A Method based on Marginal Rate of Substitution for Modeling Traveler Behavior of Choice among Different Transportation Services

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Abstract: This paper has two goals. One is to offer a new way to analyze traveler behaviors of choice of different transportation services based on the conception of marginal rate of substitution (MRS) and the other is to propose a method to calculate MRS with travel demand functions instead of marginal utility which is difficult to obtain. The MRS can be used to describe how much a traveler is willing to substitute one transportation service for one unit of another service while maintaining his/her originally achieved utility. This methodology applies to all substitution relationships among different trip modes, time periods, etc. An example is given to illustrate the calculation of the MRS between transit and private car services and the impacts of each variable or parameter on MRS are analyzed. The proposed methodology can help analyze travel behaviors of choice of transportation services and evaluate policies for travel demand management.

Keywords: Marginal Rate of Substitution, Transportation Policy Evaluation, Travel Demand Analysis, Traveler Behavior, Mode Choice

1. INTRODUCTION

In certain transportation environment, travelers have different responses to different transportation policies. Understanding travelers' characteristics and how travelers respond to different transportation policies, in other words how travel demand change, is of great importance, not only for evaluating transportation policies, but also for adjusting the policies to improve the performance of the whole traffic system. For example, in a congestion charging scheme, all private vehicles need to pay a toll. Then the demand of trips by private vehicles may decrease. At this time, it is very useful if we could know the transfer demand, since we can provide enough alternative traffic service according to this amount and ensure the service quality of the whole system.

The methods of travel demand analysis and forecasting have been studied by many scholars. From traditional four-step model to discrete choice model and activity-based model (Ben-Akiva and Lerman, 1985), they have their own characteristics and performed well in many aspects.

We'd like to provide a new view to predict travelers' behaviors and travel demands under certain policies circumstance. From a point view of market, when travelers choose a certain traffic mode, time period or transportation company, actually they are choosing a certain kind of traffic service provided by different traffic modes, time periods or transportation companies. So in the market of traffic service, we can analyze people's travel demand in the way of analyzing other common merchandizes. There are many useful indexes to analyze consumers' demand in microeconomics, one of which is called marginal rate of

substitution (MRS). It is often used to measure the rate at which the consumer is just willing to substitute one good for the other (Varian, 2010), with utility unchanged as a precondition. Producers and economists use it to predict consumers' behaviors and explain economic phenomenon. In commodity market, rational people always pursue the maximum utility with limited sources, namely determine the demand for each product in consumption bundles which satisfy themselves most with limited money. It is the same in transportation service market. Travelers need to make a decision about the demand for each kind of transportation services to get the maximum travel utility. And the MRS can be used to describe how much the traveler is just willing to substitute one kind of transportation service for another.

The literature is not much, and the limited ones show that, though the concept of MRS is not new to transportation field, it was quite limited used before. Varian (2010) described the utility for commuting by regarding each traffic mode as representing a bundle of different characteristics, such as travel time, out-of-pocket costs, waiting time, comfort and so on. For different characteristics, travelers value them differently. MRS is used here to describe the amount of one characteristic consumer would be willing to give up to gain an extra unit of another characteristic so as to keep the utility unchanged. The most widely used example is the value of time which is substantially the MRS between monetary cost and travel time. Also the MRS between any other two characteristics could be used to show the substitute relationship between them. Eboli and Mazzulla (2008) used this method to calculate the monetary value of comfort and accessibility in bus service. So we can see that, the MRS is used mainly to analyze the substitution relation between influential factors of one transportation mode.

This paper focuses on the extended use of the MRS in transportation field, which has been mentioned above, namely to describe substitution relation between different transportation services. The MRS can be very helpful in predicting different policies' effects on travelers' decisions. For example, suppose there are only two kinds of traffic modes which are transit and private vehicles, when an economic factor changes, for instance fuel price rises, then the demand for private vehicles may decrease. But people still need to travel. We may wonder how to ensure all the travelers' trips as satisfied as before. At this time, that will be very helpful if we know the MRS between public transit and private vehicles. Thus we can convert the decreased amount of private vehicles travel into the amount of public transit travel (which we need to increase), so as to keep travelers' utility unchanged. The policies could be other more realistic ones. For example, congestion charging programs in which we all know that alternative mode service is an important determiner. The MRS can help us know how much alternative service the government should provide. The service can be categorized not only according to different traffic modes, but also different time periods. Taking the congestion charging for example again, the program requires that private vehicles need to pay during peak hours. Due to the charging, private vehicles owners may be forced to switch to travel in off peak hours. So the MRS can again help us figure out the substitution relation between peak hour travel and off peak hour travel. It is reported (Mineta Transportation Institute 2009) that some cities adopt dedicated road measures in which some sections of road cannot be used by private vehicles or can be used only in certain time period. The MRS can be used here to predict private vehicles owners' choices between different routes and departure time. There are many kinds of traffic management policies and almost all of them improve the traffic system performance primarily through influencing travelers' behaviors. The MRS here provides a new perspective to help predict travelers' behaviors and evaluate traffic management policies in a microeconomic way.

The calculation of the MRS is a key point during the progress of utilizing it in transportation field, and we try to explore a new approach to get it. In classical

microeconomic, the MRS is the ratio between marginal utilities of two merchandises (Varian, 2010). So it depends on the utility function. But as we know, utility can only describe people's preference and tell us how people order different consumption bundles (Varian, 2010). If consumers prefer one to another, that means utility of former one is higher than that of the later one, but how much higher cannot be shown by utility. And it is not easy to get a utility function to measure people's wellbeing precisely. So this paper also focuses on using demand function to calculate the MRS instead of utility function.

The rest of the paper is organized as follows: after the introduction in the first section, section 2 introduces the MRS formula which uses demand function instead of utility function. As an example, a scenario with two types of transportation modes will be given in section 3 to illustrate how the MRS works between different transportation services. Section 4 will discuss some more specific indications of the utilization of the MRS in transportation field. The conclusion will be given in section 5.

2. THE MODEL

The transportation services among which existing substitution relations often provided by different modes, time period or companies. For travelers, they can only take one mode, time period or company at one time, so they need to make decisions among them. We select traffic modes as an example to show the model which we promoted to calculate the MRS between different transportation services.

Assume that there are k kinds of traffic modes in analysis. MRS_{ij} represents the marginal rate of substitution of mode service i for mode service j, namely substitute service j with service i. In microeconomic, it equals to the ratio between marginal utilities of i and j, shown below:

$$MRS_{ij} = -\frac{dJ}{dI} = \frac{MU_i}{MU_j} \tag{1}$$

Where MU_i and MU_j are the marginal utilities of mode i and j, respectively. Assume **x** to be the consumption bundle which maximizes consumers' utility and expresses how much of each good the consumer desires under some restrictions, while x_i is consumer's demand function for product i, so utility function of consumers could be $U(x)$. Then equation (1) is converted to:

$$MRS_{ij} = \frac{MU_i}{MU_j} = \frac{\dfrac{\partial u(\mathbf{x})}{\partial x_i}}{\dfrac{\partial u(\mathbf{x})}{\partial x_j}} \tag{2}$$

Utility is difficult to measure, so people usually use indirect utility function instead of direct utility in analysis. Equation (3), indirect utility function (Varian, 2010), gives the maximum utility achievable at given prices and income.

$$v(\mathbf{p}, m) \equiv u(x(\mathbf{p}, m)) \tag{3}$$

Where **p** is the price vector contains all the products' prices in consumption bundle, *m* represents consumer's income. In transportation, travelers may consider many factors rather than income and price only. Take public transit service as an example, Fu and Xin(2007) considered six level of service (LOS) measures for evaluating the quality of service of a fixed-route transit system, namely service frequency, service span, service coverage, passenger loading, service reliability and transit auto travel time difference. Just like the method to calculate value of time, the influence of these factors can be converted to monetary cost which can be considered in price. In the indirect utility function, income is considered as a representative factor, used to show traveler's social characters. Actually a variety of socio-demographic factors can influence travel behaviors. Curtis and Perkins (2006) concluded that factors such as household composition, age, gender, car ownership and income all influence the choice of travel mode and the length and duration of the journey. Here we choose income as the representative factor, since among all these factors income is continuous and also has certain relationships with other factors. For instance, higher income usually means higher car ownership. Besides, sociodemographics of different traveler groups usually correspond to different acceptable level during travel. For example, a family with a baby usually prefers traveling in a private car to crowded and muggy transit bus. That means this kind of household composition has lower acceptable comfortable level. We can also convert travelers' maximum acceptable travel time and lowest acceptable comfortable level into monetary value and be regarded as a part of their income which the travelers can spend. In sum, the framework proposed in this paper is just a basic one. With further study, it could provide richer information.

With equation (3), take the derivative with respect of different traffic mode price, we get:

$$\frac{\partial v}{\partial p_j} = \sum_{i=1}^{k} \frac{\partial u}{\partial x_i} \frac{\partial x_i}{\partial p_j} \qquad \text{j=1,2…k} \tag{4}$$

Remember that we assumed there are k traffic modes in the system, so we have *k* equations in the same form of equation (4) which can constitute an equations group with each mode's marginal utility as unknowns. In the form of matrix, the equation group is written as follows:

$$\frac{\partial u}{\partial \mathbf{x}} \mathbf{A} = \frac{\partial v}{\partial \mathbf{p}} \tag{5}$$

Where $\frac{\partial u}{\partial \mathbf{x}} = \left(\frac{\partial u}{\partial x_1} \quad \frac{\partial u}{\partial x_2} \quad \cdots \quad \frac{\partial u}{\partial x_k} \right)$;

$$\mathbf{A} = \begin{pmatrix} \dfrac{\partial x_1}{\partial p_1} & \dfrac{\partial x_1}{\partial p_2} & \cdots & \dfrac{\partial x_1}{\partial p_k} \\ \dfrac{\partial x_2}{\partial p_1} & \dfrac{\partial x_2}{\partial p_2} & \cdots & \dfrac{\partial x_2}{\partial p_k} \\ \vdots & \vdots & \cdots & \vdots \\ \dfrac{\partial x_k}{\partial p_1} & \dfrac{\partial x_k}{\partial p_2} & \cdots & \dfrac{\partial x_k}{\partial p_k} \end{pmatrix};$$

$$\frac{\partial v}{\partial \mathbf{p}} = \left(\frac{\partial v}{\partial p_1} \quad \frac{\partial v}{\partial p_2} \quad \cdots \quad \frac{\partial v}{\partial p_k} \right)$$

By mean of solving equation (5), the marginal utility of each traffic mode can be obtained:

$$\frac{\partial u}{\partial \mathbf{x}} = \frac{\partial v}{\partial \mathbf{p}} \left[\mathbf{A}^{-1} \right]_i \tag{6}$$

Matrix $\mathbf{A}$ here is used to simplify the written form of equation group. It contains demand elasticity and cross elasticity. People's elasticities of different modes change with time and are different with each other. So generally, any two columns of Matrix $\mathbf{A}$ are not related, and we can get a set of solutions from this equation. So we regard $\mathbf{A}$ as a invertible matrix. According to the definition, the MRS between any two traffic modes i and j is:

$$\frac{\frac{\partial u}{\partial x_i}}{\frac{\partial u}{\partial x_j}} = \frac{\frac{\partial v}{\partial \mathbf{p}} \left[\mathbf{A}^{-1} \right]_i}{\frac{\partial v}{\partial \mathbf{p}} \left[\mathbf{A}^{-1} \right]_j} \tag{7}$$

There is an important identity named Roy's identity (Varian, 2010) in microeconomic theory which is shown in equation (8). In Roy's identity, demand function $x_i(p,m)$ is the Marshallian demand function, and the price and income are both positive. It is widely applied to calculate the demands which can bring consumers maximal utility for consuming certain product with fixed demand and income. In transportation, it is similar. Travelers choose from different transportation services to maximize their utility with positive income and price. This identity is used here to help analysis.

$$\frac{\partial v(\mathbf{p}, m)}{\partial p_i} = -\frac{\partial v(\mathbf{p}, m)}{\partial m} x_i(\mathbf{p}, m) \quad i=1,2 \cdots\cdots, \ k \tag{8}$$

The $\frac{\partial v}{\partial \mathbf{p}}$ in equation (7) is replaced by the Roy's identity, resulting in a formula of the MRS with only demand function and its differential forms on the right hand side. Since it may lead to some confusion to give the result in specific mathematic form of the MRS, we will show it in an example with two traffic modes below.

3. SCENARIO ANALYSIS

3.1 Formula Derivation

The MRS between different traffic modes can play a part in evaluating different policies, such like congestion charging scheme, parking pricing and capacity expanding. Here we use the congestion charging scheme mentioned in the introduction as a scenario to show how to calculate the MRS between two traffic modes.

Assume that there are only two traffic modes available in traffic system including transit and private vehicles. People can only take one of them at one time to make a travel. The city now has been facing terrible congestion problem for long time and the transportation authority decides to apply congestion charging scheme. All the private vehicles need to pay according to this scheme. Higher travel cost of private vehicles certainly will make some travelers reconsider their decisions, resulting in a potential new demand pattern. For government, providing enough alternative mode service in order to guarantee the system performance is very important. The MRS can be helpful in deciding how much transit service (since there are only two modes, the alternative mode of private is transit service) the government should provide.

We denote u_i, p_i and x_i as the utility, price and demand function of mode i, respectively, and i could be chosen as c which means private vehicles or b which stands for transit bus. Based on equation (4), the indirect utilities of private vehicle and transit bus could be shown as:

$$\frac{\partial v}{\partial p_b} = \sum \frac{\partial u}{\partial x_i} \frac{\partial x_i}{\partial p_b} = \frac{\partial u}{\partial x_c} \frac{\partial x_c}{\partial p_b} + \frac{\partial u}{\partial x_b} \frac{\partial x_b}{\partial p_b} \tag{9a}$$

$$\frac{\partial v}{\partial p_c} = \sum \frac{\partial u}{\partial x_i} \frac{\partial x_i}{\partial p_c} = \frac{\partial u}{\partial x_c} \frac{\partial x_c}{\partial p_c} + \frac{\partial u}{\partial x_b} \frac{\partial x_b}{\partial p_c} \tag{9b}$$

Solving the equations group in equations (9), the direct utilities of two modes can be got:

$$\frac{\partial u}{\partial x_b} = \frac{\dfrac{\partial v}{\partial p_b} \dfrac{\partial x_c}{\partial p_c} - \dfrac{\partial v}{\partial p_c} \dfrac{\partial x_c}{\partial p_b}}{\dfrac{\partial x_b}{\partial p_b} \dfrac{\partial x_c}{\partial p_c} - \dfrac{\partial x_b}{\partial p_c} \dfrac{\partial x_c}{\partial p_b}} \tag{10a}$$

$$\frac{\partial u}{\partial x_c} = \frac{\dfrac{\partial v}{\partial p_b} \dfrac{\partial x_b}{\partial p_c} - \dfrac{\partial v}{\partial p_c} \dfrac{\partial x_b}{\partial p_b}}{\dfrac{\partial x_c}{\partial p_b} \dfrac{\partial x_b}{\partial p_c} - \dfrac{\partial x_c}{\partial p_c} \dfrac{\partial x_b}{\partial p_b}} \tag{10b}$$

Replacing corresponding parts with Roy's identity, equations (10) are converted into:

$$\frac{\partial u}{\partial x_b} = \frac{\partial v}{\partial m} \frac{x_b \dfrac{\partial x_c}{\partial p_c} - x_c \dfrac{\partial x_c}{\partial p_b}}{\dfrac{\partial x_b}{\partial p_b} \dfrac{\partial x_c}{\partial p_c} - \dfrac{\partial x_b}{\partial p_c} \dfrac{\partial x_c}{\partial p_b}} \tag{11a}$$

$$\frac{\partial u}{\partial x_c} = \frac{\partial v}{\partial m} \frac{x_b \dfrac{\partial x_b}{\partial p_c} - x_c \dfrac{\partial x_b}{\partial p_b}}{\dfrac{\partial x_c}{\partial p_b} \dfrac{\partial x_b}{\partial p_c} - \dfrac{\partial x_c}{\partial p_c} \dfrac{\partial x_b}{\partial p_b}} \tag{11b}$$

According to the definition, the MRS of using transit service to substitute private service is finally derived:

$$MRS_{bc} = \frac{x_c \dfrac{\partial x_c}{\partial p_b} - x_b \dfrac{\partial x_c}{\partial p_c}}{x_b \dfrac{\partial x_b}{\partial p_c} - x_c \dfrac{\partial x_b}{\partial p_b}} \tag{12}$$

Equation (12) shows clearly that there are only the demand function and its differential forms in its formula. It is an example of the general form of the MRS with two traffic modes between which there is substitute relation.

3.2 Numerical Calculation

3.2.1 Value setting

We have got the formula of the MRS shown in equation (12), and furthermore with specific demand functions, the MRS between two transport modes can be obtained. For calculation and analysis, a double logitharimic (or log-liner specification) of the form seen in equation (13) is used here to estimate travel demand (Button, 2010), with an assumption that the price elasticity of demand is the same at all price levels.

$$\ln Q_M = \alpha + \beta_1 \ln P_M + \beta_2 \ln Y + \beta_3 \ln P_N \tag{13}$$

In which: Q_M is the quantity of mode M demanded, P_M is the price of mode M, Y is income and P_N is the price of an alternative N. The parameters β_1, β_2 and β_3 are price elasticity, income elasticity and cross-elasticity, respectively. The price here stands for the general cost including time cost, comfort and some other travel cost. To get a specific form of this demand function, regression analysis is often used to get these parameters. As to the problem we are talking about, we can get the demand functions for both transit service and private vehicle as follow:

$$\ln x_b = \alpha + \beta_1 \ln P_b + \beta_2 \ln Y + \beta_3 \ln P_c \tag{14a}$$

$$\ln x_c = \alpha' + \beta_1' \ln P_c + \beta_2' \ln Y + \beta_3' \ln P_b \tag{14b}$$

It is not easy to get real data and calibrate these parameters through regression. But luckily, many scholars did research on this and concluded the value range of elasticity. To get some idea of the variation trend of MRS, without losing generality, we reference these values in literature. Values of the parameters we used are shown in table 1.

Table 1 Values of elasticity parameters

	Private Vehicle	Transit
Price elasticity (β_1)	[-0.3,-0.2][a]	[-0.5,-0.2][b]
Income elasticity (β_2)	[0.35,0.55][a]	[-0.4,-0.3][c]
Cross-elasticity (β_3)	[0.03,0.1][b]	[0.05,0.15][b]

Source:
a. Basso, L., Oum, T. (2007)
b. Litman, M., (2004)
c. Paulley, N., et al (2006)

For the other variables, we think that the general cost of transport mode consist of two parts: monetary cost and time cost. Here the time cost, v, is the product of travel time t and value of time (VOT). Travel time here is assumed to be unchanged, so the time cost only depends on the value of time. We use m_c and m_b to denote the monetary cost of the private vehicle and transit service. And s is travelers' income. In the analysis below in this paper, these parameters may have different assumed values in need of showing the variation trend of MRS. As to the constant term, it is easy to get the value of α and α' with real data. But here, we infer that, when the prices of both modes and the income are all around 1, then the last three items in demand function are nearly zero. And at this time, these two kinds of modes are almost the same for travelers, so there is not big difference between the demands for these two modes, then the difference between α and α' is small. Here we set the difference equals 1. We should notice that, though the values we use are assumed, when there is data, it is possible to calculate the real MRS between private vehicle and transit service in certain situation.

3.2.2 Price's influence on MRS

(1) For homogeneous groups

To have an idea of the influences of price has on MRS, we assume parameters have the values which are shown in table 2.

Table 2 Values of parameters for homogeneous groups

	Private Car	Transit
Price elasticity ($\beta 1$)	-0.25	-0.35
Income elasticity ($\beta 2$)	0.45	-0.35
Cross-elasticity ($\beta 3$)	0.06	0.1
Monetary cost	[12,22]	[0.2,5]
Value of time (vot)	24	24
Travel time (t)	1	1
Income (s)	133	133

In table 2, since the variation range is not big, we treat the elasticity as a constant in this price interval. Here we use the median of the elasticity parameters value range as the value of elasticity parameter. And the value of monetary cost changes in value interval. When the price of private car (or transit) changes, the price of transit (or private car) keeps fixed at the median value. The VOT, travel time and incomes are the same for both kinds of modes. Take

Dalian as an example, a travel survey conducted by Zhang (2010) shows that private car owners' VOT is around 24.58 yuan/h. Here we set the VOT equal to 24 yuan/h. Travel time is set to be 1h which is convenient for calculation. As to the income, 4000yuan/month is a common level in Dalian. 133 yuan/day is set here as income. With these data, we are trying to analyze the substitution relation between private car and public transit for a traveler when he/she considers one day commute trip. With the data we assumed above, the variation trends of MRS when the price of private vehicle and transit service changes respectively are shown in figure 1(a) and figure 1(b).
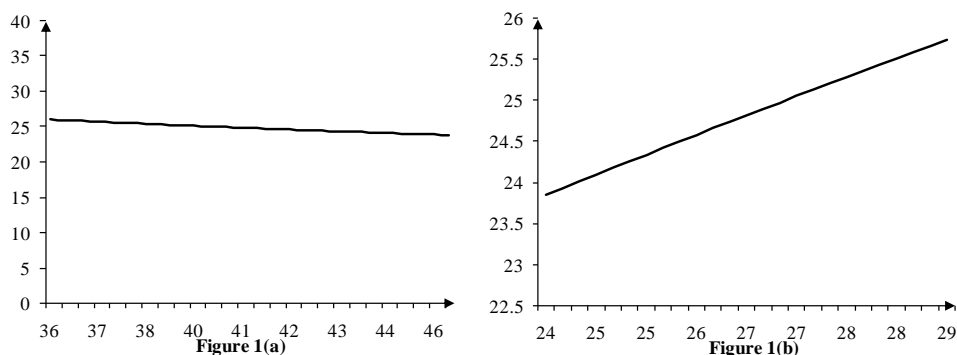

Figure 1(a)

Figure 1(b)

Figure 1(a) the variation of MRS when the price of private vehicle changes
Figure 1(b) the variation of MRS when the price of transit service changes

Figure 1(a) and 1(b) show some general trends of MRS intuitively. Figure 1(a) shows that when the price of private vehicle becomes higher, the MRS becomes lower. That means, when it is more expensive to travel by private vehicles, if travelers drop one unit of private vehicle service, they need less transit service to maintain the travel utility. Figure 1(b) shows that when the price of transit service increase, the MRS increases, too. That means when the transit service becomes more expensive, travelers need more transit service to compensate the utility reduction which is caused by one unit of private vehicle service decreasing.

(2) For heterogeneous groups

The figures above show the MRS for a homogeneous group. But we know that different groups have their own elasticity and income which may lead to a different MRS. Here we define two groups of travelers: rich one and common one. For rich group, the income and the value of time are higher than that of the common group, and the elasticity could be very different. The values of parameters used in this case are shown in Table 3. The parameters in table 3a are used to estimate the influence of private vehicle service on two groups, and the ones in table 3b are used to analysis transit services price's influence.

Table 3a Values of parameters used for analyzing private vehicle price

	Rich group		Common group	
	Private Vehicle	Transit	Private Vehicle	Transit
Price elasticity (β_1)	-0.25	-0.2	-0.3	-0.35
Income elasticity (β_2)	0.45	-0.35	0.45	-0.35
Cross-elasticity (β_3)	0.06	0.1	0.06	0.1
Monetary cost	[12,22]	2.6	[12,22]	2.6
Value of time (vot)	40	40	24	24
Travel time (t)	1	1	1	1
Income (s)	266	266	133	133

Table 3b Values of parameters used for analyzing transit service price

	Rich group		Common group	
	Private Vehicle	Transit	Private Vehicle	Transit
Price elasticity (β_1)	-0.25	-0.2	-0.3	-0.35
Income elasticity (β_2)	0.45	-0.35	0.45	-0.35
Cross-elasticity (β_3)	0.06	0.1	0.06	0.1
Monetary cost	17	[0.2,5]	17	[0.2,5]
Value of time (vot)	40	40	24	24
Travel time (t)	1	1	1	1
Income (s)	266	266	133	133

Some values in Table 3a and 3b are different from those in Table 2. The price elasticity of both group are slightly adjusted. The price elasticity of rich group is higher than common group. Since rich group has a higher income which is 266yuan/day compared with 133yuan/day of common group, they are less sensitive than common group when price changes. And the VOT of rich group is assumed to be 40yuan/hour compared with 24yuan/hour of common group. Although the values are assumed, there must be two groups which have these characters in real life. Then we can analysis the influence of price changes on different groups, as shown in figure 2(a) and figure 2(b).

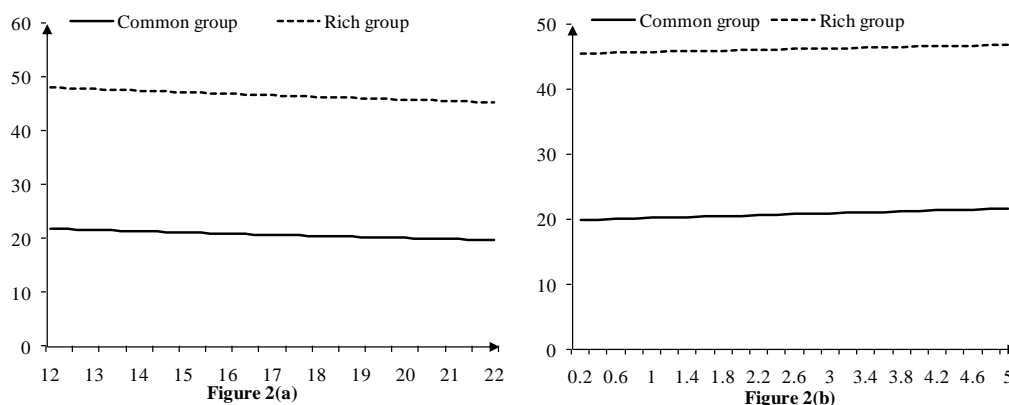**Figure 2(a)**

Figure 2(b)

Figure 2(a) the MRS of two groups when the monetary cost of private vehicle service changes
Figure 2(b) the MRS of two groups when the monetary cost of transit service changes

The overall trend is almost the same as shown in the analysis for homogeneous groups. But from figure 2(a) and figure 2(b), we can tell that the MRS of rich group is much higher than that of common group. It is easy to explain, because the rich group prefers private vehicle service which means the government needs more transit service to substitute the private vehicle service reduction.

3.2.3 Income's influence on MRS

Income also plays an important role in determining the MRS. This subsection discusses the influence of income variable. Here we assumed that the value interval of income ranges from 67 to 317 yuan/day corresponding to 2000 and 9500 yuan/month, and values of other parameters equals to the median of their own value intervals. With the value in table 4, we got the variation trend of MRS shown in figure 3. We can see from figure 3 that the MRS increases when travelers' income becomes higher. This means for people who have higher income, the government needs more transit service to substitute private vehicle service.

Table 4 values used for analysis of income's influence

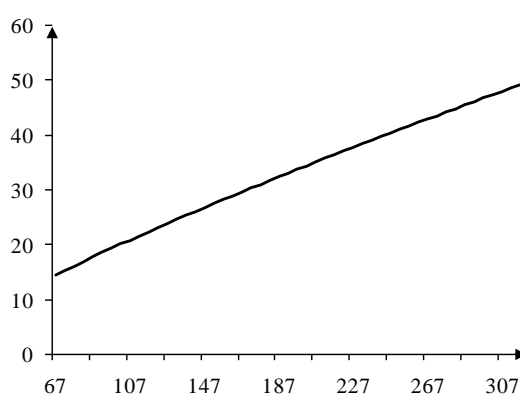|	Private Car	Transit
Price elasticity ($\beta1$)	-0.25	-0.35
Income elasticity ($\beta2$)	0.45	-0.35
Cross-elasticity ($\beta3$)	0.06	0.1
Monetary cost	17	2.6
Value of time (vot)	24	24
Travel time (t)	1	1
Income (s)	[67,317]	[67,317]

Figure 3 the variation trend of MRS when income changes

3.2.4 Elasticity parameters' influence on MRS

Different travelers groups have different elasticity parameters. To analyze the influence of elasticity parameters can help us tell the relative magnitude of MRS between different traveler groups with various characteristics. For instance, groups which have higher and lower transit service price elasticity may have different values of MRS. Numerical calculation used again here. Each parameter is analyzed separately. For analysis of each parameter, its value ranges from the lower limit to upper limit of its interval which is shown in table 1, while others have

the value of median. Other variables are the same as assumed in the analysis of homogeneous groups. Then the variation trends are shown in figures 4(a) to 4(f).
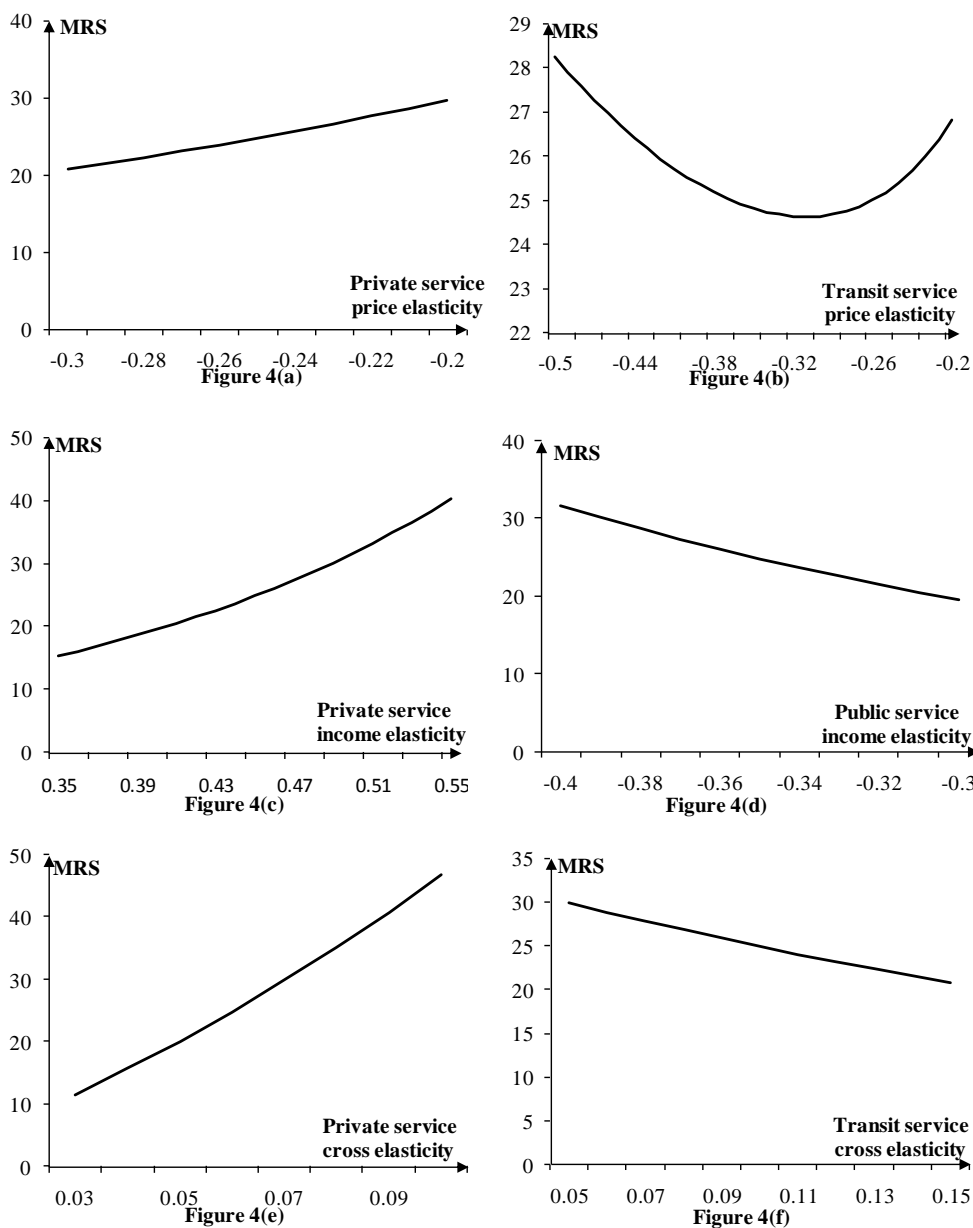

Figure 4 the variation trend of MRS when each elasticity parameter changes

From figures 4(a) to 4(f), we find that, for private service, when the price elasticity becomes smaller (according to the definition of elasticity, the elasticity is measured by its absolute value), the MRS increase. But for transit service, we can see from Figure 4(b) that when the price elasticity becomes smaller, the MRS decreases first and then increases. As to income elasticity, there is positive correlation between the value of income elasticity and MRS for both kinds of services. And the slop of curve in figure 4(c) which shows the relation between private service income elasticity and MRS becomes steeper and steeper as the value of private service income elasticity increases. At last, the MRS increases when private service cross elasticity becomes larger and decreases when transit service cross elasticity increases.

It should be noted that, due to lack of real data, the value of variables and parameters are assumed, but the relative magnitude is still meaningful. And also it is believed that in real

life we can find a group which has the characteristics just as we assumed here. With certain data, we can explain more phenomena and evaluate more policies in daily travel life.

4. POLICY INDICATION ANALYSIS

With this specific example, we may make some further analysis. The MRS can be used to analyze not only aggregate groups but also an individual traveler.

For all the citizens as a whole group, namely an aggregate group, some travelers choose transit while others choose private vehicles. So as a consumer group, its consumption bundle consists of two products, namely transit service and private vehicle service. When consumption for one product decreases due to some management measures or policies, the group may adjust its consumption bundle to maintain their utility level. The group may substitute one service with the other one. The MRS can show the substitution relation between these two kinds of services for the whole travelers group. For example, the value of MRS between transit service and private vehicle service is 60. That means, when the demand of this group for private vehicle service decreases one unit, they need another 60 units of transit service to keep the utility unchanged.

As to a single traveler, he or she has the demand for private vehicle service and transit service. These two kinds of demand can also substitute each other. For instance, the traveler usually takes bus for three times and private vehicle service four times a week, but with some transportation policy applied, he or she may switch to taking bus for five times and private vehicle service two times a week. The substitution relation also exists in this situation and could be described with MRS. For example, the MRS here equals 10. That means, when the traveler stops using private vehicle service for one time, she or he need to use transit service for 10 times to maintain her or her utility.

5. CONCLUTION

MRS is an important concept in microeconomic analysis. It is not new to transportation field but rarely used before in traveler behavior or demand analysis. This paper discusses the extended use of MRS in transportation, namely describing the substitution relationship between different transportation services. These services could be transportation modes, time periods or routes, etc. This paper aims to propose a method for calculating the MRS with demand functions instead of marginal utility that is difficult to obtain. With an example of transportation mode choices, numerical calculations are carried out to show how to calculate the MRS between transit and private car service, namely substitute private vehicle service with transit. Subsequently, this paper analyzes the trend of MRS as each variable or parameter changes. And we find that the MRS increases when the price of transit service, income level, private vehicle service or transit service income elasticity rises; and the MRS decreases when private vehicle service price, price elasticity of private vehicle service and transit service, or the cross elasticity of transit service rises. These revealed trends readily help analyze or compare the MRS of different traveler groups. As discovered previously, the MRS of the high-income-level group is much higher than that of the average-income-level group.

The significance of MRS is to predict traveler behavior of choices when some transportation policies are taken. As long as we have the travel demand functions, we can know how the demand for substitution service changes when demand for certain travel service changes. And government or transportation authority can implement some measures to

maintain travelers' utility. For example, implementing congestion charging to private vehicles in morning peak period on route A, then with MRS, we can know how much transit service in off-peak period on route B we should provide to maintain travelers' utility and the performance of the whole system. It could give some implications to policy makers and also play a role in evaluating some transportation policies.

This paper aims at introducing the concept of MRS into the analysis of travel demand and discussing the general variation trend of MRS when the parameters and variables changes. With more data, we can analyze more travelers' behaviors, evaluate specific transportation policies and explain more transportation phenomenon, such like congestion charging policy, working at home, and flexible working timetable and so on. On the other hand, by selecting demand function which contains certain economic elements, we can analyze their roles in the substitution relation between different transportation services. And as a new aspect to analyze travel demand, some comparison with classical method, like discrete choice model, could be made. Those issues will remain for future research.

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