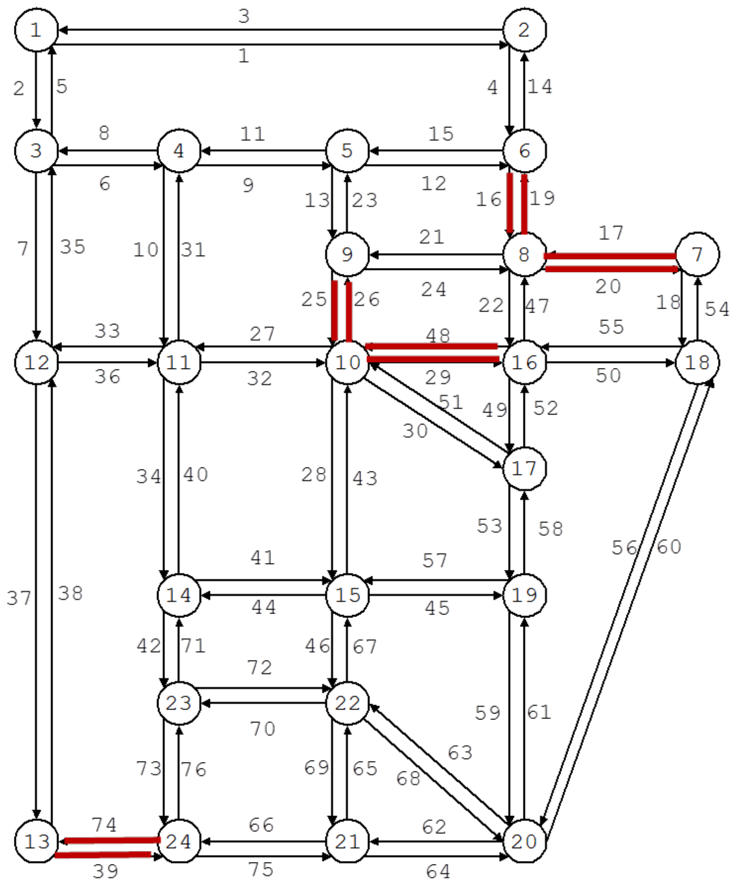
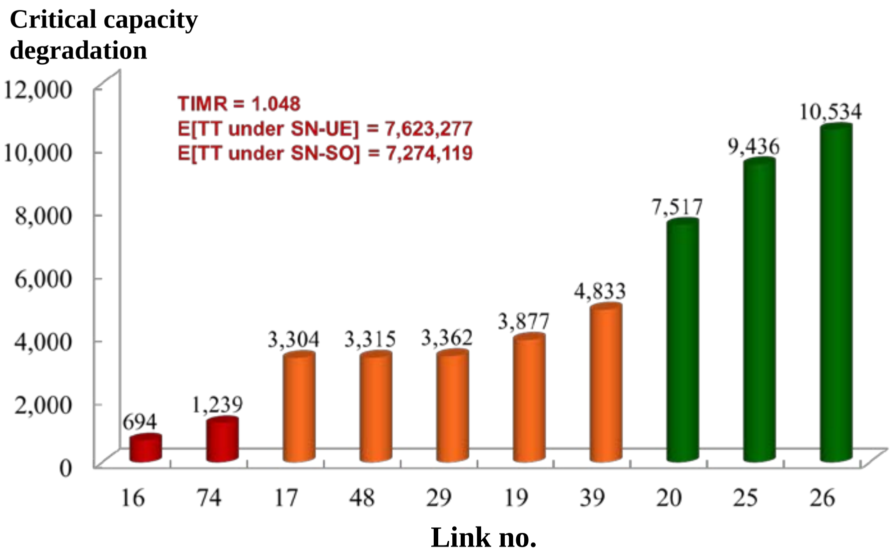
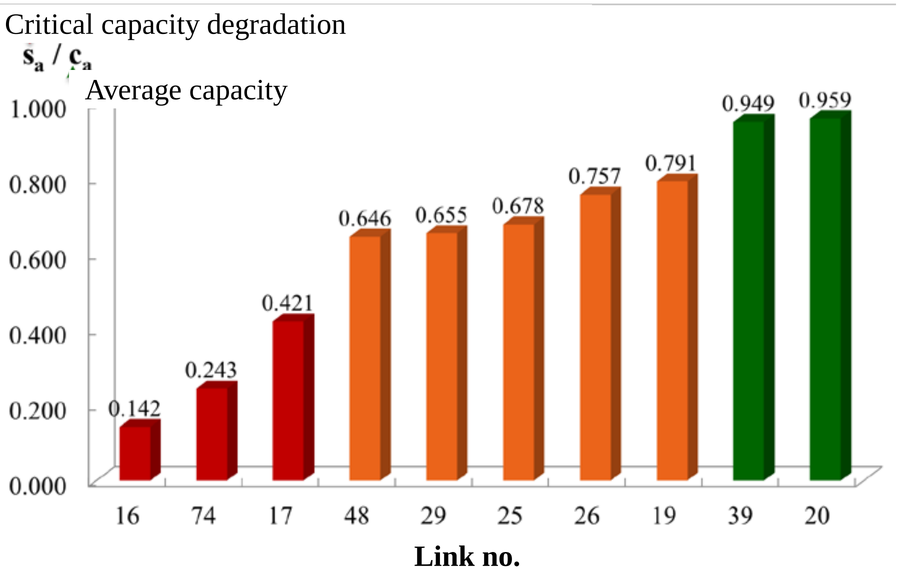


Figure 6. The Sioux Falls network

Figure 7a) shows the critical values of capacity degradations of candidate links in ascent order. Links 16 and 74 were the top two critical links required for the TIMP. However, different links have different values of the effective capacities after incidents occur (i.e. $c_a - s_a$). We thus propose the critical ratio of capacity degradation to average capacity (i.e. s_a/c_a) as a measure for identifying the critical links. Similarly, a link with low critical ratio is identified as the critical link. From Figure 7b), the top five critical links (16, 74, 17, 48 and 29, respectively) are in the same order as shown in Figure 7a). However, the last five important links are changed to 25, 26, 19, 39 and 20 in that order.



a) Critical value of link capacity degradation



b) Critical ratio of link capacity degradation to average capacity

Figure 7. Critical value and ratio of link capacity degradation

6. CONCLUSION AND DISCUSSION

This paper proposed TIMR as a new index for evaluating the performance of degradable stochastic road network without TIMP compared to that with TIMP. The TIMR was formulated by following the concept of POA, which determines the effective gap between the selfish routing (i.e. UE) and controlled routing (i.e. SO). The paper extended this concept to the degradable road network with the stochastic demand to reflect the variability and uncertainty in travel OD demands. TIML model was then proposed to find critical link capacity degradations which yield the maximize value of TIMR. The proposed model can also be used to identify the critical (important) links for long-term resource allocation of TIMP. The solution algorithm for solving the optimization model was also proposed on the basis of SQP and CCA. The numerical examples were then presented to highlight the necessity of TIMP. It was also noted that attention should focus not only on restoring traffic capacity on the critical links, but also on handling unexpected congestion incurred because of re-routing traffics on related paths or links.

The proposed model in this paper was based on the static equilibrium assignment framework. In addition, the TIMP was assumed to be perfectly efficient in dealing with traffic incidents. Future research should extend the proposed framework to the dynamic cases and apply it to large-scale transport networks.

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