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# A RECURSION EVENT-DRIVEN MODEL TO SOLVE THE SINGLE AIRPORT GROUND-HOLDING PROBLEM

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**Abstract:** The yearly congestion costs in the U.S airline industry are estimated to be more than three billions. In China, the delay problem caused by air congestion also becomes more and more serious. An effective method for reducing the delay cost in air traffic flow management is by using ground-holding policy. When numbers of flights are big, it is difficult to calculate the real-time solution of it. A new recursion event-driven model is presented in this paper considering different delay cost. Discrete-event analyze method has been used to solve the single airport ground-holding problem. The concept of delay time equivalent quantity has been presented to solve the combination optimization problem and a fast algorithm was given basing on it. The simulation results validate the feasibility of the proposed model and algorithm.
**Key Words:** Air traffic flow management, Ground-holding algorithm, Event-driven model, Delay time equivalent quantity

## 1. INTRODUCTION

The yearly congestion costs in the U.S airline industry are estimated to be more than three billions. European airlines are in a similar plight. In China, the delay problem caused by air congestion also become more and more serious. An effective method for reducing the delay cost in air traffic flow management short-term policies is to use ground-holding policy (GHP). The objective of GHP is to transfer the anticipated airborne delay to the ground delay. The ground-holding problem is the problem of determining, for a given network of airports, how long each aircraft must be held on the ground before taking off in order to minimize the total (ground plus airborne) delay cost for all flights, considering airport capacities and flight schedules. As for this problem, various models have been proposed in the literature, Andreatta et al (1987) was an earlier researcher of GHP algorithm, although the problem had been simplified. They studied the single-period GHP problem and used dynamic programming to obtain a solution. Terrab et al (1992,1993) extended these results to multi-period GHP. He studied the influence of parameter varied to the optimize result, described a set of approaches for addressing a deterministic and a stochastic version of the problem, used the minimum cost flow algorithm for the deterministic problem. Richetta et al (1993,1995) also addressed the same problem formulated as a stochastic linear program, which they obtained an optimal solution. Hoffman et al (2000) extended the problem by the addition of banking constraints to accommodate the hubbing operations of major airline. These constraints enforce the desire to land certain groups of flights, which are called “banks”, within fixed time windows, thus preventing the propagation of delays throughout entire operations. They constructed five different models of singe-airport ground-holding problem with banking constraints and evaluated them both computationally and analytically. All these models were time-driven models in which flights landing within intervals of fixed length are considered. When the number of flights is big, it is difficult to calculate the real-time result of the model. Panayiotou et al (2001) was the first who developed the event-driven model and proposed using finite perturbation analysis technique to dynamically solve this problem. Luo Xiling et al (2002) analysed the parameter effect on the model. But all these people did not take the following into account, that is, different aircraft have different delay cost.

A new recursion event-driven model and algorithm considering different delay cost are presented in this paper for the single airport ground holding problem (SGHP). The advantage of our algorithm is such, that even for the largest airports, the problem optimal result can be solved immediately by just using a personal computer. The outline of this paper is as follows: in Section 2, the recursion event-driven model has been proposed considering the airport capacities, flight schedules, different flight delay costs; the relationship of time and departure event, arrive event, and land event. In Section 3, the concept of delay time equivalent quantity are presented to translate the delay cost to delay time equivalent quantity, so the discrete-event analysis method can be used to solve this problem. In Section 4, the model in the deterministic and probabilistic situation are analyzed, and the fast algorithm basing on the delay time equivalent quantity are given. In Section 5, 100-flight-simulation are present. The simulation results validate the feasibility of the proposed model and algorithm. In Section 6, we come to a brief summary.

## 2. EVENT-DRIVED MODEL

We consider a network that is combined with some departure airports and one destination airport D, where there are a set of aircraft $f_1, f_2 \cdots f_K$ scheduled to land the airport D during time period $[0,T]$, the schedule arrival times are $YA_1, YA_2, \cdots, YA_K$, and $YA_1 < YA_2 < \cdots < YA_K$. We define the event as follows:

Departure event: aircraft departure from the airport.

Arriving event: aircraft arriving the airspace of the destination airport.

Landing event: the aircraft begin to land.

We use the following notation:

$u_k(t)$: The serve time of the $k$ th aircraft at time $t$, the length of serve time is from that the $k$ th aircraft has been allowed to land to that the $k$ th aircraft rolling out of the runway and the next aircraft may come into runway. $u_k(t)$ becomes shorter as a fine weather and vice versa;

$A_k$ : The true arriving time of the $k$ th aircraft;

$YA_k$ : The schedule arriving time of the $k$ th aircraft;

$L_k$ : The clearance- to- land time of the $k$ th aircraft;

$E_k$ : The enroute time of the $k$ th aircraft;

$S_k$ : The schedule departure time of the $k$ th aircraft;

$DG_k$ : The delay time of the $k$ th aircraft on the ground.;

$DA_k$ : The delay time of the $k$ th aircraft in the air;

$c_g(k,t)$ : Cost of delaying the $k$ th aircraft for $t$ unit period on the ground;

$c_a(k,t)$ : Cost of delaying the $k$ th aircraft for $t$ unit period in the air;

$T_k$ : The true departure time of the $k$ th aircraft.

Time relationship has shown in figure 1. Except that the $L_k$ is the decision variables, others known before.

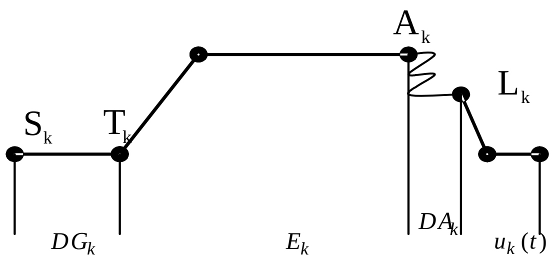


Figure 1: Departure-landing time relationship

We assume the destination airport is the only delay source, $E_k$ is known and determinate, the aircraft can not departure before its schedule departure time. In the case of the $k$ th aircraft arrives, if the runway is idle, the aircraft can land immediately, then we have

$$L_k = A_k \tag{1}$$

If the runway is busy,

$$L_k = L_{k-1} + u_{k-1}(t) \tag{2}$$

We may rewrite the two above case as following:

$$L_k = \max(A_k, L_{k-1} + u_{k-1}(t)) \tag{3}$$

The delay time is:

$$DG_k = T_k - S_k \tag{4}$$

$$DA_k = L_k - A_k \tag{5}$$

The object function is to minimize the delay cost

$$Min \quad z = \sum_{k=1}^{K} [c_g(k, DG_k) + c_a(k, DA_k)] \tag{6}$$

The constrain is $DA_k \le d$ , $d$ is the maximum value that aircraft are cleared for holding.

## 3. THE FAST ALGORITHM

In the process of solving the problem above, when numbers of aircraft are more, the combining problems make the calculation very complex. In order to solve the problem, we transfer the delay cost to corresponding delay time equivalent quantity according some relationship.

According to FAA regulation, aircraft are classified into three types by their weights, they are H type(heavy)、M type(middle)、L type(light). Delay cost of aircraft in same type are close. So, we give the defining of delay time equivalent quantity, that is to turn the delay cost into delay time equivalent quantity basing the ratio of cost of different aircraft. For example, during time period $t$, the cost ration of delay one unit is

$$DG_H(t) : DG_M(t) : DG_L(t) = a : b : 1 \tag{7}$$

That means the aircraft of type H delay one minute, just as the aircraft of type L delay $a$ minute, uniformity, the aircraft of type M delay one minute, just as the aircraft of type L delay $b$ minute. Through transferring the delay cost to delay time, we solve the problem in a easy way.

### 1.1 Determine Model

We assume the destination airport is the only constrain source, when the capacity is determined and known, the $u_k(t)$ is known. As $c_a(k,t) > c_g(k,t)$ , so we transfer all airborne delay to ground delay by making the aircraft hold on the ground for a length time. We have

$$DA_k = 0 \tag{8}$$

We assume $A_K(m)$ is the begin time of $m$ th landing event, $DG_k(m)$ is the delay time when aircraft $k$ th is assigned $m$ th departure event, then

$$A_k(m) = \max(YA_k, L_{k-1}(m) + u_{k-1}(m)) \tag{9}$$

$$DG_k(m) = A_k(m) - E_k - S_k \tag{10}$$

The initial state is:

$$DG_1(1) = 0 \tag{11}$$

$$L_1(1) = A_1 \tag{12}$$

The object function is to minimize the delay cost

$$Min \quad z = \sum_{k=1}^{K} [c_g(k, DG_k)] \tag{13}$$

The problem is to assigned the departure event for each flight to make the sum of delay time the shortest.

When we calculate the optimize permutation, basing the defination and nature of the delay time equivalent quantity, we know that the delay cost will increase when bringing the aircraft $A_{k+1}, A_{k+2} \cdots$ forward from behind the aircraft $A_k$ to ahead the aircraft $A_k$ when the type of the aircraft $A_k$ is H; when the type of the aircraft $A_k$ is M, whether the delay cost increases is connected with the location of the first aircraft, which type is H in $A_{k+1}, A_{k+2} \cdots$; when the type of aircraft $A_k$ is L, whether the delay cost increases is connected with the location of the two aircraft in $A_{k+1}, A_{k+2} \cdots$, that one is the first type H aircraft and the other is the first type M aircraft. So in each optimize process, we only need to calculate the permutation of three aircrafts.

### 3.2 Stochastic Model

Because the weather prediction is often incorrect, the capability value is stochastic. We assume the capability has $Q$ scenarios with possibility $P_q$, basing the model of expectation value, we achieve the capability $C$ is:

$$C(t) = \sum_{q=1}^{Q} C_q(t) P_q \tag{14}$$

As the serve time $u(t)$ is decided by the capability, we have

$$u_k(t) \approx 1/C(t) \tag{15}$$

Then the stochastic model is transformed to the deterministic model, and we may use the fast algorithm method to solve it. Of cause, it exists a litter airborne delay, but we may use other mature methods to permute the landing aircraft to minimize the cost.

## 4. EXPERIMENTAL RESULTS

We used the flight schedule for a typical weekday of operation at Beijing airport. The whole time period we focused on is 300 minutes. The total number of landing aircraft is 100. Assume based on the model of expectation value, the serve time $u_k(t)$ is given as figure2. Now, we analyze the parameter infection.

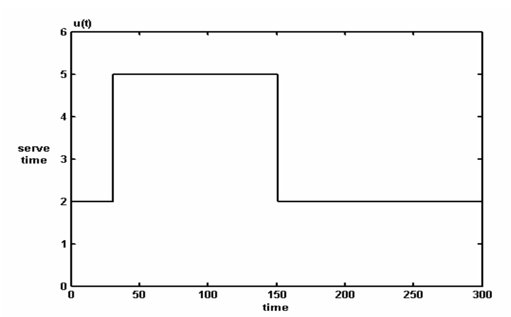


Figure 2： Serve Time

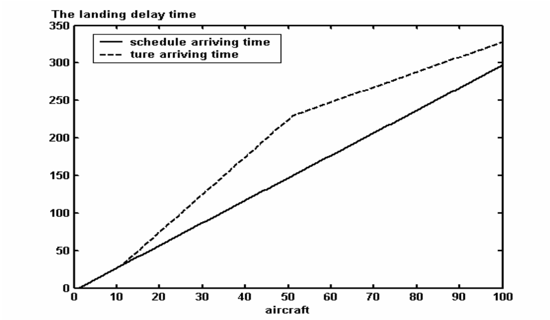


Figure 3：$u_k(t)$ Infection for Landing Delay Time

（1）$u_k(t)$ infection

From the figure3, we know, with the serve time increasing (decreasing), the delay time is increasing (decreasing).

（2）the value of $a:b:1$ infection for the optimization result

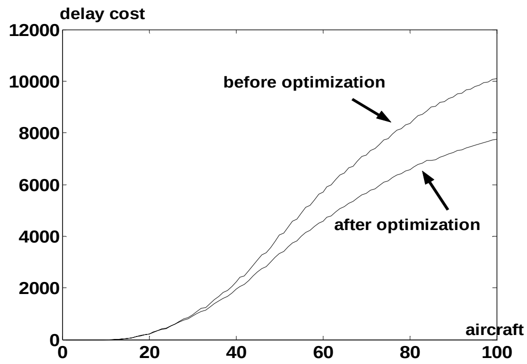


Figure 4： $a:b:1=3:2:1$

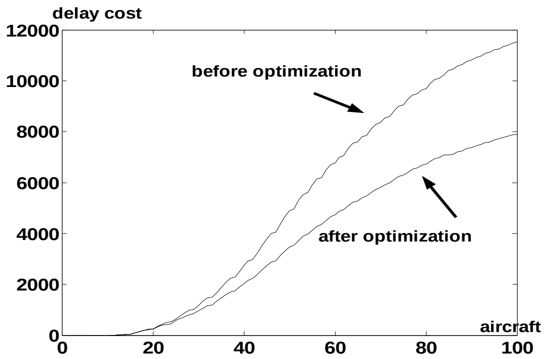


Figure 5： $a:b:1=4:2:1$

From figure4 and figure 5, we draw a conclusion that, when the rate of delay cost is higher, the optimization result becomes more obviously.

## 5. CONCLUSION

The advantage of discrete event system is only researches departure, arrival and landing of single aircraft. When the capability is determined, we may give the exact departure time of each aircraft immediately.

As the model is event-driven, the system will be optimized according to new data for every landing event, the nature of the model is dynamic.

Through importing the delay time equivalent quantity, the calculation is simplified. To make the calculation more exact, the value of $a:b:1$ may also acquire the dynamic value. This method may use for other types of aircraft.

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