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VRP BASED ON RESIDUAL LOADING CAPACITY IN REVERSE LOGISTICS

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Abstract: In this paper, the vehicle routing problem of reverse logistics with simultaneous delivery and pick-up (VRPSDP) is presented, in the meantime, the model of VRPSDP is described in detail. To overcome the shortcomings of inserting criterion based on traveling distance, by presenting and analyzing an example, two new inserting criterions with heuristic algorithm for VRPSDP are put forward, then, a comparison of effectiveness is carried out. The inserting criterion based on residual capacity combines residual capacity with traveling distance of vehicles effectively. So, the inserting criterion based on traveling distance is improved.

Key Words: Reverse logistics, Environmental logistics, VRPSDP, Heuristic algorithm, Residual capacity

1. INTRODUCTION TO REVERSE LOGISTICS

1.1 Conception of Reverse Logistics

Reverse logistics means the physical activities of products from consumer locations to original spots, including all the activities related to the recycling, substitution, return and disposal of resources. To achieve the objective of minimizing the consumption of energy and products as well as integrating it with forward logistics, reverse logistics that consists of returned and reclaimed logistics becomes the organic part of logistics system.

1.2 Reverse Logistics and Environmental Logistics

While promoting economic development, logistics activities also increase the quantity and frequency of related activities, increase energy consumption, aggravate air pollution and waste pollution, and bring many side-effects on economic sustainable development of the whole society.
The concept of environmental logistics can be described as follows: to keep a friendly environment and make good use of logistics resources, restrict the side effects on environment of logistics during operating process. Considering the theory of sustainable development, it is

very important for us to develop environmental logistics. By improving logistics stages such as transport, storage, package, load and unload as well as distributing process, the objective of reducing environment pollution and energy consumption can be attained.

Reverse logistics, as an organic part of environmental logistics, plays an important role to some extent. It can make good use of existing resources, cut down the demand on raw materials and save operating costs. However, some activities related to reverse logistics may put many side-effects on environment. Even they can destroy the positive effects brought about by recycling, remanufacturing and the reuse of new products and materials, etc. Therefore, in addition to concentrating on the saving of costs, enterprises should carefully deal with the relation between reverse logistics and environmental protection, meanwhile, strengthen research on the techniques related to reverse logistics, only by those can the side-effects on environment resulted from additional logistics activities be reduced.

2． INTRODUCTION TO VRPSDP

2.1 Concept of VRPSDP

From strategic views, in order to bring reverse logistics into full play on environment, it is necessary to coordinate the relationship between forward and reverse logistics properly so that they can be integrated in supply chain and play positive role. If we run the forward and reverse logistics independently, on the operational levels, for each of them a separate VRP has to be solved, it is difficult to operate and run against environment protection. Furthermore, in many distributing systems, operating forward and reverse logistics may lead to unnecessary consumption of vehicle capacity. If we combine pick-up and delivery at customer locations, vehicle routing problem with backhauls -VRPB- will appear. The pick-up and delivery to customers are operated separately in VRPB, that is, delivery is finished in the process of distribution and pick-up is finished in the process of backhaul.

In fact, in many environmentally motivated distributions, most customers have delivery demand as well as pick-up demand. They don not want the two activities to be operated separately just because various handling efforts are caused by both of them. Handling efforts may be reduced to a great extent if delivery and pick-up are finished simultaneously. Customers only accept the service of delivery and pick-up with just one stop. This planning situation can be called Vehicle routing problem with simultaneous delivery and pick-up (VRPSDP).

2.2 Current Research and Related Solutions on VRPSDP

At present, there are many related researches on VRP at home and abroad. But, investigations on VRP related to reverse logistics began not long ago. Up to now, there is few related literature focusing on models and algorithms of VRPSDP at home.

VRP was put forward by G. Dantzig, and Garey proved that TSP (Traveling salesman problem) is an NP-hard problem. Because TSP is a special case of VRP,so,VRP is also an NP-hard problem. Furthermore, VRPSDP is transformed from VRP and also an NP-hard problem (Zhang, T., WANG, M.G.,1996). Simultaneously, VRPB is a special instance of VRPSDP, we should take advantage of current algorithms related to VRPB to find the solutions to VRPSDP. This paper deals with VRPSDP by using an insertion-based heuristic and focuses on the improvement of traditional criterions.

3. MATHEMATICAL MODEL OF VRPSDP

3.1 Description of VRPSDP

VRPSDP can be described as follows: there is a central depot, a set of vehicles and customers, all the vehicles have the same capacity and limited traveling distance, the number of locations is confined, individual attributes of the customers are not considered, the vehicles are all located at central depot originally, the customer locations distribute in space randomly. The vehicles deliver goods to each customer, at the same time, meet their pick-up demands and go back to central depot ultimately. Each customer is served just once during the process. The objective is to minimize the general traveling distance of the vehicles and make good use of vehicle capacity.

3.2 Notations

(1)V: Set of customer locations
(2)F: Set of all vehicles
(3)C: Vehicle loading capacity
(4)L: Traveling distance limitation of all vehicles
(5) d_{ij} :Distance between locations （$i \in V, j \in V, i \neq j; d_{ii} = \infty, i \in V \setminus \{0\}, d_{00} = 0$）
(6) D_j : Delivery amount of customer j
(7) P_j :Pick-up amount of customer j
(8)M: The bigger number between total delivery amount and traveling distance, it is denoted in following equation:

$$M = \max\left\{ \sum_{j \in V \setminus \{0\}} (D_j + P_j), \sum_{i \in V} \sum_{j \in V, j \neq i} d_{ij} \right\}$$

(9)n: Number of nodes
(10) γ_k :Load of vehicle when leaving central depot, $k \in F$
(11) γ_i :Load of vehicle after having served customer *i*, $i \in V \setminus \{0\}$
(12) ρ_i :Position of location i ($i \in V \setminus \{0\}$) in the route
(13) x_{ijk} :binary variable (0,1)

$x_{ijk} = 1$ （vehicle k travels directly from location *i* to location j）

$x_{ijk} = 0$ (other situations)

3.3 Description of Model

$$Min\sum_{i\in V}\sum_{j\in V}\sum_{k\in F} d_{ij}x_{ijk}$$ (Minimize total traveling distance) (1)

Subject to:

$$\sum_{i\in V}\sum_{k\in F} x_{ijk} = 1 \quad (j \in V \setminus \{0\})$$ (Each customer is served once only) (2)

$$\sum_{i\in V} x_{isk} = \sum_{j\in V} x_{sjk} \quad (s \in V \setminus \{0\}, k \in F)$$

(Only one vehicle arrives at and leaves a customer location) (3)

$$\gamma_k = \sum_{i\in V}\sum_{j\in V\setminus\{0\}} D_j x_{ijk} \quad (k \in F)$$ (Initial vehicle load) (4)

$$\sum_{i\in V}\sum_{j\in V\setminus\{0\}} d_{ijk} \le L \quad (k \in F)$$ (Traveling distance of vehicles can not excess L) (5)

$$\max d_{0j} \le L/2 \quad (j \in V \setminus \{0\})$$ (Meet the need of the farthest customer) (6)

$$\gamma_i \ge \gamma_k - D_i + P_i - M(1 - x_{0ik})$$ (Vehicle load after served the first customer) (7)

$$\gamma_j \ge \gamma_i - D_j + P_j - M(1 - \sum_{k\in F} x_{ijk}) \quad (i \in V \setminus \{0\}, j \in V \setminus \{0\}, j \ne i)$$

(Vehicle load after served customer j) (8)

$$\gamma_k \le C \quad (k \in F)$$ (Vehicle initial load can not overrun vehicle capacity) (9)

$$\gamma_i \le C \quad (i \in V)$$ (Vehicle initial load can not overrun vehicle capacity after served customer *i*) (10)

$$\rho_j \ge \rho_i + 1 - n(1 - \sum_{k\in F} x_{ijk}) \quad (i \in V \setminus \{0\}, j \in V \setminus \{0\}, j \ne i)$$ (11)

$$\rho_i \ge 0 \quad (i \in V)$$ (12)

(Prevent sub-tour and guarantee the continuity of routes)

4. NEW INSERTION CRITERION

4.1 Insertion Criterion Based on Traveling distance and Its Shortages

It is necessary to point out that insertion criterion is a key to the quality of the solutions observed by using algorithms. A simple greedy algorithm only calculates additional traveling distance when inserting customer *s* between *i* and *j*. It is denoted by ψ_{TD} as follows:

$$\psi_{TD} = C_{is} + C_{sj} - C_{ij}$$ (13)

In general, choosing an insertion approach with the smallest additional traveling distance has two main shortages. First, it does not consider the influences of vehicle capacity and the degree of freedom for future insertion. Second, at relatively late stages, far customers can be inserted into current route, resulting extra traveling distance. Apparently, it just inserts customers into existing routes and can not solve VRPSDP efficiently. Therefore, it is necessary for TC to be modified and improved.

4.2 Insertion Criterion Based on Net Load

Here we present an instance with four customer points. Let P_i and D_i represent pick-up and delivery amount of customer *i* respectively. We suppose that each vehicle has the same capacity. The amount of pick-up and delivery of the four customers are as follows: D_1=20, P_1=10; D_2=10, P_2=50; D_3=10,P_3=20;D_4=40,P_4=30. Here, we let the maximum vehicle capacity C=100 and the maximum vehicle traveling distance L=1000. Distances between the four locations are given in table 1.

Table 1. Distances between locations

i \ j (d_{ij})	0	1	2	3	4
0	∞	70	90	80	110
1	70	∞	40	100	90
2	90	40	∞	60	50
3	80	100	60	∞	80
4	110	90	50	80	∞

To make up the shortage of the insertion criterion based on net load nl_i ($nl_i = P_i - D_i$) at each inserting stage and make good use of vehicle capacity, we put up a new criterion based on net load at each inserting step. Net load equals the difference between the amount of pick-up and delivery at each customer waiting to be inserted. Then, all these net loads are sorted by size. First, insert the customers points with the smallest net load, then the second smallest until the constraints upon do not permit. The needs of those residual customer points that can not be inserted are met by other vehicles, so other routes need to be constructed.

Then, during the inserting process, when the net loads of several customer points to be inserted are the same, we take the distances between customer points and the preceding point inserted just now into consideration, the points with relatively small distances are inserted. The illustration of the instance upon is as follows:

The sorting result of net loads of customer points is [nl_4 (-10), nl_1 (-10), nl_3 (10), nl_2 (40)]. Considering the restriction of vehicle capacity, single vehicle can only finish the needs of three customers (1, 3and 4). The need of pick-up and delivery of customer 2 has to be met by another vehicle. Two routes are constructed in the figure below：

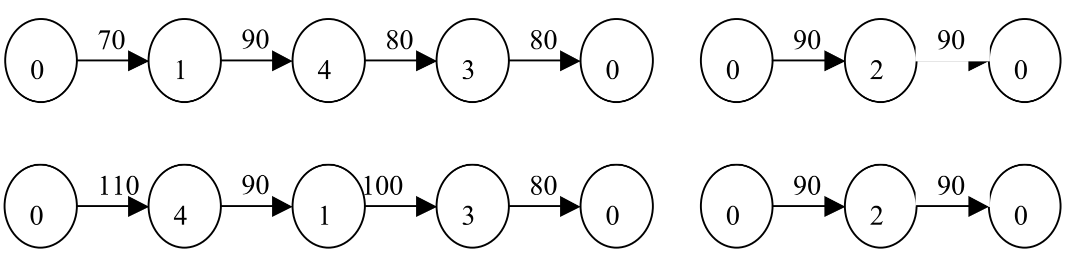

Figure 1 Routes Got by Using the Criterion Based on Net Load

As shown in figure 1, if distance is considered, the first route is the best solution with relatively small vehicle traveling distances of 500. Total load of vehicle 1 is 60.Based on traveling distances, this criterion is effective when solving problems with little amount of customer points.

4.3 Insertion Criterion Based on Residual Capacity

4.3.1 Illustration of instance

To make up the shortages of TC, we combined residual capacity with traveling distance. It is apparent that the bigger the vehicle residual capacity is, the bigger the degree of future insertion is. To obtain a satisfying insertion criterion, an instance for further explanation is presented(Vigo, D.,1996).

$R_d(i)$ represents the residual delivery capacity from the central depot to customer j (customer *j* is inserted just after customer *i*) after serving customer i; $R_d(i)$ represents the biggest possible pick-up capacity that the customer which is inserted after customer *i*. Q（$\{t\}$）represents the set of existing customers. First, insertion situation with only two locations is illustrated, laying a solid foundation for further computation.

As shown in figure 1, we take customer 1 as “first customer”, then, the residual capacities can be computed recursively as follows:

$$R_d(0) = C - \sum_{t \in Q} D_t = 100 - 20 = 80$$

$$R_d(1) = Min\{R_d(0), C - r_1\} = Min\{80, 100 - 10\} = 80$$

$$R_p(1) = C - \sum_{t \in Q} P_t = 100 - 10 = 90$$

$$R_P(0) = Min\{R_P(1), C - r_0\} = Min\{90, 100 - 20\} = 80$$

If $D_S \le R_d(0) = 80$ and $P_S \le R_P(0) = 80$, it is feasible to insert customer s behind central depot. Similarly, if $D_s \le R_d(1) = 80$ 且 $P_s \le R_P(1) = 90$, customer s can be inserted behind customer 1, in other words, customer *s* has two optional insertion positions. In this example, customer 2 has two insertion positions, as given below:

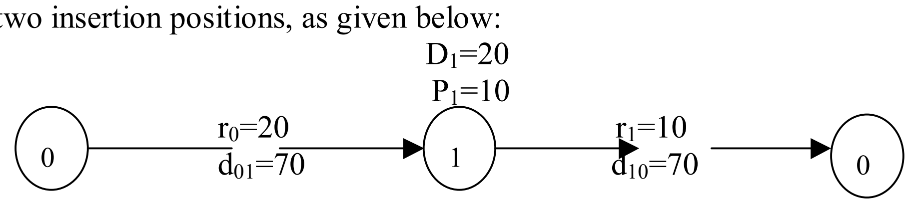

Figure 2. Route Base on “First Customer” 1

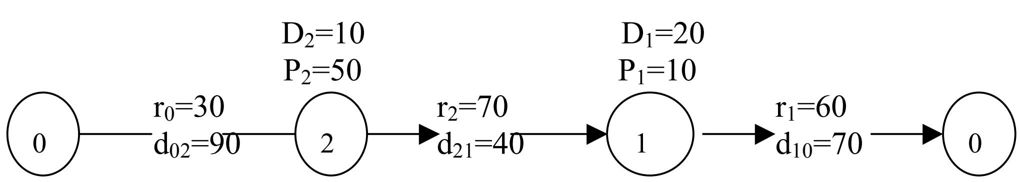

Figure 3. Route Illustration after Customer 2 Is Inserted behind Central Depot

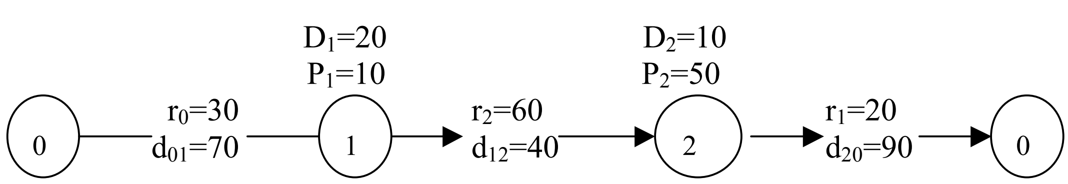

Figure 4. Route Illustration after Customer 2 Is Inserted behind “First Customer” 1

(1)Customer 2 is inserted behind central depot
Based on the above computation, if we insert customer 2 behind central depot, as shown in figure 2, the load and traveling distance meet restrictions(C and L), computing process is as follows:

$$R_d(0) = C - \sum_{t \in Q} D_t = 100 - (20 + 10) = 70$$
$$R_d(2) = Min\{R_d(0), C - \gamma_2\} = Min\{70, 100 - 70\} = 30$$
$$R_d(1) = Min\{R_d(2), C - \gamma_1\} = Min\{30, 100 - 60\} = 30$$
$$R_p(1) = C - \sum_{t \in Q} P_t = 100 - (10 + 50) = 40$$
$$R_P(2) = Min\{R_P(1), C - \gamma_2\} = Min\{40, 100 - 70\} = 30$$
$$R_P(0) = Min\{R_P(2), C - \gamma_0\} = Min\{30, 100 - 30\} = 30$$

Customer 3(D_3=10,P_3=20)has three positions to be inserted in this insertion approach.

(2)Customer 2 is inserted behind "first customer" 1
As shown in figure 3, if we insert customer 2 behind "first customer" 1, computing results are as follows: $R_d(0)$ =70, $R_d(1)$ =40 $R_d(2)$ =40; $R_p(2)$ =40, $R_P(1)$ =40, $R_p(0)$ =40.It is apparent that customer 3（D_3=20,P_3=20） can be inserted behind such locations as customer 1,customer 2 and central depot. Computing method is the same as situation 1.

4.3.2 Representation of new insertion criterion

From the example above, we can see that by inserting customers with relatively big delivery amount and small pick-up amount firstly and inserting customers with relatively small delivery amount and big pick-up amount secondly, bigger residual capacity can be obtained. Here, $D_d(t)$ represents the traveling distance from central depot to customer t along existing route, on the contrary, $D_P(t)$ denotes the traveling distance from customer to central depot along existing route. We let $NIA(t)$ represents the set of the un-immediate successors of customer i. As shown in following equations, *TRD* and *TRP* denote the general residual pick-up capacity and delivery capacity respectively (Dethloff, J., 2001).

$$TRD = \left[\sum_{t \in Q \cup \{0\}} R_d(t) * D_d(NIA(t)) \right] \Big/ \left[\sum_{t \in Q \cup \{0\}} D_d(NIA(t)) \right] \tag{14}$$
$$TRP = \left[\sum_{t \in Q \cup \{0\}} R_p(t) * D_p(t) \right] \Big/ \left[\sum_{t \in Q \cup \{0\}} D_p(t) \right] \tag{15}$$

For route 0-2-1-0, computing process is as follows:
TRD=[70*90+30*(90+40)+30*(90+40+70)]/420=38.57;
TRP=[40*70+30*(70+40)+30*(70+40+90)/380=31.84;

For route 0-1-2-0, *TRD*=45.53 and *TRP*=40;

For the customers that have not been inserted into the route yet, if their accumulated delivery demands (AD_u) and pick-up demands (AP_u) are high enough, residual vehicle capacity should be given priority. Here, by combining the used proportion of vehicle capacity and residual vehicle capacity, and let *TUC* represents the total used capacity, we put forward a scale to measure vehicle capacity, as shown in equation (16):

$$TUC = \frac{AD_u * (1 - TRD / C)}{\sum_{t \in Q} D_t} + \frac{AP_u * (1 - TRP / C)}{\sum_{t \in Q} P_t} \tag{16}$$

Our ultimate objective is to meet the delivery and pick-up demands of customers with possibly least vehicle capacity. Therefore, we integrate *TUC* with vehicle traveling distance and put forward a new insertion criterion based on both residual vehicle capacity and vehicle traveling distance, as shown in equation (17).

$$\psi_{RC} = \psi_{TD} + \lambda \cdot TUC \tag{17}$$

Here, ψ_{RC} is used as the measurement of vehicle traveling distance and used vehicle capacity. $0 \le \lambda \le 1$, when λ equals 0, the insertion criterion will become the one only based on traveling distance. Under other conditions, on the base of the TC, the value of λ should be given according to the influences on practical insertions resulted from vehicle residual capacity.

Based on the example, we explicate the application of insertion criterion. According to the representation of the instance, under the condition of meeting the restrictions of vehicle traveling distance and capacity, for the given routes, customer 3 has six kinds of insertion approaches in all. Here we let λ= 0.7 and customer 4 (D_4=40, P_4=30) is not inserted yet. According to the computing process given above, we compute the related performances of the six routes respectively and list them in table 2.

Table 2. Results

Routes	*TRD*	*TRP*	*TUC*	ψ_{TD}	ψ_{RC}
0-1-2-3-0	33	20	0.97	50	50.68
0-1-3-2-0	43.8	20	0.86	120	120.60
0-2-1-3-0	24.74	20	1.05	110	110.74
0-2-3-1-0	17.41	10.89	1.16	120	120.81
0-3-2-1-0	39.24	20	0.91	110	110.64
0-3-1-2-0	24.77	11.17	1.09	50	50.76

As shown in table 2, when customer 3 is inserted into existing route, seen from the computed results, up to the current stage, ψ_{RC} of route 0-1-2-3-0 is 50.68, it is clear that route 0-1-2-3-0 is the best solution, which is shown in figure 5 It considers traveling distance and residual vehicle capacity simultaneously, that is to say, while minimizing traveling distance, it takes good advantage of vehicle capacity. At current stage, since $R_p(t) \le P_4 = 30(t \in Q)$, customer can not be inserted into the route. For other instances in which there are a relatively large number of customers waiting to be inserted, computing processes is the same as above.

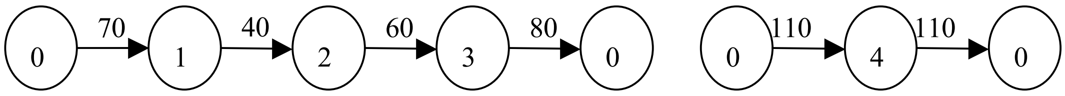

Figure 5 Best Solution by Using the Criterion Based on Residual Capacity

As shown in figure 5, the best solution need two vehicles with total traveling distance of 470. Compared with the best solution got by using criterion based on net load, with the same number of vehicles, traveling distance is smaller. At the same time, from the view of the using rate of vehicle capacity, the criterion also gets a better result, total load in route 1(as shown in the figure 5) is 80, larger than 60 got by using the criterion based on net load.
In all, through the illustration of the two criterions, apparently, the insertion criterion implements efficient combination of traveling distance and residual vehicle capacity. However, it is important to point out that the insertion-based heuristic in this paper needs a great deal of computation, if the customers is too many for us to construct vehicle route by hand, and if conditions are permitted, we can program by using professional computer software to relieve computing load.

5. CONCLUSIONS AND DISCUSSION

As an efficient method for increasing after-sale service quality, protecting environment and reducing logistics costs, people pay more and more attentions to reverse logistics. At the same time, the combination between forward logistics and reverse logistics is becoming more and more important. Therefore, on operational levels, research on VRPSDP is being increasingly strengthened.

This paper puts forward two new insertion criterions with heuristic algorithm, by simple comparison, the one based on residual vehicle capacity is better. It improves TC to a great extent. However, when it comes to the priorities of pick-up and delivery demands of customers as well as capacity restrictions of central depot, VRPSDP will become extremely complex and need further research.

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