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FUTURE PREDICTION OF PAVEMENT CONDITION USING MARKOV PROBABILITY TRANSITION MATRIX

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Abstract: Application of pavement management systems requires the ability of system to predict future pavement conditions. There are several methods available, and one of them is Markovian technique which requires probability transition matrix. Historical data of pavement conditions are required to build appropriate matrix. In case of pavement conditions are represented by Pavement Condition Index, it is possible to assign ten different pavement conditions with 10 points range. Matrix 10 by 10 has to be constructed, however, the solution is not unique. Each matrix solution will predict pavement condition, and has some deviation from actual condition. Pavement condition prediction from all matrix solutions are evaluated. Matrix which predicts pavement condition with the least deviation is the most appropriate matrix solution. From the most appropriate matrix, a deviation of 5.5 up to 10.6% of total length is obtained. This deviation level is still within an acceptable level for network level of pavement management system.

Keywords: Markov probability transition matrix, pavement management system

1. INTRODUCTION

In addition to the adverse effect of environment, new pavement structure deteriorates gradually after it opens to traffic. Degree of pavement deterioration can be expressed in either pavement structure condition or pavement roughness. New pavement structure should be smooth and having no visible distress. As cumulative passage of traffic continuously increases, it shows increasing level of load and environment related distress, such as alligator cracking, rutting, corrugation, raveling. At the beginning, the rate of pavement distress is very small, however in the later stage, it increases rapidly. Those pavement distresses reduce number of remaining load repetitions before the failure of pavement structure. While traffic users on that particular pavement structure will feel an increasing roughness and reduced riding comfort and safety. In order to maintain its riding comfort, safety, and pavement distress at an acceptable level, pavement maintenance is a must. As the total cost for pavement maintenance is usually beyond the available budget, then the organization that maintain pavement is urgently required an objective standard procedure to optimize spending on the limited budget. For this purpose, long term and life cycle cost analysis are required. Various maintenance scenarios must be examined.

Standard procedure for these purpose is called pavement management system. The system requires inventory of pavement structures, assessment of their current and projected pavement conditions, analysis of budget needs to maintain pavement conditions above an acceptable level, identification of maintenance work required, prioritization of road segement, and optimization of spending. Capability to predict future pavement condition is very important. There are several methods currently available such as straight line extrapolation, regression technique, and Markovian (Shahin, 1994). Markovian technique uses current pavement conditions to predict future condition by using probability transition matrix. The matrix manipulates data of current conditions to predict future pavement conditions. This paper demonstrates procedure to build Markov probability transition matrix, and how to choose the most appropriate Markov probability transition matrix could be used for a set of data available.

2. PAVEMENT MANAGEMENT SYSTEM

Currently pavement structures in Indonesia are simply maintained and are not managed. Some highway segments are not maintained properly as budget available is very limited. Those highway segments become badly deteriorated. Badly deteriorated pavement costs more to the institution for its maintenance as well as to the road user. There are two factors affected the effectiveness of the maintenance, i.e. proper type of maintenance work, and time of its application. Pavement maintenance engineer manually selects maintenance work alternatives based on current pavement condition and according to his best knowledge and experience. It is an *ad hoc* process, and the best and most cost effective option may be not be selected. As the scarce maintenance fund available, an optimum selection of maintenance work and its application is a must.

A systematic procedure and analysis of current and future pavement condition is provided by pavement management system. There are two levels of pavement management system, i.e. network level, and project level. At network level, all road segments are regularly surveyed. At project level, selected road segments that get priority for maintenance work is surveyed. This project level survey is costly and time consuming. Selection of road segments that get priority is the result of process in network level of pavement management system.

At network level, surveyed pavement condition data is updated to database of the entire pavement network. This database is used to predict future pavement condition. Analysis of different scenarios of maintenance and rehabilitation can be performed. Each scenario has total maintenance and rehabilitation cost. As annual budget available is almost always less than the minimum required fund, then there will be a process to make prioritization. Road segments within the network will have a certain type of maintenance and rehabilitation work. These decisions should be the best type of maintenance work and the most appropriate time to apply the work for the entire network.

|At network level of pavement management system, its analysis primarily depends on the ability and accuracy of future pavement condition. As this is an important part of the system, the accuracy of prediction of future pavement condition is a must. There are several methods available for this purpose, and one of them is Markovian technique. Markovian technique differs from others as it applies probabilistic instead of deterministic of pavement condition. It is almost

impossible to predict the future pavement condition to be precise as in deterministic theory. In this sense, Markovian probabilistic technique is more appropriate.

3. MARKOV PROBABILITY TRANSITION MATRIX

Pavement condition can be evaluated based on its structural condition, riding comfort, or a combination of them. Assessment of pavement condition for network level based on riding comfort may be suitable, as the process of data collection is fast, cheap, and reliable. Response type road roughness measuring system such as Mays meter can be used for this purpose. Other method of current pavement condition assessment also can be used for Markov technique, as long as parameter of pavement condition is numerical.

For instance, one can use Pavement Condition Index (PCI) with a numerical value of pavement condition between 1 – 100 (Shahin, 1994, and Butt et al., 1994). The best pavement condition is 100, and 1 is the worst. Then pavement condition is classified into states (classes or level). As an example there are 10 states with 10 points range. State 1 is between 91 and 100, State 2 is between 81 and 90, etc. After one duty cycle (one year or one season), the state of pavement condition will change into a better state only if type of maintenance work applied improves pavement structure, such maintenance work as overlay, or reconstruction, otherwise pavement condition is still the same at the current state or even drop to the next lower state. The probability that pavement condition is still at current state (i) is p(i). Hence, the probability it will go to the next lower state (i+1) is q(i) with q(i) = 1 – p(i). The expression of probability of being at the current state or at the next lower state for all 10 states is in Markov probability transition matrix, as shown in Figure 1.

From database, each road segment has information about its current condition. Data from all of those segments in the network is compiled and produce information about the total length of pavement with State (i), i=1, ... , 10. Mathematically it can be written as $\tilde{p}(0)$ a vector with 1 row and 10 columns. Total length of each state after one duty cycle can be predicted as $\tilde{p}(1) = \tilde{p}(0) \times P$. In general, the future condition after j duty cycle is $\tilde{p}(j) = \tilde{p}(0) \times P^j$.

$$P = \begin{pmatrix} p(1) & q(1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q(6) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p(7) & q(7) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p(8) & q(8) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p(9) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Figure 1. Markov Probability Transition Matrix

4. DEVELOPMENT OF MARKOV PROBABILITY TRANSITION MATRIX

Rate of pavement deterioration depends on type of pavement structure (such as rigid or flexible pavement, flexible pavement with asphaltic concrete or penetration macadam), and road classification (such as arterial, collector, or local). As Markov probability transition matrix is a tool to predict future pavement condition, the matrix is specific for a particular pavement structure and road classification. At the beginning when an insitution develops a pavement management system, probably pavement condition data is not available. In this case, the value of each cell in the matrix is constructed with rule of thumb. At the later stage of pavement management system development, as survey of pavement condition is routinely collected and a good database is developed, there is time to improvc the accuracy of future pavement condition by developing an appropriate Markov probability matrix. As explained earlier, there is no such data available in Indonesia. In order to explain the process of constructing the matrix, a hypothetical data is used.

For example there are a set of pavement condition data for 11 duty cycle. The first 10 duty cycle data will be used as input. The output will be the last 10 duty cycle data. Output matrix is the product of input matrix and Markov probability transition matrix. In the case it is assumed that Markov probability transition matrix is still unknown. However, both input and output matrices are known. Solving this unknown of 10 by 10 matrix is the way to construct Markov probability transition matrix. There are 100 cell values have to be determined. Imposing boundary condition to the cell values, most of them are zero, and 19 of them are greater than zero. As $p(i) + q(i)$ equals 1, there are 9 cells have to determined (and later other 9 cells are calculated) and $p(10)$ equals 1.0 (as shown in Figure 1).

After the matrix multiplication there are 100 equations. Gaussian elimination process is used. Boundary condition is applied for backward substitution process. As the solution is not an exact solution, the solution is not unique. Every matrix solution will produce deviation. The deviation is the difference between value of each cell in the output matrix and the product of input and transition matrix. Hence, it is interested to obtain the Markov probability transition matrix which has a minimum deviation, i.e. the minimum sum of square deviation. This matrix is called as the most appropriate Markov probability transition matrix. Later, this matrix could be used to predict of future pavement condition, as a part of pavement maagement system.

5. PAVEMENT CONDITION DATA

As there are no actual pavement condition data available in Indonesia, it is used hypothetical data as Table 1 for total network of 1000 kms. The first 10 years is the input matrix, while the last 10 years is the output matrix. The complete matrix multiplication is shown in Figure 2. Multiplication of input matrix with the first column of Markov probability transition matrix is shown in Table 2. The multiplication with other nine columns of transition matrix gives other 90 equations.

Gaussian elimination process of the first 10 equations are shown in Tables 3 to 6. Gaussian elimination for the other 90 equations is not shown here (see Pitaloka, 2003). Once the Gaussian elimination process for all equations is completed, backward substitution process begins.

Applying the boundary condition of Markov probability transition matrix, i.e. $X_{i,1}$=0 for i=2 to 10, then $X_{1,1}$=8/9=0.9 (with the accuracy of one decimal). The first solution for p(1) is the same as $X_{1,1}$. By definition, q(1) = 1 – 0.9 = 0.1.

Table 7 shows the result of matrix multiplication with second column of Markov transition matrix. Applying the boundary condition and finally $X_{2,2}$=13.889/5.56. As value of any X should be ≤1.0, then its result from this equation is not valid. As q(1)=$X_{1,2}$=0.1, then $X_{2,2}$={145-90(0.1)}/175=0.8, and q(2)=$X_{2,3}$=1-0.8=0.2. One of the solution obtained is shown in Figure 3. With one decimal accuracy, there are 89 other solutions available as shown in Pitaloka (2003). More solution will be available for accuracy of two or more decimals.

Table 1. Length of Each State Pavement Condition for the Last Eleven Years (in kms)

Year	States									
	1 (91-100)	2 (81-90)	3 (71-80)	4 (61-70)	5 (51-60)	6 (41-50)	7 (31-40)	8 (21-30)	9 (11-20)	10 (1-10)
0	100	200	250	450	0	0	0	0	0	0
1	90	175	215	375	120	25	0	0	0	0
2	80	145	195	310	190	65	10	5	0	0
3	70	125	170	260	225	110	35	5	0	0
4	65	110	145	220	230	145	65	20	0	0
5	60	90	130	190	225	170	90	35	10	0
6	55	80	110	160	210	185	120	55	20	5
7	50	70	95	135	195	190	140	80	35	10
8	45	60	85	115	170	190	155	100	50	30
9	40	50	75	100	155	180	165	120	65	50
10	35	45	65	90	140	170	165	130	85	75

Table 2. The First Ten Equations of Matrix Multiplication

$x_{1,1}$	$x_{2,1}$	$x_{3,1}$	$x_{4,1}$	$x_{5,1}$	$x_{6,1}$	$x_{7,1}$	$x_{8,1}$	$x_{9,1}$	$x_{10,1}$		
100	200	250	450	0	0	0	0	0	0	=	90
90	175	215	375	120	25	0	0	0	0	=	80
80	145	195	310	190	65	10	5	0	0	=	70
70	125	170	260	225	110	35	5	0	0	=	65
65	110	145	220	230	145	65	20	0	0	=	60
60	90	130	190	225	170	90	35	10	0	=	55
55	80	110	160	210	185	120	55	20	5	=	50
50	70	95	135	195	190	140	80	35	10	=	45
45	60	85	115	170	190	155	100	50	30	=	40
40	50	75	100	155	180	165	120	65	50	=	35

Table 3. The Equations Used in Gaussian Elimination

$x_{1,1}$	$x_{2,1}$	$x_{3,1}$	$x_{4,1}$	$x_{5,1}$	$x_{6,1}$	$x_{7,1}$	$x_{8,1}$	$x_{9,1}$	$x_{10,1}$		
90	175	215	375	120	25	0	0	0	0	=	80
100	200	250	450	0	0	0	0	0	0	=	90
80	145	195	310	190	65	10	5	0	0	=	70
70	125	170	260	225	110	35	5	0	0	=	65
65	110	145	220	230	145	65	20	0	0	=	60
60	90	130	190	225	170	90	35	10	0	=	55
55	80	110	160	210	185	120	55	20	5	=	50
50	70	95	135	195	190	140	80	35	10	=	45
45	60	85	115	170	190	155	100	50	30	=	40
40	50	75	100	155	180	165	120	65	50	=	35

Table 4. The First Step of Gaussian Elimination

	$x_{1,1}$	$x_{2,1}$	$x_{3,1}$	$x_{4,1}$	$x_{5,1}$	$x_{6,1}$	$x_{7,1}$	$x_{8,1}$	$x_{9,1}$	$x_{10,1}$			
R1	90	175	215	375	120	25	0	0	0	0	=	80	
R2	100	200	250	450	0	0	0	0	0	0	=	90	R2-R1(100/90)
R3	80	145	195	310	190	65	10	5	0	0	=	70	R3-R1(80/90)
R4	70	125	170	260	225	110	35	5	0	0	=	65	R4-R1(70/90)
R5	65	110	145	220	230	145	65	20	0	0	=	60	R5-R1(65/90)
R6	60	90	130	190	225	170	90	35	10	0	=	55	R6-R1(60-90)
R7	55	80	110	160	210	185	120	55	20	5	=	50	R7-R1(55/90)
R8	50	70	95	135	195	190	140	80	35	10	=	45	R8-R1(50/90)
R9	45	60	85	115	170	190	155	100	50	30	=	40	R9-R1(45/90)
R10	40	50	75	100	155	180	165	120	65	50	=	35	R10-R1(40/90)

Table 5. The Result of First Step Gaussian Elimination

$x_{1,1}$	$x_{2,1}$	$x_{3,1}$	$x_{4,1}$	$x_{5,1}$	$x_{6,1}$	$x_{7,1}$	$x_{8,1}$	$x_{9,1}$	$x_{10,1}$		
90.00	175.00	215.00	375.00	120.00	25.00	0.00	0.00	0.00	0.00	=	80.00
0.00	5.56	11.11	33.33	-133.33	-22.22	0.00	0.00	0.00	0.00	=	1.11
0.00	-10.56	3.89	-23.33	83.33	47.22	10.00	5.00	0.00	0.00	=	-1.11
0.00	-11.11	2.78	-31.67	131.67	94.44	35.00	5.00	0.00	0.00	=	2.78
0.00	-16.39	-10.28	-50.83	143.33	130.56	65.00	20.00	0.00	0.00	=	2.22
0.00	-26.67	-13.33	-60.00	145.00	156.67	90.00	35.00	10.00	0.00	=	1.67
0.00	-26.94	-21.39	-69.17	136.67	172.78	120.00	55.00	20.00	5.00	=	1.11
0.00	-27.22	-24.44	-73.33	128.33	178.89	140.00	80.00	35.00	10.00	=	0.56
0.00	-27.50	-22.50	-72.50	110.00	180.00	155.00	100.00	50.00	30.00	=	0.00
0.00	-27.78	-20.56	-66.67	101.67	171.11	165.00	120.00	65.00	50.00	=	-0.56

$$\begin{pmatrix} 100 & 200 & 250 & 450 & 0 & 0 & 0 & 0 & 0 & 0 \\ 90 & 175 & 215 & 375 & 120 & 25 & 0 & 0 & 0 & 0 \\ 80 & 145 & 195 & 310 & 190 & 65 & 10 & 5 & 0 & 0 \\ 70 & 125 & 170 & 260 & 225 & 110 & 35 & 5 & 0 & 0 \\ 65 & 110 & 145 & 220 & 230 & 145 & 65 & 20 & 0 & 0 \\ 60 & 90 & 130 & 190 & 225 & 170 & 90 & 35 & 10 & 0 \\ 55 & 80 & 110 & 160 & 210 & 185 & 120 & 55 & 20 & 5 \\ 50 & 70 & 95 & 135 & 195 & 190 & 140 & 80 & 35 & 10 \\ 45 & 60 & 85 & 115 & 170 & 190 & 155 & 100 & 50 & 30 \\ 40 & 50 & 75 & 100 & 155 & 180 & 165 & 120 & 65 & 50 \end{pmatrix} \times \begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} & x_{1,5} & x_{1,6} & x_{1,7} & x_{1,8} & x_{1,9} & x_{1,10} \\ x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} & x_{2,5} & x_{2,6} & x_{2,7} & x_{2,8} & x_{2,9} & x_{2,10} \\ x_{3,1} & x_{3,2} & x_{3,3} & x_{3,4} & x_{3,5} & x_{3,6} & x_{3,7} & x_{3,8} & x_{3,9} & x_{3,10} \\ x_{4,1} & x_{4,2} & x_{4,3} & x_{4,4} & x_{4,5} & x_{4,6} & x_{4,7} & x_{4,8} & x_{4,9} & x_{4,10} \\ x_{5,1} & x_{5,2} & x_{5,3} & x_{5,4} & x_{5,5} & x_{5,6} & x_{5,7} & x_{5,8} & x_{5,9} & x_{5,10} \\ x_{6,1} & x_{6,2} & x_{6,3} & x_{6,4} & x_{6,5} & x_{6,6} & x_{6,7} & x_{6,8} & x_{6,9} & x_{6,10} \\ x_{7,1} & x_{7,2} & x_{7,3} & x_{7,4} & x_{7,5} & x_{7,6} & x_{7,7} & x_{7,8} & x_{7,9} & x_{7,10} \\ x_{8,1} & x_{8,2} & x_{8,3} & x_{8,4} & x_{8,5} & x_{8,6} & x_{8,7} & x_{8,8} & x_{8,9} & x_{8,10} \\ x_{9,1} & x_{9,2} & x_{9,3} & x_{9,4} & x_{9,5} & x_{9,6} & x_{9,7} & x_{9,8} & x_{9,9} & x_{9,10} \\ x_{10,1} & x_{10,2} & x_{10,3} & x_{10,4} & x_{10,5} & x_{10,6} & x_{10,7} & x_{10,8} & x_{10,9} & x_{10,10} \end{pmatrix} =$$

$$\begin{pmatrix} 90 & 175 & 215 & 375 & 120 & 25 & 0 & 0 & 0 & 0 \\ 80 & 145 & 195 & 310 & 190 & 65 & 10 & 5 & 0 & 0 \\ 70 & 125 & 170 & 260 & 225 & 110 & 35 & 5 & 0 & 0 \\ 65 & 110 & 145 & 220 & 230 & 145 & 65 & 20 & 0 & 0 \\ 60 & 90 & 130 & 190 & 225 & 170 & 90 & 35 & 10 & 0 \\ 55 & 80 & 110 & 160 & 210 & 185 & 120 & 55 & 20 & 5 \\ 50 & 70 & 95 & 135 & 195 & 190 & 140 & 80 & 35 & 10 \\ 45 & 60 & 85 & 115 & 170 & 190 & 155 & 100 & 50 & 30 \\ 40 & 50 & 75 & 100 & 155 & 180 & 165 & 120 & 65 & 50 \\ 35 & 45 & 65 & 90 & 140 & 170 & 165 & 130 & 85 & 75 \end{pmatrix}$$

Figure 2. Output Matrix and Multiplication of Input and Markov Probability Transition Matrix

Table 6. The Final Result of Gaussian Elimination

$x_{1,1}$	$x_{2,1}$	$x_{3,1}$	$x_{4,1}$	$x_{5,1}$	$x_{6,1}$	$x_{7,1}$	$x_{8,1}$	$x_{9,1}$	$x_{10,1}$		
90.000	175.000	215.000	375.000	120.000	25.000	0.000	0.000	0.000	0.000	=	80.000
0.000	5.556	11.111	33.333	-133.333	-22.222	0.000	0.000	0.000	0.000	=	1.111
0.000	0.000	25.000	40.000	-170.000	5.000	10.000	5.000	0.000	0.000	=	1.000
0.000	0.000	0.000	-5.000	35.000	45.000	25.000	0.000	0.000	0.000	=	4.000
0.000	0.000	0.000	0.000	-16.500	164.000	113.500	15.500	0.000	0.000	=	13.800
0.000	0.000	0.000	0.000	0.000	654.242	453.485	54.242	10.000	0.000	=	58.455
0.000	0.000	0.000	0.000	0.000	0.000	16.358	12.795	14.370	5.000	=	0.091
0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.275	4.308	0.244	=	0.128
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.970	14.837	=	0.238
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	92.929	=	-0.040

Table 7. Final Gaussian Elimination for Second Column of Markov Transition Matrix

$x_{1,2}$	$x_{2,2}$	$x_{3,2}$	$x_{4,2}$	$x_{5,2}$	$x_{6,2}$	$x_{7,2}$	$x_{8,2}$	$x_{9,2}$	$x_{10,2}$		
90.000	175.000	215.000	375.000	120.000	25.000	0.000	0.000	0.000	0.000	=	145
0.000	5.556	11.111	33.333	-133.333	-22.222	0.000	0.000	0.000	0.000	=	13.889
0.000	0.000	25.000	40.000	-170.000	5.000	10.000	5.000	0.000	0.000	=	22.5
0.000	0.000	0.000	-5.000	35.000	45.000	25.000	0.000	0.000	0.000	=	2.5
0.000	0.000	0.000	0.000	-16.500	164.000	113.500	15.500	0.000	0.000	=	11.75
0.000	0.000	0.000	0.000	0.000	654.242	453.485	54.242	10.000	0.000	=	52.652
0.000	0.000	0.000	0.000	0.000	0.000	16.358	12.795	14.370	5.000	=	6.1909
0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.275	4.308	0.244	=	-3.781
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.970	14.837	=	-5.384
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	92.929	=	-28.283

$$
P_1 = \begin{pmatrix}
0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$

Figure 3. One of Possible Solution for Markov Probability Transition Matrix

Each of those solution is not the exact solution, and each of them has deviation. The complete solution is available at Pitaloka (2003). From those 90 matrices, it is evaluated which has the minimum of sum of square deviation. The solution of 81st matrix from Pitaloka (2003) produces the minimum deviation of predicted future pavement condition. The matrix is shown in Figure 4.

In order to describe error level obtained from this most appropriate matrix, the hypothetical output data is used. The input matrix data is multiplied with the best Markov probability transition matrix. The result of output matrix is the predicted pavement condition. Accurate solution will have both matrices exactly the same. However, the result shows some differences as shown in Table 8. The difference between actual and predicted pavement condition is also shown in Figure 5. The last column of Table 8 shown the percentage of deviation for every year of pavement condition predicted. The deviation is between 5.5 – 10.6%, it depends on the how many duty cycle of pavement condition predicted.

$$P_{81} = \begin{pmatrix} 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Figure 4. The Matrix with Minimum Sum of Square Deviation

Table 8. The Difference Between Actual and Predicted Pavement Condition

Year	Data Length of Pavement Condition (kms)										Predicted Length of Pavement Condition (kms)										% Deviation
	State										State										
	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	
1	90	175	215	375	120	20	0	0	0	0	90	170	215	390	135	0	0	0	0	0	5.5
2	80	145	195	310	190	65	10	5	0	0	81	149	186	327	209	39	10	0	0	0	8.1
3	70	125	170	260	225	110	35	5	0	0	72	124	166	276	245	77	31	7.5	2.5	0	8.5
4	65	110	145	220	230	145	65	20	0	0	63	107	144	233	258	111	62	20	2.5	0	8.7
5	60	90	130	190	225	170	90	35	10	0	59	95	124	198	250	133	91	43	10	0	9.0
6	55	80	110	160	210	185	120	55	20	5	54	78	109	172	237	147	113	63	23	5	9.8
7	50	70	95	135	195	190	140	80	35	10	50	70	93	145	216	153	134	88	38	15	9.2
8	45	60	85	115	170	190	155	100	50	30	45	61	81	123	197	153	146	110	58	28	10.6
9	40	50	75	100	155	180	165	120	65	50	41	53	72	106	171	148	154	128	75	55	9.4
10	35	45	65	90	140	170	165	130	85	75	36	44	63	93	154	139	155	143	93	83	9.0

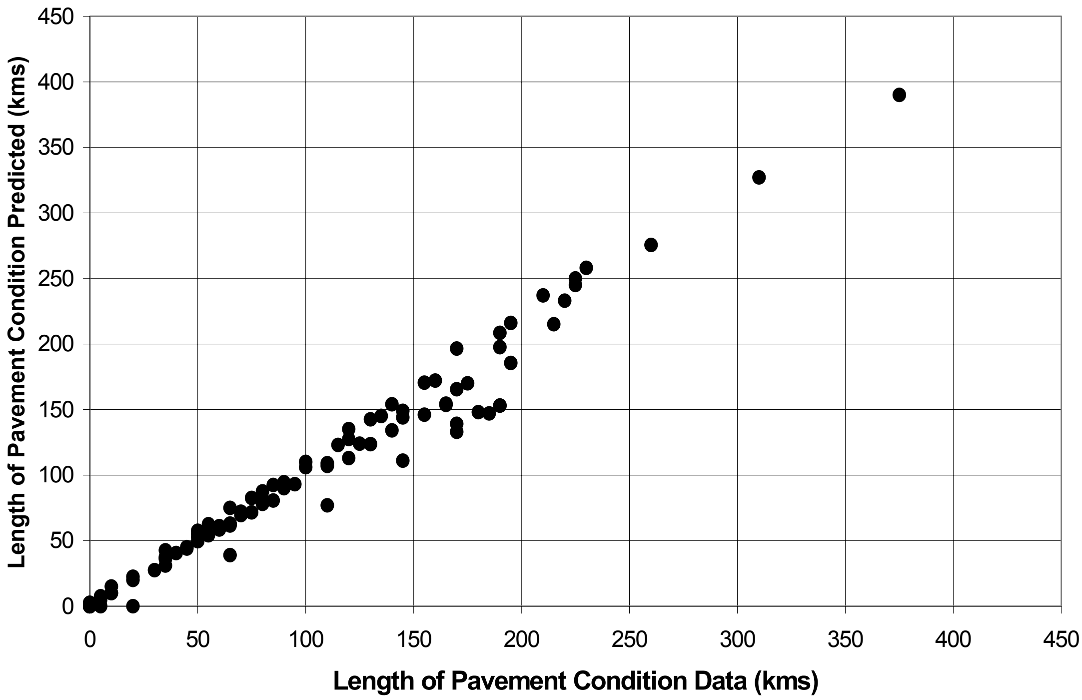

Figure 5. Actual and Predicted Length of Pavement Condition

6. CONCLUSIONS

1. A non unique solution still can be solved by Gaussian elimination, and imposing boundary condition of the Markov matrix to the backward substitution process. Each of the Markov probability transition matrix solution gives deviation. All possible solutions (in this case with 1 decimal accuracy there are 90 different solutions available) must be evaluated before the most appropriate Markov probability transition matrix with the minimum sum of square deviation can be determined.
2. The most appropriate Markov probability transition matrix (in this case the 81st solution) has deviation between 5.5 – 10.6 % of the actual total length for each year predicted. For network level of pavement management system, such deviation is still acceptable. The most appropriate matrix assures that predicted future pavement condition is within the range of acceptable error.

REFERENCES

Butt, A.A., M.Y. Shahin, S.H. Carpenter, J.V. Carnahan (1994) Application of Markov Process to Pavement Management Systems at Network Level. **Proceedings of Third International Conference on Managing Pavements**. Vol. II. Transportation Research Board, National Research Council. Washington, DC.

Pitaloka, D (2003) Analisis Matriks Transisi Probabilitas pada Prediksi Kondisi Perkerasan dengan Menggunakan Metode Markov. In Indonesian. **Undergraduate Thesis**, Civil Engineering School, Universitas Katolik Parahyangan. Bandung, Indonesia

Shahin, M.Y. (1994) **Pavement Management for Airports, Roads and Parking Lots**, Chapman and Hall, New York, NY.