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GLS ESTIMATION OF OD MATRIX WITH TRAFFIC COUNTS AND INFORMATION FROM ATIS

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Abstract: With the development of science and technology, Automated Transportation Information System (ATIS) is actively recommended in communications management and the traffic control. The ATIS observers is provided with the information, such as, traffic counts of roads implemented by the sensor of traffic flow and both the location and movement of vehicle recorded by ATIS. The inputs of the estimation are the traffic counts on part of roads and the information of the vehicle movement recorded by ATIS. The output is the Origin-Destination (OD) matrix estimated by the use of GLS method. The initial sample sub-OD matrix from the vehicle with ATIS is estimated first and further the link choice proportion is obtained. Hence, the estimate of OD matrix is extracted from the partial information of the sub-OD matrix and the means of population information of traffic counts by using a statistical model. Its implementation is demonstrated by a numerical example.

Key Words: OD matrix, Automated transportation information system, Estimation

1. INTRODUCTION

Origin-Destination matrix is the basic data for the traffic planning and management. It is demand of traffic flows from origins to destinations, which is expressed as a matrix to explore the movement of space flow. Statistical techniques have become popular in the estimation or updating of OD matrix from traffic counts. There are many famous experts, such as Bell, Cascetta, Snikers, Nguyen, Willumsen, Weibul, Van Zuylen, Maher and McNeil etc, who do a lot of important work about OD matrix estimation from traffic counts. General commenting, previous statistical researches on OD matrix are based on traffic counts, link choice proportion and prior OD matrix. The link choice proportion is usually obtained by the traffic assignment model. Different traffic assignment model produces different link choice proportion. The original source of link choice proportion makes significant impact on the method and accuracy of OD matrix estimation. The prior OD matrix is generally originated from the survey with high cost and long term. However, the proposed method in the article can overcome the shortcoming that the prior OD matrix is generally originated from the survey and research method can not be updated in short time. It is worthful noted that the method can obtain the real-time and reliable data and update the OD matrix simultaneously. Especially, the proposed algorithms can be effective and improve precision of estimation of OD matrix.

The rest of this paper is organized as follows. Statistical models with traffic counts and information from ATIS are firstly described in Section 2. Discussion of the model and algorithm are set in Section 3 and a numerical example is given in Sections 4. Finally, conclusions are given in Section 5.

2. THE MODEL OF OD MATRIX

Let G(N,L) be a transportation network, N is the set of node and L is the set of link. O is the set of origin of the network and the number of the node is h. D is the set of destination of the network and the number of node is *l*. O and D are the subset of N. T is the set of origin-destination pairs of the network and the number of OD pairs is n. M is the set of the observed link and the subset of L. The number of the observed link is m.

2.1 The Data Collection Using ATIS

Suppose we need estimate one day or one week's OD matrix in the network. x_{gk} （g=1,2,…,h；k=1,2,…,l）is denoted as the number of vehicles observed using ATIS from the origin g to the destination k. Let X' be sub-OD matrix of the vehicles observed using ATIS, $X' = \left[x_{gk} \right]_{h \times l}$. r_{ij} （i=1,2,…,m；j=1,2,…,n）is denoted as the link choice proportion, which is the traffic counts using ATIS on the observed link i between the OD pairs j and the traffic counts of the OD pairs j using ATIS. Let R is the matrix of link choice proportion using ATIS, $R = \left[r_{ij} \right]_{m \times n}$. p_{ij}（i=1,2,…,m；j=1,2,…,n）is denoted as the link choice proportion, which is the traffic counts on the observed link i between the OD pairs j and the traffic counts of the OD pairs j. Let P is the matrix of link choice proportion in the network, $P = \left[p_{ij} \right]_{m \times n}$.
P and R have the following relationship

$$R = P + \beta . \tag{1}$$

β is random error, $\beta = \left[\beta_{ij} \right]_{m \times n}$, and $E(\beta) = 0$. Random errors are independent.

Hence, $$E(R) = E(P) + E(\beta) = E(P). \tag{2}$$

Let $\bar{R} = E(R), \bar{R} = \left[\bar{r}_{ij} \right]_{m \times n}, \bar{P} = E(P), \bar{P} = \left[\bar{p}_{ij} \right]_{m \times n}$.

Hence, $$\bar{P} = \bar{R} . \tag{3}$$

2.2 The Model

y_{gk} （g=1,2,…,h；k=1,2,…,l）is denoted as the number of vehicles from the origin g to the destination k. Let Y' be OD matrix of the network, $Y' = \left[y_{ij} \right]_{h \times l}$. a_g （g=1,2,…,h）is denoted as the factor, which influencing the proportion of Y' to X' by the factor of the original g. Let A' be the origin factor matrix, $A' = diag(a_1, a_2, ..., a_h)$. b_k （k=1,2,…,l）is denoted as the factor, which influencing the proportion of Y' to X' by the factor of the destination k. Let B' be the destination factor matrix, $B' = diag(b_1, b_2, ..., b_l)$. Y' and X' have the following relationship

$$Y' = A'X'B' + \alpha' . \tag{4}$$

α' is random error, $\alpha' = \left[\alpha_{ij} \right]_{h \times l}$, and $E(\alpha') = 0$. The random errors are independent. In order to formulation, we can rewrite the equation (4) as

$$Y = AXB + \alpha . \tag{5}$$

Y is another form of OD matrix, $Y = diag\left[y_{11}, y_{12}, .., y_{hl}\right]$. X is another form of OD matrix of vehicles observed using ATIS, $X = diag\left[x_{11}, x_{12}, ..., x_{hl}\right]$. A is another form of the origin factor matrix, $A = \begin{bmatrix} a_1, 0, ..., 0, ... \\ 0, a_1, ..., 0, ... \\ .... \\ 0, ..., a_1, ..., 0 \\ 0..., 0, a_2, ..., 0 \\ .... \\ 0, ..., 0, ..., a_2, ... \\ .... \\ 0, ..., ..., 0, ..., a_h \end{bmatrix}$. B is another form of the destination factor matrix,

$B = \begin{bmatrix} b_1, 0, ..., 0, ... \\ 0, b_2, ..., 0, ... \\ .... \\ 0, ..., b_l, ..., 0 \\ 0, ..., 0, b_1, ..., 0 \\ .... \\ 0, ..., 0, ..., b_l, ... \\ .... \\ 0, 0, ..., ..., 0, b_l \end{bmatrix}$. α is another form of random error, $\alpha = diag\left[\alpha_{11}, \alpha_{12}, ..., \alpha_{hl}\right]$.

Hence, $E(Y) = E(AXB + \alpha) = E(AXB) + E(\alpha) = AE(X)B$. (6)

v_i (i=1,2,…,m) is denoted as the traffic counts on the link i. Let V be traffic counts matrix of the link observed, $V = (v_1, v_2, ..., v_m)^T$. V and Y have the following relationship

$V = PYI + \eta$. (7)

I=$[1, 1, ..., 1]^T$, $\eta = [\eta_1, \eta_2, \eta_3, ..., \eta_m]^T$, and $E(\eta) = 0$. The P is independent to Y.

Hence, $E(V) = E(PYI) + E(\eta) = E(P)E(Y)E(I) = E(P)E(Y)I$. (8)

Let $\overline{V}$ be the average traffic counts matrix of the link observed during a certain period, $\overline{V} = (\overline{v}_1, \overline{v}_2, ..., \overline{v}_m)^T$.

Hence, $e = \overline{V} - E(V) = \overline{V} - E(P)E(Y)I$

$= \overline{V} - E(P)E(AXB + \alpha)I$

$= \overline{V} - E(P)AE(X)BI$. (9)

Let $\overline{X} = E(X)$, $\overline{X} = diag\left[\overline{x}_{11}, \overline{x}_{12}, .., \overline{x}_{hl}\right]$.

Hence, $e = \overline{V} - E(P)AE(X)BI$

$= \overline{V} - \overline{P}A\overline{X}BI$

$= \overline{V} - \overline{R}A\overline{X}BI$. (10)

Hence, $\overline{V} = \overline{R}A\overline{X}BI + e$. (11)

The e is random error, and $E(e)=0$. The random error and explanatory variable are independent.
The estimations $\hat{Y}=AXB$ can be obtained by minimizing the objective function

$$\min_{A,B} e^T e=\left(\bar{V}-\bar{R}A\bar{X}BI\right)^T\left(\bar{V}-\bar{R}A\bar{X}BI\right). \tag{12}$$

In addition to the statistical information of different periods, we can also establish an improved objective function

$$\min_{A,B}\sum_{t=1}^{\theta}\lambda_t\left(\bar{V}_t-\bar{R}_tA\bar{X}_tBI\right)^T\left(\bar{V}_t-\bar{R}_tA\bar{X}_tBI\right). \tag{13}$$

λ_t (t=1,2,…, θ) is the weight of different periods and θ is number of different independent periods.

3. DISCUSSION OF THE MODEL AND ALGORITHM

3.1 Discussion of Two Different Special Conditions of the Model

3.1.1 B Is Constant Matrix and A Is Unknown Matrix

While B is constant matrix and A is unknown matrix, we discuss the solutions of the model. Let

$$\begin{cases} w_{11}=\left(r_{11}b_1\bar{x}_{11}+\ldots+\bar{r}_{1l}b_l\bar{x}_{1l}\right), w_{12}=\left(\bar{r}_{1l+1}b_1\bar{x}_{21}+\ldots+\bar{r}_{12l}b_l\bar{x}_{2l}\right),\ldots, w_{1h}=\left(\bar{r}_{1n-l+1}b_1\bar{x}_{h1}+\ldots+\bar{r}_{1n}b_l\bar{x}_{hl}\right) \\ w_{21}=\left(\bar{r}_{21}b_1\bar{x}_{11}+\ldots+\bar{p}_{2l}b_l\bar{x}_{1l}\right), w_{22}=\left(\bar{r}_{2l+1}b_1\bar{x}_{21}+\ldots+\bar{p}_{22l}b_l\bar{x}_{2l}\right),\ldots, w_{2h}=\left(\bar{r}_{2n-l+1}b_1\bar{x}_{h1}+\ldots+\bar{p}_{2n}b_l\bar{x}_{hl}\right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ w_{m1}=\left(\bar{r}_{m1}b_1\bar{x}_{11}+\ldots+\bar{r}_{ml}b_l\bar{x}_{1l}\right), w_{m2}=\left(\bar{r}_{ml+1}b_1\bar{x}_{21}+\ldots+\bar{r}_{m2l}b_l\bar{x}_{2l}\right),\ldots, w_{mh}=\left(\bar{r}_{mn-l+1}b_1\bar{x}_{h1}+\ldots+\bar{r}_{mn}b_l\bar{x}_{hl}\right) \end{cases},$$

$W=\left[w_{ig}\right]_{m\times h}$.

Hence, $e=\bar{V}-WA'I$. (14)

Hence, $\bar{V}=WA'I+e$. (15)

The estimations $\hat{Y}=AXB$ can be obtained by minimizing the objective function

$$\min_{A} e^T e=\left(\bar{V}-WA'I\right)^T\left(\bar{V}-WA'I\right). \tag{16}$$

Hence, $W^TW\hat{A}'I=W^T\bar{V}$. (17)

Hence, $\hat{A}'I=\left(W^TW\right)^{-1}W^T\bar{V}$. (18)

According to the mentioned above, it proves that $\hat{A}$ is the best linear unbiased estimates.

Theorem1. As to the least square estimate on AX=b, if the rank of A=$\left(a_{ij}\right)_{m\times n}$ is n(m>n), that is if the columns of A are not linearly relative, there is only one solution to the problem of least squares estimate.

As all the columns of W are independent to each other, W is not linearly relevant. Therefore, there is only one solution to (16).

3.1.2 A Is Constant Matrix and B Is Unknown Matrix

While A is constant matrix and B is unknown matrix, we discuss the solutions of the model.

Let

$$\begin{cases} u_{11}=\bar{r}_{11}a_1\bar{x}_{11}+...+\bar{r}_{1n-l+1}a_h\bar{x}_{h1}, u_{12}=\bar{r}_{12}a_1\bar{x}_{12}+...+\bar{r}_{1n-l+2}a_h\bar{x}_{h2},...,u_{1l}=\bar{r}_{1l}a_1\bar{x}_{1l}+...+\bar{r}_{1n}a_h\bar{x}_{hl} \\ u_{21}=\bar{r}_{21}a_1\bar{x}_{11}+...+\bar{r}_{2n-l+1}a_h\bar{x}_{h1}, u_{22}=\bar{r}_{22}a_1\bar{x}_{12}+...+\bar{r}_{2n-l+2}a_h\bar{x}_{h2},...,u_{2l}=\bar{r}_{2l}a_1\bar{x}_{1l}+...+\bar{r}_{2n}a_h\bar{x}_{hl} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ u_{m1}=\bar{r}_{m1}a_1\bar{x}_{11}+...+\bar{r}_{mn-l+1}a_h\bar{x}_{h1}, u_{m2}=\bar{r}_{m2}a_1\bar{x}_{12}+...+\bar{r}_{mn-l+2}a_h\bar{x}_{h2},...,u_{ml}=\bar{r}_{ml}a_1\bar{x}_{1l}+...+\bar{r}_{mn}a_h\bar{x}_{hl} \end{cases},$$

$U=\left[u_{ik}\right]_{m\times l}$.

Hence, $e=\bar{V}-UB'I$. (19)

Hence, $\bar{V}=UB'I+e$. (20)

The estimations $\hat{Y}=AXB$ can be obtained by minimizing the objective function

$$\min_B e^T e=\left(\bar{V}-UB'I\right)^T\left(\bar{V}-UB'I\right). \quad (21)$$

Hence, $U^T U\hat{B}'I=U^T\bar{V}$. (22)

Hence, $\hat{B}'I=\left(U^T U\right)^{-1}U^T\bar{V}$. (23)

According to the mentioned above, it proves that $\hat{B}$ is the best linear unbiased estimates and there is only one solution to (21).

3.2 Quality of Solutions of the Model

Let $f=\left(\bar{V}-\bar{R}A\bar{X}BI\right)^T\left(\bar{V}-\bar{R}A\bar{X}BI\right)$. (24)

3.2.1 Proposition 1. (24) is pseudo convex function to variable matrix A and B.

Proof. Let B be constant matrix. Hence, (24) is convex function. Hence, $\forall\vec{a},\vec{a}'\in\vec{R}^+$,

Hence, $\left(\vec{a}'-\vec{a}\right)^T\dfrac{\partial f\left(\vec{a},\vec{b}\right)}{\partial\vec{a}}\geq 0$ and $\left(\vec{a}'-\vec{a}\right)^T\dfrac{\partial f\left(\vec{a},\vec{b}'\right)}{\partial\vec{a}}\geq 0$.

That is $f\left(\vec{a}',\vec{b}\right)\geq f\left(\vec{a},\vec{b}\right)$ and $f\left(\vec{a}',\vec{b}'\right)\geq f\left(\vec{a},\vec{b}'\right)$. (25)

Let A be constant matrix. Hence, (24) is convex function. Hence, $\forall\vec{b},\vec{b}'\in\vec{R}^+$,

Hence, $\left(\vec{b}'-\vec{b}\right)^T\dfrac{\partial f\left(\vec{a},\vec{b}\right)}{\partial\vec{b}}\geq 0$ and $\left(\vec{b}'-\vec{b}\right)^T\dfrac{\partial f\left(\vec{a}',\vec{b}\right)}{\partial\vec{b}}\geq 0$.

That is $f\left(\vec{a},\vec{b}'\right)\geq f\left(\vec{a},\vec{b}\right)$ and $f\left(\vec{a}',\vec{b}'\right)\geq f\left(\vec{a}',\vec{b}\right)$. (26)

Hence, $\forall\vec{a},\vec{a}',\vec{b},\vec{b}'\in\vec{R}^+$,

$$f\left(\vec{a}',\vec{b}'\right)\geq f\left(\vec{a},\vec{b}\right). \quad (27)$$

Hence, (24) is also pseudo convex function to variable matrix A and B.

Theorem2. If $f(x)$ is pseudo convex function, $f(x)$ is strictly quasic convex function.

Theorem3. Let R be convex set, $f(x)$ is strictly quasic convex function, thus, the partial optimum solutions of the programming problem

$$\min f(x)$$
$$x \in R$$

is the global optimum solutions.

Hence, the partial optimum of (12) is the global optimum solution.

3.2.2 Proposition 2. The Estimations $\hat{Y}$ Obtained By (12) Is Unique.

Proof. Let A_1, B_1 and A_2, B_2 be the global optimum solutions of (12). Hence, $\hat{Y}_1 = A_1 \overline{X} B_1$ and $\hat{Y}_2 = A_2 \overline{X} B_2$. Put A_1 into (12). (12) translate (16). Since A_1, B_1 is the global optimum solutions of (12), B_1 is the global optimum solutions of (16). Hence, A_1, B_1 is also the global optimum solutions of (17). Put A_1, B_1 into (17). By formulating, we can obtain $A_1 \overline{X} B_1 I = \left(R^T R\right)^{-1} R^T \overline{V}$. Hence, as mentioned above, Put A_2, B_2 into (17). By formulating, we can also obtain $A_2 \overline{X} B_2 I = \left(R^T R\right)^{-1} R^T \overline{V}$. Hence, $A_1 \overline{X} B_1 I = A_2 \overline{X} B_2 I$. Hence, $\hat{Y}_1 = \hat{Y}_2$. Hence, the estimations $\hat{Y}$ obtained by (12) is unique.

3.3 Algorithm

It's difficult to estimate parameters of such a complicated model directly with normal traditional algorithms. So, a new algorithm is designed here. The advantage of the algorithm is that it makes the calculation easier by reducing dimensions, especially for estimating the parameters of complicated model. Meanwhile, the algorithm overcomes the shortcomings of using the non-linear least square directly by making use of the advantage of linear least squares to estimate the OD matrix. The procedures of the proposed algorithm are shown as follows.

Step1. A_0 and B_0 as original values are given and set the parameter τ of control.
Calculate $f_0 = \left(\overline{V} - \overline{R} A_0 \overline{X} B_0 I\right)^T \left(\overline{V} - \overline{R} A_0 \overline{X} B_0 I\right)$.

Step2. A_1 is generated by $\min e_0^T e_0 = \left(\overline{V} - \overline{R} A \overline{X} B_0 I\right)^T \left(\overline{V} - \overline{R} A \overline{X} B_0 I\right)$. Calculate $f_1 = \left(\overline{V} - \overline{R} A_1 \overline{X} B_0 I\right)^T \left(\overline{V} - \overline{R} A_1 \overline{X} B_0 I\right)$. If the condition $|f_0 - f_1| \le \tau$ is satisfied, let $f_0 = f_1$, $A_0 = A_1$ and go to step 4. Otherwise, go to Step 3.

Step3. B_1 is generated by $\min e_1^T e_1 = \left(\overline{V} - \overline{R} A_1 \overline{X} B I\right)^T \left(\overline{V} - \overline{R} A_1 \overline{X} B I\right)$.Calculate $f_2 = \left(\overline{V} - \overline{R} A_1 \overline{X} B_1 I\right)^T \left(\overline{V} - \overline{R} A_1 \overline{X} B_1 I\right)$. Let $A_0 = A_1$, $B_0 = B_1$, $f_0 = f_2$.
If the condition $|f_1 - f_2| \le \tau$ is satisfied, go to step 4. Otherwise, go to Step 2.

Step4. Calculate $\hat{Y} = A_0 \overline{X} B_0$ .The process terminates.

4. NUMERICAL EXAMPLE

To demonstrate the use of the proposed method, suppose the following sampled data are available for the small synthetic network described. The proposed method is described by the example. The network example is shown with 4 OD pairs in Figure1. The data is presented in Table1.

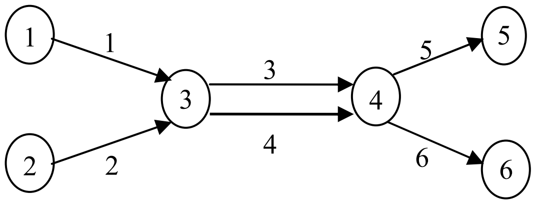

Figure1. A Simple Network

Table1. Data of the Network Example

OD Pairs	P_1	P_2	P_3	P_4	P_5	P_6	Sub-OD	Actual OD
From 1 to 5	1	0	0.4	0.6	1	0	100	120
From 1 to 6	1	0	0.5	0.5	0	1	60	80
From 2 to 5	0	1	0.8	0.2	1	0	50	60
From 2 to 6	0	1	0.7	0.3	0	1	80	100
Traffic Counts	200	160	210	160	180	180		

Table2.Two Methods of OD Matrix Estimation

Estimation Methods	y_{11}	y_{12}	y_{21}	y_{22}	Error Precision
the New Estimation Method	122	79	59	102	0.86%
the Old Estimation Method	115	79	62	95	1.66%

Programs of OD matrix estimation have been written in the Matlab programming language. The result of OD matrix estimation is showed in Table2. The error precision is measured by average relative error, which is

$$E = \frac{\sqrt{\sum_{ij}\left(\frac{y_{ij} - y_{ij}^{0}}{y_{ij}^{0}}\right)^2}}{n} \times 100\% \quad (28)$$

In order that the new least square method compare with old least square methods, we take the sub-OD matrix as the prior OD matrix to estimate OD matrix. We select a traditional least square method, which is

$$\min e^T e = \left(V - PY''\right)^T \left(V - PY''\right) + (Y'' - Y_0'')^T (Y'' - Y_0''), \quad (29)$$

$$Y'' = [y_{11}, y_{12}, \ldots, y_{hl}].$$

The result of OD matrix estimation is showed in Table 2. Comparing the error precision of the two methods, the results show that the error precision of the new method is 0.86% and the error precision of the old method is 1.66%. The error precision of the new method is less. Therefore, the error precision is quite acceptable. It also illuminates performance of the new method is very good.

Moreover, we can change the sub-OD matrix to test the new method. The new sub-OD matrix can be generated by random numbers of normal distribution. We can obtain a group of data of the sub-OD matrix and the result is showed in Table 3. The average error precision of the new method is 0.76% and the error precision of the old method is 5.61%. The error precision of the new method is sensational. The results show that the new method has very good stable performance.

Table3. OD Matrix Estimation by Changing the Sub-OD Matrix

Sub-OD				the New Method					the Old Method				
x_{11}	x_{12}	x_{21}	x_{22}	y_{11}	y_{12}	y_{21}	y_{22}	E	y_{11}	y_{12}	y_{21}	y_{22}	E
60	40	30	50	121	80	60	100	0.44%	102	82	65	85	5.59%
60	39	29	49	121	79	59	101	0.60%	103	82	65	85	5.48%
59	39	30	49	120	81	60	100	0.50%	102	82	65	85	5.73%
60	40	31	49	120	81	61	99	0.89%	102	82	66	85	5.86%
60	40	30	49	120	81	60	100	0.56%	102	82	65	85	5.72%
60	40	28	49	122	79	59	101	0.69%	103	82	64	85	5.44%
61	41	28	49	121	79	59	101	0.61%	103	82	64	85	5.43%
56	35	29	52	123	77	57	103	1.74%	102	80	64	87	5.19%
55	31	33	47	121	80	60	101	0.43%	101	80	67	85	6.16%
60	43	24	47	122	78	58	102	1.14%	103	83	64	85	5.51%
				Average Error				0.76%	Average Error				5.61%

Table4. OD Matrix Estimation by Reducing the Observed Links

	Average Error of the New	Average Error of the Old
Reducing One Observed Link	1.05%	1.93%
Reducing Two Observed Links	1.94%	2.44%

Further, we can also study the result of OD matrix estimation by reducing different observed links. The result of OD matrix estimation is showed in Table4. When we reduce a different observed link one by one, we can obtain the error precision of six conditions. The average error precision of the new method is 1.05% and the average error precision of the old method is 1.93%. The error precision of the new method is less. In addition, when we reduce two different observed links one by one, we can obtain the error precision of fifteen conditions. The average error precision of the new method is 1.94% and the average error precision of the old method is 2.44%. The error precision of the new model is also less. It can testify the model has good quality. Therefore, it validates the new OD matrix estimation method is steady and reliable.

5. CONCLUSION

The paper proposes the OD matrix estimation model by the use of GLS method. The new approach is advantageous in three aspects: (1) The link choice proportion is obtained by using ATIS. It offers a new method for getting the original source of link choice proportion. And it can be updated in short time and improves accuracy of OD matrix estimation. (2) Due to the sub-OD matrix replacing the prior OD, the proposed method can obtain the real-time and reliable data of OD matrix in short time and update the OD matrix simultaneously. (3) The proposed method makes better use of related data and information. Not only that results obtained from different periods of time can be combined and utilized for the estimation of OD matrix, but additional information related to the reliability and variations of the observed values can also be incorporated into the model. In addition, this paper proves the uniqueness of solutions of OD matrix estimation. And by the numerical example, it demonstrates the applicability of the new model and it shows that the new model can be effective and improve precision of OD matrix estimation.

Some future extensions of research work are suggested. The algorithm should be further studied. The model for a large-scale network can be explored. The model can also be extended to a dynamic framework.

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REFERENCES

Bell, M.G.H. (1983) The estimation of an origin-destination matrix from traffic counts, **Transportation Science, Vol.17, No.2**, 198–217.

Bell, M.G.H. (1991) The estimation of origin-destination matrices by constrained generalized least squares, **Transportation Research Part B, No.25**, 13-22.

Cascetta, E. (1984) Estimation of trip matrices from traffic counts and survey data: a generalized least squares estimation, **Transportation Research Part B, No.18**, 289-299.

Cascetta, E. and Nguyen, S. (1988) A unified framework for estimating or updating origin-destination matrices from counts, **Transportation Research Part B, No. 22**, 437-455.

Chen, X.R. and Wang, S.G. (2003) Least squares estimate of linear model, **Shanghai Science Technology Press**, Shanghai.

Lo, H.P., ZHANG, N. and Lam, W.H.K. (1996) Estimation of an origin-destination matrix with random link choice proportion: a statistical approach, **Transportation Research Part B, No.30**, 309-324.

Lo, H.P., Zhang, N. and Lam, W.H.K. (1999) Decomposition algorithm for statistical estimation of OD matrix with random link choice proportions from traffic counts, **Transportation Research Part B, No.33**, 369-385.

Li, J. and Peng, G.X. (2001) A study on estimating OD trips from observed link flows—based on the method of iterative procedure to calculate OD on the multi-path assignment model, **Journal of Traffic and Transportation Engineering, Vol.1, No.2**, 78-82.

Michel, B. (2002) The total demand scale: a new measure of quality for static and dynamic origin-destination trip tables, **Transportation Research Part B, No.36**, 837-850.

Wang J.M. (2002) The study and analysis of model algorithm for dynamic origin-destination matrix estimation and prediction, **Ningxia Engineering Technology, Vol.1, No.4**, 362-365.

Wei, Q.L. and Yan, H. (2003) General Optimization Theory and Model, **Science Press**, Beijing.

Yang, Q., Wang, W., Lu, L. and Ge, S.M. (1999) Research on traffic counting locations for OD matrix estimation, **China Journal of Highway and Transport, Vol.12, Sup.**, 81-87.