# STUDY ON BI-LEVEL PLANNING MODEL & ALGORITHM

## **OPTIMIZING HIGHWAY NETWORK LAYOUT**

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**Abstract:** With the rapid development of socioeconomy and communications and transportation, the demand for highway network planning and construction continuously increases. The single objective optimization model was adopted in the past, which can not reflect the objective and overall demand of highway constructions and users. Based on this, the demand of provincial or urban level highway network layout is firstly analyzed in this paper, then through optimizing the continuous variables and introducing the bi-level planning model, this paper establishes bi-level planning model which is fit for provincial or urban level highway network layout optimization. Finally the solution algorithm is studied and the simulated annealing algorithm is applied to highway network layout optimization problem in this paper, applying program language of Visual Basic 6.0, this paper developed corresponding algorithm program. The algorithm is especially apporipriate for solving large scale network optimization problems.

**Key works:** highway network, layout optimization, bi-level planning model, simulated annealing algorithm

## **1.INTRODUCTION**

Optimizing highway network layout is a plane outline design of highway network, it will be a process of the to-be highway network plane layout project after we have researched and analyzed the status of highway network, forecasted the socioeconomy and transportation demand, depended on definite optimizing objective and restricted condition, connected the selected controlling crunodes by appropriate methods and programming routes.

The optimizing project was often founded with single objective optimization model when the position of route was optimized in the past. With the rapid development of socioeconomy and communications and transportation, the demand for highway network planning and construction continuously increases, the constructing of highway can ease the pressure and satisfy the demand of transportation, at the same time, people continuously seek the shortcut for coming out and comfort, security, shortcut, economize for traveling. That is to say, the aim of the highway network layout optimization should be increasing, and the single

objective optimization should be enlarged to multi-objective optimization. The multi-objective optimization can reflect the desire and intention of the erectors and the messengers more external than the single objective optimization, the fact has been proved from the analysis to overseas highway network layout optimization and internal highway network layout optimization. So it is necessary that we erect multi-objective optimization model of the highway network layout.

Based on this, the function and trait of highway network were analyzed in this paper, and at the same time, we have studied the aim of the highway network layout optimization and this aim is appropriate, and we also erected relevant multi-objective layout optimization model.

## 2.FOUNDATION OF BI-LEVEL PLANNING MODE

For provincial or urban level highway, the keystone of layout should be the analysis of distribution between the crunodes of traffic and pursue the best managing effect of overall highway network, and based on this it is carried out. Current layout should not be microscopic but microcosmic lay.

Contrasting to national level highway, the functional and technical grading for every route that should be studied are more complex in the provincial or urban level highway, the grading structure of highway network and the collocation of every route take on very important effect on the overall highway network. So that, when carrying out optimizing for the provincial or urban level highway network, we must take into account the issue of technical grading and specially layout of the highway network. We should ensure that crunodes is connected or not, what is the technical grading, the crunodes that have been connected have been rebound or not and the grading, these are the primary duties to layout. Then, the content of the layout concludes two sides: the first, ensure new routes have been disposed or not and existing routes have been rebound or not; the second, what is the technical grading of these.

## 2.1 Current Situation of Domestic and International Studies

Yang and Meng used bi-level planning model to study BOT network design; Gwo-Hshiung Tzeng et al. used multi-projective maths planning to design continuous network model, utilizing bi-level planning idea to explain and discuss network improvement with multi-projective decision-making, established nonlinear bi-level planning mode; Sanjay Melkote and Mark S. Daskin brought forward mixed integer planning model, which was used to compute establishment layout and network design optimization; Zhang Xiao-ning presented a unified description of the transportation bi-level optimization problems with user equilibrium constraints. Specifically, the network link capacity expansion problem, the road toll pricing problem and the optimal signal timing problem were discussed.

## 2.2 Discretizing the Continuous Variables

In course of highway network optimizing at overseas, increasing new route and rebuilding

existing route are analyzed and studied singly, that is to say, do not take count of influence in each other. When optimizing the highway network in China, most of people go along to increasing new route lonely, and take account of that local rebuilding of existing route affect highway network layout hardly. Actually, so you say, the highway network layout not only includes rebuilding existing route but also lays new route for provincial or urban level highway network. Layout optimization is based on existing highway network, rebuilding existing route will affect the location, the scope and laying or not laying of the new route importantly. Then, during highway network optimization, we must take account of rebuilding existing route and laying new route. In this way, highway network layout optimization of provincial or urban level highway network has become mixed network design problem (MNDP). The MNDP is very complex and very difficult in modeling and computing, so it attracts few people's eyes. Because of this, there is no study in this field presently.

For modeling and predigesting complexity & difficulty, connected network design problem (CNDP) of mix network design problem (MNDP) will be translated into discrete network design problem (DNDP). The mean is to translate the variable of CNDP into the variable of DNDP. In this way, the problem will become DNDP that concludes discrete variable only.

### 2.3 Aim of Optimization

We should analyze microcosmic provincial or urban level highway network, and we can carry out from the angle of network messengers and network erecters, study and confirm the aim of layout optimization. From the angle of messengers, the charge or time for travel should be minimal; from the angle of erecters, the total charge or time for travel should be minimal; at the same time, we should pursue the maximal network erecting benefit (including socio-benefit and economy benefit), exclusion of this, we should try our best to making crunodes connected. Because erecting benefit and total travel charge of network system are relevant with each other, reducing the time of travel spells increasing the benefit of erecting, the reducing of erecting benefit contrast to the increasing of travel time. So we can analyze and optimize one of the two. Because the erecting benefit can hardly analyze and compute in ration, we regard the travel total charge as an optimization aim in this paper. In this way, we can say that the optimization aim is: the travel charge of single messenger is minimal, the travel charge or time of network system is minimal and the connected limitation of network system is maximal.

#### 2.4 Introducing Bi-level Planning Mode

Because the factual programming and decision-making are all very huge or complex, and it concludes all kinds of factors, and it involves the benefits of many departments, units and people, so the decision-making that is adopted should be not single arrangement but multi-arrangement. Generally speaking, decision-making institution is an administer institution that is grading and layered, in the precondition of consistent aim, all levels have their own or conflict aims. So, if we consider all kinds of opinions that involve all levels departments, units or people, and which may single or effective each either, it is certainly to make a scientific and final decision, do our best to make the decision of overall system to

become the most excellent.

The mathematic method that contrasts to the multi-system decision is multi-levels programming. The decision-makers optimizing their own functions optimize the decision-variable that can be controlled after the high-grade decision-makers have gotten the cost of decision-variable, in this way, they can get the most excellent effect. Bi-level planning mode is an especial example of the multi-level planning. There are only two arrangements and two kinds of decision-maker. Due to involve the reciprocity between the government and public or their associated action when deciding traffic investment, this is a representative Bi-level decision problem, so Bi-level planning mode is an ideal tool when we depict the course of traffic investment.

Bi-level planning model is better than traditional single-level planning model. The material behaves as the following (Huang, 1994): 1). It can analyze two different and conflict each either aims in course of decision. 2) The decision-making mean of Bi-level planning multi-merit rule approach practice furthermore. 3) The function between government and public can be denoted definitely.

Highway network layout optimization problem involves two kinds of decision-makers, that is, network messenger and the people of network planning, that they have different objective function. So bi-level planning model is used to depict the highway network layout optimization problem of the provincial or urban level highway, thereby, the substrate problem is: the travel behavior of network messenger is in accordance with user most excellent rule; the upper problem is: considering from the system angle (that is from the angle of upper decision-maker), in condition of satisfying the restriction of investment, making the time for travel minimal and the network connected limitation maximal.

### 2.5 Stochastic Messenger Balance Model

In the bi-level planning model of provincial or urban highway network layout optimization, the core of the substrate model is balanceable-mix-flow model of the highway network, the balanceable-mix-flow model can be classified two kinds of models, the first model is from the angle of users and the second model is from the angle of highway network system. And the user balanceable-mix-flow model can be classified into two kinds of models: Determinate User Equation (DUE) and Stochastic User Equation (SUE). According to the trait of the model, SUE is used for substrate optimization model of highway network layout bi-level planning model. The form of the model is:

$$\min Z(f) = -\sum_{ij} q_{ij} \cdot E[\min_{k \in K_{ij}} \{c_k^{ij}\} | c^{ij}(f)] + \sum_a f_a c_a(f_a) - \sum_a \int_0^{f_a} c_a(x) dx$$
(1)

In the formula: Z(f)—function of optimization objective,  $q_{ij}$ —the OD quantity of travel between section *i* and section *j*,  $E[\min\{c_k^{ij}\} | c^{ij}(f)]$ —the impedance of highway expectation comprehend,  $f_a$ —the flowing keK quantity of *a* path,  $c_a(x)$ —the charge or impedance of *a* path.

#### 2.6 The Form of Bi-level Planning Model

According to aforesaid, the highway network layout bi-level planning model that can be erected is: upper optimization model (the total time of highway network system is the minimal, and the connecting limitation of highway network system is the maximal.).

$$\begin{cases} \min Z(X_a^l, Y_a^l) = \sum_{a \in A} t_a(v_a(X_a^l, Y_a^l), X_a^l, Y_a^l)v_a(X_a^l, Y_a^l) \\ \sum_{a \in A}^n m_i \\ \max J = \frac{i = i}{n} \end{cases}$$

$$\sum_{\substack{a \in A \\ l \in \varphi_a}} (C_a^l X_a^l + g_a^l Y_a^l) \leq B$$

$$\sum_{\substack{a \in A \\ l \in \varphi_a}} (L_a^l X_a^l + L_{yy}) \leq L_0$$

$$l_i^* \leq \sum_{\substack{a \in A \\ l \in \varphi_a}} L_a^l \leq L_i^* \quad (l = 0, 1, 2, 3, 4, 5)$$

$$X_a^l = \begin{cases} 1 & a \\ 0 & \text{orelse} \end{cases}$$

$$further \ more \ X_a^l \in \{0, 1\} \quad \forall a \in A \\ y_a^l = \begin{cases} 1 & \text{in the widen section of the highway}, \sigma \ is \ l \ form \\ 0 & \text{or else} \end{cases}$$

$$further \ more \ Y_a^l \in \{0, 1\} \quad \forall a \in A, \quad \forall l \in \{0, 1, 2, 3, 4, 5\} \end{cases}$$

$$(2)$$

The substrate optimization model (Stochastic User Equation: the travel time of the single user is the minimal.)

$$\min Z(f) = -\sum_{ij} q_{ij} \cdot E[\min_{k \in K_{ij}} \{c_k^{ij}\} | c^{ij}(f)] + \sum_a f_a c_a(f_a) - \sum_a \int_0^{f_a} c_a(x) dx$$
(3)

The meaning of every last letter: Z——the total travel time of highway network, J——The connecting limitation of highway network,  $t_a(v_a(X_a^l, Y_a^l), X_a^l, Y_a^l)$ —The travel time in a section of highway is the function of traffic  $v_a$  and the decision variable  $X_a^l$  or  $Y_a^l$ ,  $m_i$ —The number of border near the No. I crunode, n—The number of crunode that must be connected in the planning area,  $c_a^l$ —How much is the cost of a newly-built section of highway pre-path while the section of highway is No. l grading (ten thousand yuan / km),  $g_a^l$ —How much is the cost of a extent section of highway pre-path while the section of highway pre-path while the section of highway is No. l grading (ten thousand yuan / km),  $g_a^l$ —How much is the cost of a extent section of highway pre-path while the section of highway is No. l grading (ten thousand yuan / km),  $g_a^l$ —The mileage of a section of highway and l render the grading of a, km,  $L_{yy}$ —Intrinsic total mileage of the highway network, km,  $L_0$ —Total planning mileage of the highway network, km,  $l_i^*$ —The mileage floor level cost of l grading highway in the highway network, km,  $l_i^*$ —The mileage upper limit cost of l grading highway in the highway network, km,

# Z(f), $q_{ij}$ , $E[\min_{k \in K_{ij}} \{c_k^{ij}\} | c^{ij}(f)]$ , $f_a$ , $c_a(x)$ The meanings idem.

### **3. SOLVING AND COMPUTING THE MODEL**

Bi-level planning model is a multi-level planning model, in the multi-objective optimization problem, it does not exist to get the most excellent cost and the most excellent solution, especially the objectives conflict each either. At this time, we often use the efficient concept, the optimizing concept to one solution is that we can not mend any objective function if we do not give up the best objective function.

As we all known, there are many finer solution about the single-objective optimization problem, so, in solving multi-objective optimization problem, we often translate multi-objective optimization problem into single-objective optimization problem. The same means is adopted in this paper.

### 3.1 Transform of the Objective Function:

In multiply and division, the upper model objective function of highway network layout bi-level planning model can be translated into:

$$\min Z(X_a^l, Y_a^l) = \sum_{a \in A} t_a(v_a(X_a^l, Y_a^l), X_a^l, Y_a^l)v_a(X_a^l, Y_a^l)n \bigg/ \sum_{i=1}^n m_i$$
(4)

### 3.2 Solving the Substrate Optimization Model

The solving course of highway network layout bi-planning model is: the first, solving substrate optimization model; the second, applying the result of substrate optimization model to solving upper-optimization model; finally, we can get the most excellent result of the whole optimization problem.

The substrate model can be solved with the fast descend theory of non-restraint teeny problem, the teeny numerical value problem is solving with average arithmetic one after another in this paper. This arithmetic is applied extensively at present, thus, the arithmetic about this need not be introduced anymore.

# **3.3** Solving Upper Optimization Model——the Applying of the Simulated Annealing Algorithm

Simulated Annealing (SA) is an universal searching arithmetic with that, it has more excellent generally action (Kang, L.S. *et al.*, 2000). The idea of the arithmetic is brought forward by N.Metoplis and others in 1953. This arithmetic develops generally in 1980s, and it has become an arithmetic of a huge combination optimization problem (Daniela, M. *et al.*, 2002). It based on the conformation of the huge optimization problem solving course and the physics annealing coursing, the simulate annealing course can be carried on with Metropolis rule and

control the descend of the temperature, thereby getting the complete optimization result, this arithmetic has been used in huge combination optimization problem (Yu, 2001), for example, attempt, compositor and so on (Eulogio, 1998).

## (1) Arithmetic depicting

In the searching policy, the simulated annealing and the traditional stochastic searching arithmetic are different, the simulated annealing introduces the natural mechanism of the physics system annealing coursing, in the alternating era coursing, it does not only accept the feeler of making the objective function to become "better", but accept the feeler of make the objective function to become "worse", in some probability, and accepts the probability reducing with the temperature descending. The searching policy of simulate annealing arithmetic is good to avoid the abuse that slumping local optimization in the searching coursing, and the security of getting the whole most excellent result is brought forward.

Supposing: an cost i of combination optimization problem and objective function f(i) equip a microcosmic state i of solid and its energy  $E_i$  respectively. Regarding the controlling parameter t of the stochastic coursing of descending cost as the temperature T of sold annealing coursing, then, any cost of the controlling parameter t, and the continuing arithmetic iteration coursing, "producing new unlay-judging-accepting / rejection", corresponds the going heat balance coursing of sold at a certain constant temperature, that is to say: we have carried on Metropolis arithmetic once. It is resemble to Metropolis arithmetic that computing time evolvement coursing of computing system from a certain early state and solving the ending state of the system, the simulated Annealing arithmetic can solve the relative optimal solution of giving controlling parameter unlay and combination optimization problem through translating many unlay, if this, then we must start at the beginning of unlay. Then minish the unlay of controlling parameter t, repeating Metropolis arithmetic, and solving the total optimal solution of the combination optimization problem when the controlling parameter t going zero. Because descending temperature is "slow and slow" in the sold annealing, this can ensure the heat balance in any temperature, and going to minimum ground state finally, and the cost of the parameter must damp slowly, only this we can ensure the annealing arithmetic going to the optimal solution of combination optimization problem.

Simulated annealing arithmetic produce list of combination optimization problem unlay with Metropolis arithmetic, furthermore simulated annealing arithmetic and corresponding shift probability  $P_t$  of Metropolis rule ensure accepting the transferring from current unlay I to new unlay j or not.

$$P_{t}(i \Rightarrow j) = \begin{cases} 1 & whenf(j) \le f(i) \\ \exp(\frac{f(i) - f(j)}{t}) & orelse \end{cases}$$
(5)

In the formular,  $t \in R^+$  denote controlling parameter. *T* should adopt to quite unlay (corresponding to dissolution temperature of sold), after transferring amply, minishing unlay *t* slowly (corresponding to descending temperature "slowly and slowly"), again and again,

stopping the arithmetic until satisfying a certain stopping rule. Thus, simulated annealing arithmetic can be regarded as the alternation of Metropolis arithmetic and Metropolis arithmetic is cost in descending and controlling.

## (2) Computing flow of the arithmetic

The figure 1 denotes the flow of simulated annealing arithmetic. From the figure 1, we can see that simulated annealing arithmetic basing on Metropolis rule accepts new cost, thus, it does not only accept the optimizing cost but also accept worsen cost in a certain scope, this is the essential difference between simulated annealing arithmetic and local searching arithmetic. Cost t is big in the beginning, it may accept worsen cost of a sort; with minishing of the cost t, it only accepts preferable worsen cost; finally, when the cost t going zero, does not accept any worsen cost. From this coursing, simulated annealing arithmetic can jump from the "trap" of local optimization, the possibility of whole optimization cost of solving combination optimizing problem is more, and this coursing takes on briefness and currency. Thus, for the most combination optimizing problem, simulated annealing arithmetic is better corresponding to local searching arithmetic.

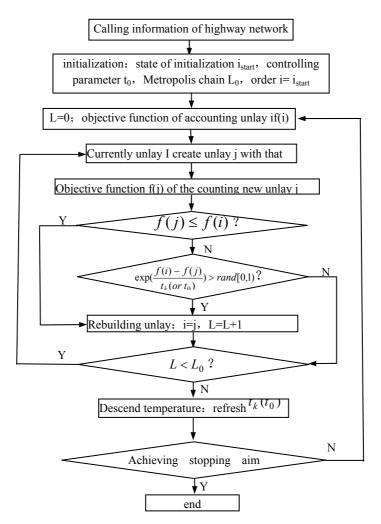


Figure1 Simulated Annealing Formality

(3)The mostly idea of solving highway network layout optimization problem with simulated

## annealing arithmetic

According to the idea and principle of simulated annealing, the method of solving highway network layout optimization problem has been exploited simulated annealing arithmetic owner formality and modules in Visual basic 6.0 in this paper.

The mostly means of solving highway network layout optimization problem in simulated annealing arithmetic is: the first is that transferring highway network layout optimization problem into discrete single objective combination optimization problem; the second is that ensure preparing channel collection that should be cost space, designing creating mechanism of new cost and computing flow of objective function. Through repeating alternation, comparing the objective function cost of every new cost, the optimization cost will be ensured finally.

## 4. EXAMPLE

## 4.1 Current Highway Network

In this paper, we chose Harbin urban zone highway network as an example in 2004. It's shown in Figure 2.

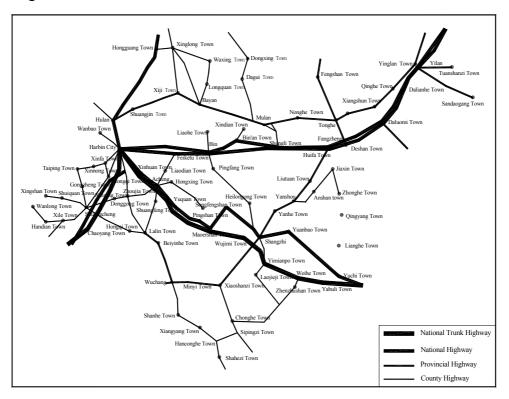


Figure2 Current Situation of Harbin Urban Zone Highway Network

## 4.2 Selecting for Network Nodes

We chose Harbin city, another 12 counties and the main towns contained as the highway network nodes, whose number was 89 totally. For the first, the area was made off 13 traffic zones, shown in table 1, and then, we got the exchange volume from traffic distributions after

Table 1 Number of Traffic Zone					
traffic zone number	name of nodes	traffic zone number	name of nodes		
1	Harbin	8	Bin		
2	Hulan	9	Acheng		
3	Bayan	10	Shangzhi		
4	Mulan	11	Yanshou		
5	Tonghe	12	Wuchang		
6	Yilan	13	Shuangcheng		
7	Fangzheng				

forecasting traffic generations, shown in table 2.

Table 2 Traffic OD of Harbin Urban Zone Highway Network

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	360	389	542	299	206	657	388	2064	657	853	825	896
2	360	0	361	455	860	632	452	681	445	708	399	263	656
3	389	361	0	869	528	655	389	467	786	803	815	312	394
4	542	455	869	0	689	718	735	464	368	653	561	803	342
5	299	860	528	689	0	636	424	451	530	424	424	504	762
6	206	632	655	718	636	0	716	478	507	597	478	478	567
7	657	452	389	735	424	716	0	569	379	403	474	379	379
8	388	681	467	464	451	478	569	0	598	399	424	499	399
9	2064	445	786	368	530	507	379	598	0	569	379	403	474
10	657	708	803	653	424	597	403	399	569	0	568	509	497
11	853	399	815	561	424	478	474	424	379	568	0	287	364
12	825	263	312	803	504	478	379	499	403	509	287	0	968
13	896	656	394	342	762	567	379	399	474	497	364	968	0

## 4.3 Choosing Values of Parameters

The value of parameter n was 89. According to the future planning of Harbin highway network, at the end of "ten fifth" planning, there will be about 140 hundreds million RMB used in highway construction, at that time, the total length of highway network will extent 9663 km. The cost of newly-built highway is shown as follows, 16 millions RMB per kilometer for expressway, 7 millions RMB per kilometer for 1 grading highway, 3.2 millions RMB per kilometer for 2 grading highway, 2.7 millions RMB per kilometer for 3 grading highway, and 0.6 millions RMB per kilometer for 4 grading highway. The cost of highway reconstruction and extension were shown in Table 3. We got the optimization model of Harbin urban highway network layout after taking those parameters into bi-level planning model of Highway network layout.

Table 3 Cost of Highway Reconstruction and Extension

grade before	grade after	cost(10 thousand RMB/km)
Two Grading highway	Expressway	1300
Three Grading highway	one grading highway	630
Three Grading highway	two grading highway	260
Four Grading highway	two grading highway	280
Four Grading highway	three grading highway	220

Expressway	(adding two lanes) expressway	800
Two Grading highway	(adding two lanes) two grading highway	250
One Grading highway	(adding two lanes) one grading highway	350

## 4.4 Computing the Model

## (1)Deciding spare routes

Every possible reconstruction scheme of current routes, such as heightening 1 or 2 grading, reconstructing lanes, used as different spare routes, at the same time, newly-built spare routes were decided according to connectivity of current nodes. However, when central city didn't have out circle line or had circle line but some sections needed reconstruction, so the out circle line should be a spare route, and we could decide the spare routes.

(2)The result of layout optimization

We could get the result of highway layout optimization, after accomplishing the simulated annealing formality to the layout optimization model of Harbin highway network. Those were 207.6 km newly-built highway, 568.1 km reconstruction highway, 188.9 km extension highway. Highway network layout optimization was shown in Figure 3.

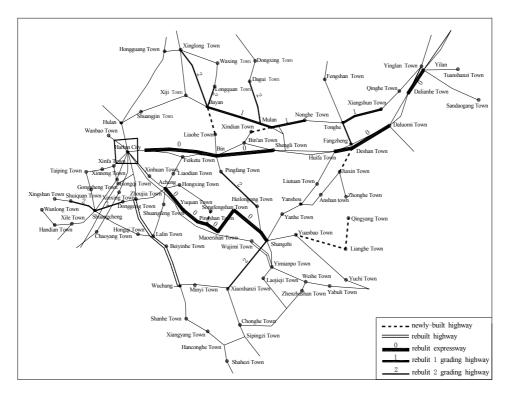


Figure3 Optimization Scheme of Harbin Urban Highway Network Layout

# 4. CONCLUSION

Highway network layout optimization is an important content of highway network planning. According to the trait and need of provincial or urban highway network layout, we have enlarged traditional single objective optimizing model and set up highway network layout bi-level planning model fitting provincial or urban highway network in this paper, at the same time, the simulated annealing arithmetic has been applied to solve model and exploited arithmetic formality in Visual Basic6.0, this arithmetic agrees with solving huge highway network optimization problem especially, this arithmetic has solved the localization that general arithmetic can not be applied with cosmically network optimization problem.

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