Study on External Collecting and Distributing Network Collapse Path Model and Algorithm of Chinese Urban HSR Passenger Station

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Abstract: As an important part of the connection between railway passenger station and urban transportation, railway passenger station external distribution network is a key link in operation management of passenger collecting and distributing. Firstly, it distributes the adaptability agent network topology based on the complex adaptive system theory. Secondly, it defines the collapse path of the railway passenger station external distribution network and sets up the maximum collapse path model. According to the particularity of the model, it converts the model into the shortest path problem and uses Lagrangian relaxation algorithm to solve it. Finally, it takes the Beijing south station collecting and distributing network as an example to verify the practicability and effectiveness.

Keywords: Urban railway passenger station, Collapse path, Lagrangian relaxation algorithm, Collecting and distributing network

1. INTRODUCTION

The fast development of High speed railway (HSR) has an important influence on intercity transportation system and urban transportation system. In urban transportation system, only railway passenger station has perfect external collecting and distributing system, can it be play the role of main nodes of urban traffic network. As an important part that joins railway passenger station and the urban road, the structure of collecting and distributing network of urban railway passenger station is more and more complex. And as a complex system, it has very important practical significance to study collapse path [1] .For the problem of external collecting and distributing network of railway passenger station, the research method and view in recent years mainly focus on the following aspects: hang Yu-biao[2] deeply analyzed the relationship between passenger station of HSR and urban traffic from the point of the transportation planning, and also has the research on traffic integration planning of HSR area from two aspects of traffic demand forecast and the integration planning; Based on the research of railway hub and urban traffic modes, Xu Wei[3] studied the coordination scheme ,as well as the design, integration and optimization for joint, and determined attracting range and connection mode of railway sites during integrating; Tang Huai-hai[4] put forward definition and boundary collecting and distributing transportation system of HSR, and using three visual model technology, he analyzed the simulation of static structure, controlling elements and information change of collecting and distributing transportation system. In the aspect of connection between railway passenger station and urban traffic, Du Heng[5] compared the network structure of the area of urban railway hub at home and abroad, and drew the conclusion that there exists inadequacy of the density, level and evacuation model of the region of domestic railway passenger transport hub. Based on research of typical network layout of borderline-high-speed railway passenger transport hub, Li Xing[6]calculated preponderant travel distance of different road-level under kinds of demand, calculated transportation turnover investigation for each road grade using electron cloud model, and get the network distribution calculation model for hub area. In the aspect of model and algorithm for ， established evolution direction discriminant model for complicated system to determine the direction of the evolution, gave qualitative research on the relations of constraints for the complicated system, and established a preliminary mathematical model. Wu Hong-mei[8]found the collapse path of the complex system, and found the shortest time that may make whole complex system collapse according to the brittle excitation delay. Lin De-ming[9]established empowerment graph model of a complex system brittleness, define the minimum and maximum collapse path for system, put forward ant colony algorithm for solving the problem of the biggest collapse path.

External collecting and distributing network of railway passenger station is an urban traffic channel system which means that passenger flow travel together from starting point to the railway station, or evacuates from railway station to the starting point. It is mainly composed by railway passenger distribution channel and distribution system surrounding railway station, and is an important part of the connection between railway passenger station and urban traffic.

When dividing boundary of external collecting and distributing network of railway passenger station, it must be according to China's national conditions and the actual operational and management mode of the railway station. According to the size of cohesion region, distribution system is divided to 3 layers from low to high, including cohesion with surrounding area of railway passenger station (walking 5-10 minutes), cohesion with the district and the city center area (walking 10-15 minutes), cohesion with surrounding large towns (walking more than 15 minutes).The distribution network studied in this paper includes the first layer and second layer. The network structures, network density, node load degree, correlation coefficient of distribution network seriously influence the operation of distribution network. According to the arrival passenger flow volume, traffic flow assigned to distribution network can be forecast, congestion path of the distribution network can be found out, and the traffic can be managed and controlled pertinently, which can make distribution network play the distribution effect and promote prosperity of the city.

2. THE LARGEST COLLAPSE PATH OF DISTRIBUTION NETWORK MODELING

2.1 The Definition for Largest Collapse Path of Distribution Network

When a subsystem of a complicated system suffered an attack of enough external force, including physical external force as well as external factors such as information and material flow, the original ordered state was destroyed, while the new disordered state was setting up, it was called that subsystem collapse. And its collapse will make other subsystem’s order destroy, at last lead to collapse. The property of complicated system is called the brittleness. According to the complicated system brittleness theory[7],each node is given a certain initial entropy value and the critical threshold in adaptive Agent diagram of the distribution network. When the node entropy value of railway passenger station is more than the threshold value caused by the interference and led collapse, the node absorb the negative entropy from neighbor node according to certain strategies, which lead to a new round of entropy redistribution. When this process repeated, the affected nodes gradually spread, and finally lead to the collapse of the whole distribution network.

There are high grade road intersection or overpass on the boundary of distribution network layers, which is defined the main nodes(General for 4) of distribution network. The scale of traffic stream is 8 OD from railway passenger station node to other four main node.

1) $V(h_i) \subset V(D)$
2) $E(h_i) \subset E(D)$
3) In distribution network, when there is no relationship between 8 OD of supply and demand, distribution network is collapsing. There is a breakdown path $h_i (i = 1, 2, \cdots, n)$ between each OD which satisfies the condition, it is called the biggest collapse path for distribution network.

$v_i (i = 1, 2, \cdots, n)$: Distribution network node, as the railway passenger station

$u_l (l = 1, 2, \cdots, m)$: High grade road intersection or overpass on the boundary of distribution network layers

$h_i (i = 1, 2, \cdots, t)$: There is a breakdown path between each OD

$V(h_i)$, $E(h_i)$: Nodes and sections set passed by collapse path

$\omega(e)$: The weight of edge of the road

Z^*: Product for road section weight

2.2 The Biggest Collapse Path Modeling

Based on the premise of the known OD demand, use the sampling method of Monte Carlo for random sampling, allocate wagon-flow for the whole distribution network through the stochastic equilibrium assignment (SUE) model [11]. The weight $w(\mathrm{e}{:}v_i\text{-}v_j)$ of road section $e \in < v_i, v_j >$ is defined as the proportion in the total flow of transferring traffic from v_i to v_j:

$$\omega(e) = f_{v_i v_j} \big/ f_i \qquad \text{equ(1)}$$

Aiming at research on external collecting and distributing network of railway passenger station, suppose that transfer direction of a distribution network collapse is $v_{i_1} \to v_{i_2} \to \cdots \to v_{i_n}$,the collapse behavior of each node has relations with node which has contact in brittle, according to the conditional probability formula:

$$\begin{aligned}
\omega_{h_i} &= P(B_{i_n} B_{i_{n-1}} \cdots B_{i_1} \mid B_{i_1}) = \frac{P(B_{i_n} B_{i_{n-1}} \cdots B_{i_1})}{P(B_{i_1})} \\
&= \frac{P(B_{i_2} B_{i_1})}{P(B_{i_1})} \frac{P(B_{i_3} B_{i_2} B_{i_1})}{P(B_{i_2} B_{i_1})} \cdots \frac{P(B_{i_n} \cdots B_{i_1})}{P(B_{i_{n-1}} \cdots B_{i_1})} \\
&= P(B_{i_2} \mid B_{i_1}) P(B_{i_3} \mid B_{i_2} B_{i_1}) \cdots P(B_{i_n} \mid B_{i_{n-1}} \cdots B_{i_1}) \qquad \text{equ (2)} \\
&= P(B_{i_2} \mid B_{i_1}) P(B_{i_3} \mid B_{i_2}) \cdots P(B_{i_n} \mid B_{i_{n-1}}) \\
&= \omega_{i_1 i_2} \omega_{i_2 i_3} \cdots \omega_{i_{n-1} i_n}
\end{aligned}$$

Set the number of OD of the adaptive Agent chart is m, evaluate t collapse path between node v_p and node $u_l (l = 1, 2, \cdots, m)$,satisfy the following conditions:

$$Z^* = \max_{h \in E(h_i)} \Big(\prod_{e \in h_i} \omega(e) \Big) � (i = 1, 2, \cdots, n) \qquad \text{equ(3)}$$

$$s,t\sum_{j:ij\in E}x_{ij}-\sum_{j:ji\in E}x_{ji}=\begin{cases}1, & i=o\\ 0, & i\in N-\{o,d\}\\ -1, & i=d\end{cases}$$ equ(4)

m : the number of OD of distribution network

x_{ij} : dualism variable, $x_{ij}=1$ indicates section $e\in<v_i,v_j>$ contains solution path, $x_{ij}=0$ is not included. Take $h_{\max}$ as the biggest collapse path for OD of distribution network.

The collapse path of the distribution network reflects the possible order and movement direction that brittle acts. The weight of collapse path for distribution network reflects the possibility of probability caused by the starting point collapse. The weight of collapse path is the product of the weight of all sections.

3. PATH COLLAPSE OF DISTRIBUTION NETWORK BASED ON LAGRANGIAN RELAXATION ALGORITHM

3.1 Optimization Ideas

Assume that the biggest collapse path for each OD of a distributed network is the directed routing from node v_p to u_l . The node number is $v_p\to v_{i_1}\to v_{i_2}\to\cdots\to v_n\to u_l$ or $u_l\to v_{i_n}\to v_{i_{n-1}}\to\cdots\to v_{i_1}\to v_p$ in order. The arc weights for collapse path is $\omega_{pi_1},\omega_{i_1i_2},\cdots,\omega_{i_nl}$ or $\omega_{li_n},\cdots,\omega_{i_1i_2},\omega_{i_1p}$ .In the distribution network D, if $i=j\in V(\mathrm{D})$, then $\omega_{ij}=1$;when $i,j\notin V(\mathrm{D})$,then $\omega_{ij}=0$

In order to use the solving ideas of the shortest circuit diagram in empowerment, we can do the following conversion:

$$c_{ij}=\begin{cases}-\ln\omega_{ij}, & 0<\omega_{ij}<1\\ \text{inf}, & \omega_{ij}=0\\ 0, & \omega_{ij}=1\ or\ i=j\end{cases}$$ equ(5)

c_{ij} :The logarithm of weight ω_{ij}

3.2 Lagrangian Relaxation Algorithm

According to the equ (3) and equ (5),the problem of The largest collapse path for distribution network can be converted into the most short circuit problem.

We use Lagrangian Relaxation Algorithm to get the biggest collapse path more accurately. Not only can Lagrangian Relaxation Algorithm be used to evaluate the effect of the algorithm, it can also improve the efficiency of the algorithm. It contains two parts: providing lower bound and Lagrangian relaxation heuristic algorithm.

(1)Considering mean value of c_{ij} ,the deviation degree (standard deviation) between the weights and mean value for each section in the objective function, in order to get a better biggest result.

$$Z_0^*=\min\sum_{ij\in A}\bar{c}_{ij}x_{ij}+\beta\sqrt{\mathrm{var}(c_p)}$$ equ(6)

$$s.t\sum_{j:ij\in E}x_{ij}-\sum_{j:ji\in E}x_{ji}=b$$ equ(7)

β : Weigh coefficient

$$b = \begin{cases} 1, & i = o \\ 0, & i \in N - \{o, d\} \\ -1, & i = d \end{cases} \qquad \text{equ(8)}$$

Constraint indicates the equilibrium state of traffic flow in distribution network

Introduce a nonnegative auxiliary variable y, and set $y = \sum_{ij \in A} \sigma_{ij}^2 x_{ij}$, then equ (6) is converted into:

$$Z^* = \min \sum_{ij \in A} \bar{c}_{ij} x_{ij} + \beta \sqrt{y} \qquad \text{equ (9)}$$

$$s.t \sum_{ij \in A} \sigma_{ij}^2 x_{ij} = y \qquad \text{equ (10)}$$

$$\sum_{j:ij \in E} x_{ij} - \sum_{j:ji \in E} x_{ji} = b \qquad \text{equ (11)}$$

$$0 \le y \le y' \qquad \text{equ (12)}$$

y' is related to variance expectations of correlation coefficient of traffic flow between nodes; $U(y) = \beta \sqrt{y}$ is monotonically increasing concave function.

According to Lagrangian relaxation theory, relax the constraint $\sum_{ij \in A} \sigma_{ij}^2 x_{ij}$, and it is changed into:

$$\sum_{ij \in A} \sigma_{ij}^2 x_{ij} \le y \qquad \text{equ(13)}$$

Join Lagrangian multiplier $\mu(\mu \ge 0)$ into the objective function ,and get the following dual function:

$$Z^* = \min \sum_{ij \in A} \bar{c}_{ij} x_{ij} + \beta \sqrt{y} + \mu(\sum_{ij \in A} \sigma_{ij}^2 x_{ij} - y) \qquad \text{equ (14)}$$

$$s.t \sum_{j:ij \in A} x_{ij} - \sum_{j:ji \in A} x_{ji} = b \qquad \text{equ (15)}$$

$$0 \le y \le y' \qquad \text{equ (16)}$$

Merge similar terms for the target function:

$$Z^* = \min \sum_{ij \in A} \left(\overline{c_{ij}} + \mu \sigma_{ij}^2 \right) x_{ij} + \beta \sqrt{y} - \mu y \qquad \text{equ (17)}$$

Command:

$$L(\mu) = \min \left\{ \sum_{ij \in A} \left(\overline{c_{ij}} + \mu \sigma_{ij}^2 \right) x_{ij} + \beta \sqrt{y} - \mu y : \sum_{j:ij \in A} x_{ij} - \sum_{j:ji \in A} x_{ji} = b, 0 \le y \le y' \right\},$$

$\mu(\mu \ge 0)$:Lagrangian multiplier.

(2)Decompose Lagrangian Dual problem:

$$L(\mu) = L_x(\mu) + L_y(\mu) \qquad \text{equ (18)}$$

$$L_x(\mu) = \min \left\{ \sum_{ij \in A} \left(\overline{c_{ij}} + \mu \sigma_{ij}^2 \right) x_{ij} : \sum_{j:ij \in A} x_{ij} - \sum_{j:ji \in A} x_{ji} = b \right\},$$

Obviously $L_x(\mu)$ is typical of the most short circuit problem, whose objective function is linear. So more mature method for he most short circuit algorithm can be used, $L_y(\mu) = \min \left\{ \beta \sqrt{y} - \mu y : 0 \le y \le y' \right\}$. The objective function for $L_y(\mu)$ is concave function.

According to the theory of optimum, the optimal solution for $L_y(\mu)$ can be got in the pole of the feasible region (showed in figure 1).Therefore, $L_y(\mu) = \min\left\{0, \beta\sqrt{y'} - \mu y'\right\}$

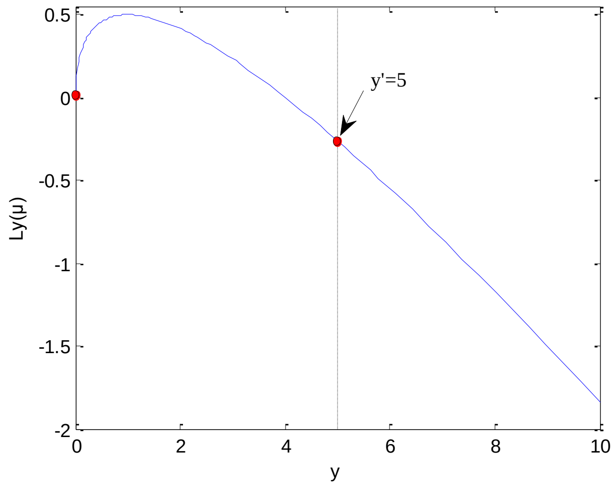

Figure 1. While β=1�μ=0.5, optimal solution for $L_y(\mu)$

(3)Subgradient method

According to Lagrangian relaxation theory, $L^* = \max\limits_{\mu} L(\mu)$ is the the lower bound LB of original problem, and the upper bound UB is obtained the feasible region directly.

$$Z_0^* = \min \sum_{ij \in A} \bar{c}_{ij} x_{ij} + \beta\sqrt{\mathrm{var}(c_p)} \qquad \text{equ (19)}$$

$$s.t \sum_{j:ij \in E} x_{ij} - \sum_{j:ji \in E} x_{ji} = b \qquad \text{equ (20)}$$

In order to get search direction for μ, the subgradient for $L(\mu)$ must be solved.

$$\nabla L(\mu) = \sum_{ij \in A} \sigma_{ij}^2 x_{ij} - y \qquad \text{equ (21)}$$

Update Lagrangian multiplier:

$$\mu^{k+1} = \mu^k + \theta^k \left(\sum_{ij \in A} \sigma_{ij}^2 x_{ij}^k - y^k \right) \qquad \text{equ (22)}$$

k is referred to iterations, and θ^k is referred to the step size. In order to ensure that Lagrangian multiplier μ is non-negative θ^k can be measured By the following type

$$\theta^k = \frac{\lambda^k \left(UB - L\left(\mu^k\right)\right)}{\left\| \sum_{ij \in A} \sigma_{ij}^2 x_{ij}^k - y^k \right\|^2}$$ ��λ can be in valued from 0 to 2� , equ23)

In conclusion, termination conditions of Lagrangian relaxation algorithm can be controlled by the two methods. The first one is the common iteration steps k; The second is in the limit range of error $\varepsilon = UB - L^*$.The proportional error is as follows:

$$\varepsilon' = \frac{UB - LB}{UB} \qquad \text{equ (24)}$$

UB—Evolution algorithm upper bound ;

LB —Evolution algorithm lower bound

3.3 A Concrete Implement Steps for Algorithm

Step1：Initialization for Algorithm parameters initialization

Give any Lagrangian Multiplier $\mu > 0$, and set the initial iterations $k=0$.Give an initial feasible solution as the upper bound UB of the model.

Step2：Solve two son functions of Lagrangian dual function

Firstly, use Dijkstra Algorithm to solve $L_x(\mu)$, then work out $L_y(\mu)=\min\left\{0, \beta\sqrt{y'} - \mu y'\right\}$ which is more easily solved. At last, calculate error limit $\varepsilon = UB - LB$

Step3：Updated Lagrangian multiplier μ

Use $\mu^{k+1} = \mu^k + \theta^k \left(\sum_{ij\in A} \sigma_{ij}^2 x_{ij}^k - y^k \right)$ to update Lagrangian multiplier $\mu > 0$, and $\theta^k = \dfrac{\lambda^k \left(UB - L\left(\mu^k\right)\right)}{\left\| \sum_{ij\in A} \sigma_{ij}^2 x_{ij}^k - y^k \right\|^2}$.

Step4：Judgment termination conditions

Give a maximum iterations $Iter$ and permissible error limit ε' ,if $k > Iter \ or \ \varepsilon \le \varepsilon'$,then stop calculation; otherwise, back to Step2.

4.EXAMPLE CALCULATION

Passenger railway station is as the center of distribution network ,which can be divided into gathered network and divergent network according to the direction of traffic flow. Take external divergent network of Beijing south station as an example, the biggest collapse path of important nodes is studied. It is the City borderline passenger station from the relative relationship of the city space layout.

The known parameters for Beijing south station include walking-time for free flow, distribution capacity of road section and traffic volume of each OD. Walking-time for free flow can be got by the section length and speed limit. The passenger flow ratio between four main nodes of Beijing south station in rush hour is 4:3:2:1.Define that traffic flow of node in unit of time is $f_i(t)$, the traffic occurrence and attract quantity of each OD is just as the following Table1.

Table1.Traffic occurrence and attract quantity of distribution network for Beijing south station

	D_1	D_2	D_3	D_4	Total PUC
O	1806	903	1354	452	4515

The capacity of distribution is that of Beijing south station which removed through traffic flow.Passing rate for express way is 83.8%,arterial road is 62.23% ,and central branch is 4.97.Use SUE thrust method [11]to distribute the traffic flow of each section for distribution network.Using GAMS modeling software, $\theta = 0.1$,sampling obey normal distribution.After 100 times SUE thrust was made, results and weights of each section of Beijing south station distribution network were got.Using Lagrangian Relaxation Algorithm,solve the the biggest collapse path problem of OD,thereinto,node 10 is for O,node 1,3,18,21 are for D;node 10 is for D,node 1,3,18,21 are for O.The results are showed in the following Figure2.

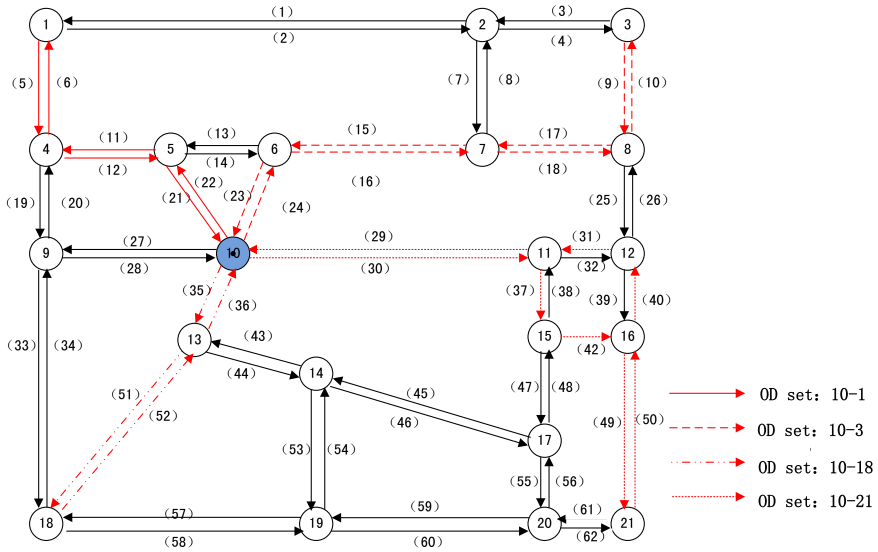

Figure2. Beijing south station collapse path diagram

As is shown in figure2,node 5、4、 6、7、8、13、11、12、16 are crossings whose management should be focused on. Traffic management can be improved by traffic controlling and traffic inducing, which can improve the efficiency of Beijing south station passenger flow distribution.

While $\varepsilon' = \dfrac{UB - LB}{UB} \geq \dfrac{Z_0^* - L^*}{Z_0^*}$, it means that the relative error is greater than actual error between the optimal solution and the dual problem. Through the Lagrangian Relaxation algorithm,100 times iteration were given. ε' floats from 0.01 to 0.07.And this verify practicability and effectiveness of Lagrangian Relaxation algorithm to solve the problem of short circuit .

5.CONCLUSION

As the joint part connecting railway passenger station and urban traffic, congestion and delay of distribution network has brought great challenges for operation and management of railway passenger station. According to the biggest collapse path problem of railway passenger distribution network, take the product of weights for each section as the target function. As he short circuit problem need additive weights, the shortest circuit problem should be converted, and then Lagrangian Relaxation Algorithm is used to solve the problem. The results show that the algorithm has better feasibility and efficiency, which can provide theoretical support of operation and management for external distribution network of railway passenger station.

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