A Study on Effect of Subsidy on Location Choice of Multi-unit Firms

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Abstract: People living in mega cities suffer from heavy congestion in their commuting. For mitigating the heavy commuting, redistribution of population in a central business district to other less-crowded areas is expected as a countermeasure. For a local region, the redistribution of population brings about increment of tax revenue and employment creation. This study proposes a model representing location choice behavior of a private firm considering introduction of a satellite office. A private firm and a household behave as minimizing costs for producing goods and as maximizing the utility, respectively. When labor market reaches to an equilibrium state, the optimal wage rate of satellite office is obtained. By considering both costs and the utility, a private firm chooses an appropriate location for its satellite office. We formulate this decision-making process by a nested-logit model. We demonstrate our proposed model by numerical calculations. Finally, we conclude this article and show future perspectives.

Keywords: location choice behavior, teleworking, satellite office, heavy commuting

1. INTRODUCTION

High concentration of population in mega city is one of major problems to be solved in the world. Tokyo metropolitan area, one of the most crowded areas in the world, suffers from heavily-congested commuting. Average commuting time from home to workplace in Tokyo metropolitan area is 67.7 minutes (MLIT, 2017), so most of workers spend more than two hours per day for their commuting. Speaking of the congestion state inside trains, average congestion rate among Tokyo metropolitan area is announced as 163% in 2017FY (MLIT, 2018).

For mitigating such heavy congestion of commuting in metropolitan area in Japan, Japanese national government has urged a private firm in metropolitan areas to introduce their offices in less-crowded local areas. For example, recent policies by national government are listed as: promotion of telework, relocation of some parts of national government and headquarters functions of major enterprises out of Tokyo. Recent high improvement of telecommunication technology, a private firm can separate offices in a central business district (CBD) to those in other area with small costs.

Then, national and local governments of Japan have introduced some policies for

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relocating workers in CBD to local areas recently. As motivations of a private firm for relocation of workers, improvement of living standard of workers can be expected. This is affected by increment of disposable time per day, or reduction of commuting time. If workers have much disposable time in less-crowded area than in CBD, they can use it for leisure or some other activities.

This study focuses on a case that a private firm introduces a “satellite office (SO)” in less-crowded area. In special, we define a SO as a work place where workers do not spend much time for commuting. The objective of this article is to model location choice behavior of a private firm that considers introduction of a SO. This study concentrates on comparative statics of subsidy from governments to a private firm. The subsidy motivates the private firm to introduce a SO to local cities.

Basically, this study bases on theory of urban economics. As a primary study of urban economics, Alonzo (1964) applied the model of von Thünen () to urban structures and proposed bid rent theory. The conventional studies of urban economics have interests in comprehensive urban policies of land use or regulation. Ota and Fujita (1993) proposed a general equilibrium model considering the case that the offices of a private firm can be divided in CBD and in non-CBD. Ota and Fujita (1993) analyzes the formation of bid rent curve in a city, in the case that separated offices can communicate each other due to the development of telecommunication. However, this study does not consider decision-making process of location choice behavior of an individual firm.

There are some empirical studies about location choice behavior in urban cities. For example, Alpkokin et al. (2007) analyzed dynamic process of rank-size distribution of cities in Asian-Pacific region by using empirical data. Their research interest focuses on poly-centricity of employment. Malaitham et al. (2013) examined location choice behavior of residence in urban region. They analyzed stated preference data collected in Bangkok Metropolitan Region.

As far as the authors know, there are few studies just dealing with location choice behavior of a private firm considering introduction of a SO. Usami et al. (2015) proposes a model for evaluating the effect on economic welfare of a private firm and of a household by introduction of SO. However, the model cannot represent location choice behavior of an individual firm. Thus, our proposed model relaxes the assumption and represents the location choice decision by using a nested-logit model.

This study formulates behaviors of a private firm and a household, as an individual economic unit, when considering introduction of SO in local cities. We assume that goods market and bid rent are given exogenously. It means that behaviors of a private firm and a household cannot change the price and the demand of goods and the bid rents. A firm behaves as minimizing costs for producing goods. On the other hand, a household behaves as maximizing the utility. Then, when candidate locations for a SO are given, a private firm choose an appropriate location subject to the costs for producing goods and utilities of a household. In this study, we assume the business of a private firm which does not constrain to the location of its office with the telecommunication technology. Our proposed model considers not only location choice behavior but also the decision-making behavior for introducing SO by a private firm. Thus, we formulate two kinds of firm behaviors as a nested-logit model. By using a nested-logit model, the effect of independence from irrelevant alternatives among their behaviors should be avoided.

The rest of this article is structured as follows. Section 2 describes the proposed methodology for representing firm location choice behavior. Behaviors of a private firm and a household are formulated as minimizing costs for producing goods and maximizing the utility, respectively. The location choice behavior of a private firm is formulated by a nested-logit model. Section 3 demonstrates the methodology described in Section 2 by numerical calculations. Finally, we conclude this article and show some future perspectives.

2. THE MODEL

2.1 Assumptions

The overall structure of our proposed model is shown in Figure 1.

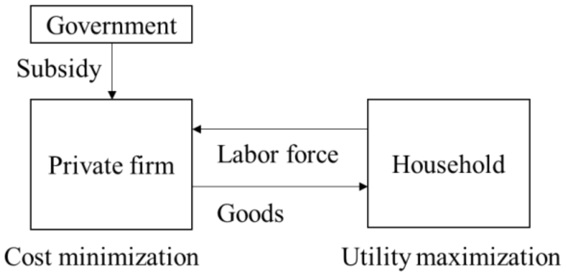

Figure 1. The overall structure of proposed model

Assumptions employed in this study are given below.

- A private firm has activities which can be transferable from a headquarter office in CBD to other arbitrary location.
- We consider two economic units, a private firm and a household. A household is a consumer of goods which a private firm produces.
- Locations of offices and residences of a household are situated in two areas, in the vicinities of CBD and a local area.
- Commuting time of a household who works in SO is negligible small compared with that in the headquarter office.
- A private firm produces two kinds of goods by inputting labor force and capital. All the goods are consumed by a household. A firm produces good 1 by inputting labor forces in CBD and SO, and good 2 by inputting labor forces in SO.
- The demand and price of goods are given exogenously, so that a private firm can produce goods by minimizing the costs for the production.
- Perfect competitive market of goods is assumed, and all goods are consumed in the corresponding good markets.
- Each consumer allocates his/her disposable time to leisure and commuting, continuously.
- A household can change its location without using any time and costs.

2.2 Notations

The following notations are used in this article. Note that locations c and s imply CBD and SO, respectively.

ω_i	Wage rate of area $i \in \{c, s\}$
$\widehat{\omega}$	Synthetic wage rate
$l_{i,j}$	Labor input of area $i \in \{c, s\}$ for producing good $j \in \{1, 2\}$
$\hat{l}$	Synthetic labor input
α_i	Rate of labor input at area $i \in \{c, s\}$
σ	Surrogate parameter of CES function

c_j	Total cost for producing good $j \in \{1,2\}$
r_j	Price of capital for producing good $j \in \{1,2\}$
k_j	Capital input for producing good $j \in \{1,2\}$
$x_{i,j}$	Consumption of good $j \in \{1,2\}$ at area $i \in \{c,s\}$
x_j	Total consumption of good $j \in \{1,2\}$
β_0	Parameter indicating economics of scale of a private firm
$\beta_{1,j}, \beta_{2,j}$	Parameters of production function of good $j = \{1,2\}$
u_i	Utility of labor who lives at area $i \in \{c,s\}$
t_{li}	Leisure time of labor who lives at area $i \in \{c,s\}$
T_t	Disposable time of a household
t_{c_i}	Commuting time of a household who lives at area $i \in \{c,s\}$
q_i	Inventory of land at area $i \in \{c,s\}$
π_i	Land rent at aria $i \in \{c,s\}$
p_j	Price of good $j \in \{1,2\}$
$\chi_{1,j}, \chi_2, \chi_3$	Parameters of utility function ($j \in \{1,2\}$)
φ	Ratio of wage which local government subsidizes
$\tilde{V}_Y$	Deterministic term of utility of a private firm when introducing a SO
U_Y	Utility of a private firm when introducing a SO
λ_1, λ_2	Parameters of logit model
K	Set of candidate locations for a SO
V_k	Deterministic term of utility of a private firm choosing candidate location *k* as a SO
U_k	Utility of a private firm choosing candidate location *k* as a SO
P_Y	Probability of a private firm introducing a SO
$P_k(k\|Y)$	Probability of a private firm which introduce a SO choosing a candidate location *k*
c_k	Cost for producing a unit of good when a private firm introduces a SO at city *k*
v_k	Utility of a household when a private firm introduces a SO at candidate location *k*
c_t	Total subsidies

2.3 Firm Behavior without Introduction of SO

2.3.1 Cost minimization of a private firm

A firm is assumed to have a Cobb-Douglas production function. The inputs are labor force and capital. The amount of inputs is determined by minimizing total cost for producing goods subject to a given demand of goods. A cost for producing good *i* is determined by solving the following minimization problem.

$$c_1(\omega, x) = \min_{l,k_1} \omega \cdot l + r_1 \cdot k_1 \tag{1}$$

s.t.

$$x_1 = \beta_0 \cdot l^{\beta_{1,1}} \cdot k_1^{\beta_{2,1}} \tag{2}$$

By solving above problem, the optimum amount of synthetic labor and capital inputs for good *i* are respectively represented as:

$$l_1^* = \frac{x_1}{\beta_0}\left(\frac{\beta_{1,1}}{\beta_{2,1}} \cdot \frac{r_1}{\omega}\right)^{\beta_{1,2}} \tag{3}$$

$$k_1^* = \frac{x_1}{\beta_0}\left(\frac{\beta_{1,1}}{\beta_{2,1}} \cdot \frac{r_1}{\widehat{\omega}}\right)^{-\beta_{11}} \tag{4}$$

2.3.2 Utility maximization of a household

A household is assumed to have a Cobb-Douglas utility function in which utility level is determined by amount of goods consumed, leisure time and inventory of land. The amount of consumptions is determined by maximizing household's utility subject to a given income. Disposable time of a household is consumed by leisure, labor and commuting. The utility maximizing problem is represented as:

$$\max_{x,t_c,q_1} u = \prod_{j=1}^{2} x_j^{\chi_j} \cdot t_l^{\chi_2} \cdot q^{\chi_3} \tag{5}$$

s.t.

$$p_1 \cdot x_1 + p_2 \cdot x_2 + \pi_c \cdot q_c = \omega_c \cdot (t_c - t_l) \tag{6}$$

where

$$t_l = T_t - t_c \tag{7}$$

By solving above problem, optimum consumption of good, inventory of land and leisure time are respectively represented as:

$$x_j^* = \frac{\chi_{1,j} \cdot \omega_c \cdot t_c}{p_j} \tag{8}$$

$$q^* = \frac{\chi_3 \cdot \omega_c \cdot t_c}{\pi_c} \tag{9}$$

$$t_l^* = \chi_2 \cdot t_c \tag{10}$$

2.4 Firm Behavior with Introduction of SO

2.4.1 Labor input cost minimization

A firm is assumed to have a CES production function in which inputs are labors at CBD and at SO. A synthetic labor input is determined by minimizing the following labor costs.

$$\widehat{\omega} \cdot \hat{l} = \min_{l_{c,1},l_{s,1}} \omega_c \cdot l_{c,1} + \omega_s \cdot l_{s,1} \tag{11}$$

s.t.

$$\hat{l} = \left(\alpha_c^{\frac{1}{\sigma}} \cdot l_{c,1}^{\frac{o-1}{o}} + \alpha_s^{\frac{1}{\sigma}} \cdot l_{s,1}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{12}$$

By solving above cost minimization problem, optimal labor inputs of CBD and SO are respectively obtained as:

$$l_{c,1}^* = \frac{1}{\left(\frac{\alpha_c^{\sigma}}{\omega_c^{\sigma-1}} + \frac{\alpha_s^{\sigma}}{\omega_s^{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}}} \cdot \left(\frac{\alpha_c}{\omega_c} \right)^{\sigma} \cdot \hat{l} \tag{13}$$

$$l_{s,1}^* = \frac{1}{\left(\frac{\alpha_c^{\sigma}}{\omega_c^{\sigma-1}} + \frac{\alpha_s^{\sigma}}{\omega_s^{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}}} \cdot \left(\frac{\alpha_s}{\omega_s} \right)^{\sigma} \cdot \hat{l} \tag{14}$$

A synthetic labor wage for good 1 is represented as:

$$\widehat{\omega} = A \cdot \left(\sum_{i \in \{c,s\}} (\omega_i^{1-\sigma} \cdot \alpha_i) \right) \tag{15}$$

where

$$A = \left(\sum_{i \in \{c,s\}} \alpha_i \cdot \omega_i^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} \tag{16}$$

Note that minimization of labor input cost for good 2 is not conducted, because good 2 is produced only in CBD.

2.4.2 Total cost minimization

A firm is assumed to have a CES production function in which inputs are synthetic labor inputs and capitals. The inputs are determined by minimizing total cost subject to a given demand of good. A cost minimization problem with respect to synthetic wage, synthetic labor input, price of capital and capital input is formulated as:

$$c_1(\widehat{\omega}, x_1) = \min_{\hat{l}, k_1} \widehat{\omega} \cdot \hat{l} + r_1 \cdot k_1 \tag{17}$$

s.t.

$$\begin{aligned} x_1 &= x_{c,1} + x_{s,1} \\ &= \beta_0 \cdot \hat{l}^{\beta_{1,1}} \cdot k_1^{\beta_{1,2}} \end{aligned} \tag{18}$$

By solving above problem, optimum synthetic labor input and capital input are respectively obtained as:

$$\hat{l}^* = \frac{x_1}{\beta_0} \left(\frac{\beta_{1,1}}{\beta_{1,2}} \cdot \frac{r_1}{\widehat{\omega}} \right)^{\beta_{1,2}} \tag{19}$$

$$k_1^* = \frac{x_1}{\beta_0} \left(\frac{\beta_{1,1}}{\beta_{1,2}} \cdot \frac{r_1}{\widehat{\omega}} \right)^{-\beta_{1,1}} \tag{20}$$

2.4.3 Household's behavior

A household in area i ($\in \{c, s\}$) has a Cobb-Douglas utility function and behaves as maximizing the utility obtained by consuming goods, leisure time and inventory of land subject to a given income. Disposal time of a household is consumed by leisure, labor and commuting (for a household working in CBD). A utility maximization problem of a household is formulated as:

$$\max_{x_{i,1} x_{i,2}, t_{l_i}, q_i} u_i = \prod_{j=1}^{2} x_{i,j}^{\chi_{1,j}} \cdot t_{l_i}^{\chi_2} \cdot q_i^{\chi_3} \quad \forall i \in \{c, s\} \tag{21}$$

s.t.

$$p_1 \cdot x_{i,1} + p_2 \cdot x_{i,2} + \pi_i \cdot q_i = \omega_i \cdot \left(T_t - t_{c_i} - t_{l_i} \right) \tag{22}$$

where

$$t_c = \begin{cases} t_c (> 0) & \text{if } i = c \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

By solving above problem, optimum consumptions of good, land and leisure time are respectively obtained as:

$$x_{i,j}^* = \frac{\chi_{1,j} \cdot \omega_i \cdot \left(T_t - t_{c_i} \right)}{p_j} \tag{24}$$

$$q_i^* = \frac{\chi_3 \cdot \omega_i \cdot \left(T_t - t_{c_i} \right)}{\pi_i} \tag{25}$$

$$t_{l_i}^* = \chi_2 \cdot \left(T_t - t_{c_i} \right) \tag{26}$$

Here, when behavior of a private firm is based on profit maximization in complete competition market, an equilibrium price of each good is represented as:

$$p_1 = \frac{1}{\beta_0} \cdot \frac{\widehat{\omega}^{\beta_{1,1}} \cdot r_1^{\beta_{1,2}}}{\beta_{2,1}^{\beta_{2,1}} \cdot \beta_{1,1}^{\beta_{1,1}}} \tag{27}$$

$$p_2 = \frac{1}{\beta_0} \cdot \frac{\omega_1^{\beta_{1,1}} \cdot r_2^{\beta_{2,2}}}{\beta_{2,2}^{\beta_{2,2}} \cdot \beta_{1,2}^{\beta_{1,2}}} \tag{28}$$

2.5 Consideration of Subsidies

2.5.1 Minimization of labor cost

We consider the case that central or local governments subsidize a private firm which intend to introduce their SOs. Here, central or local governments are assumed to subsidize fixed percentage of wage of a household who works in SO. A labor input decision problem for a private firm is formulated as:

$$\widehat{\omega} \cdot \hat{l} = \min_{l_{c,1}, l_{s,1}} \omega_c \cdot l_{c,1} + (1-\varphi) \cdot \omega_s \cdot l_{s,1} \tag{29}$$

s.t.

$$\hat{l} = \left(\alpha_c^{\frac{1}{\sigma}} \cdot l_{c,1}^{\frac{\sigma-1}{\sigma}} + \alpha_s^{\frac{1}{\sigma}} \cdot l_{s,1}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{30}$$

By solving above problem, labor inputs of a private firm introducing SO are respectively given as:

$$l_{c,1}^* = \frac{1}{\left(\frac{\alpha_c^\sigma}{\omega_c^{\sigma-1}} + \frac{\alpha_s^\sigma}{\left((1-\varphi) \cdot \omega_s\right)^{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}}} \cdot \left(\frac{\alpha_c}{\omega_c} \right)^\sigma \cdot \hat{l} \tag{31}$$

$$l_{s,1}^* = \frac{1}{\left(\frac{\alpha_c^\sigma}{\omega_c^{\sigma-1}} + \frac{\alpha_s^\sigma}{(1-\varphi) \cdot \omega_s^{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}}} \cdot \left(\frac{\alpha_s}{(1-\varphi) \cdot \omega_s} \right)^\sigma \cdot \hat{l} \tag{32}$$

Composite labor wage for good 1 is given as:

$$\widehat{\omega} = A' \cdot \left(\omega_c^{1-\sigma} \cdot \alpha_c + (1-\varphi) \cdot \omega_s^{1-\sigma} \cdot \alpha_s \right) \tag{33}$$

where

$$A' = \left(\alpha_c \cdot \omega_c^{1-\sigma} + \alpha_s \cdot (1-\varphi) \cdot \omega_s^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} \tag{34}$$

Note that minimization of labor input cost for good 2 is not conducted, because good 2 is produced only in CBD.

2.5.2 Minimization of total cost

Cost minimization of a private firm with respect to synthetic wage, synthetic labor input, price of capital and capital input is formulated as:

$$c_1(\widehat{\omega}, x) = \min_{\hat{l}, k_1} \widehat{\omega} \cdot \hat{l} + r_1 \cdot k_1 \tag{35}$$

s.t.

$$\begin{aligned} x_1 &= x_{c,1} + x_{s,1} \\ &= \beta_0 \cdot \hat{l}^{\beta_{1,1}} \cdot k_1^{\beta_{1,2}} \end{aligned} \tag{36}$$

By solving above problem, synthetic labor and capital inputs are respectively given as:

$$\hat{l}^* = \frac{x_1}{\beta_0}\left(\frac{\beta_{1,1}}{\beta_{1,2}} \cdot \frac{r_1}{\widehat{\omega}}\right)^{\beta_{1,2}} \quad (37)$$

$$k_1^* = \frac{x_1}{\beta_0}\left(\frac{\beta_{1,1}}{\beta_{1,2}} \cdot \frac{r_1}{\widehat{\omega}}\right)^{-\beta_{1,1}} \quad (38)$$

2.5.3 Household's behavior

In the same manner as **2.3.2**, a utility maximization problem of a household is formulated as:

$$\max_{x,t_l,q_i} u_i = \prod_{j=1}^{2} x_{i,j}^{\chi_{1,j}} \cdot t_{l_i}^{\chi_2} \cdot q_i^{\chi_3} \;\; \forall i \in \{c,s\} \quad (39)$$

s.t.

$$p_1 \cdot x_{i,1} + p_2 \cdot x_{i,2} + \pi_i \cdot q_i = \omega_i \cdot \left(T_t - t_{c_1} - t_{l_i}\right) \quad (40)$$

where

$$t_{c_1} = \begin{cases} t_c(>0) & if\ i = c \\ 0 & otherwise \end{cases} \quad (41)$$

By solving above problem, optimum consumptions of good, land and leisure time are given as:

$$x_{i,j}^* = \frac{\chi_{1,j}^* \cdot \omega_i \cdot \left(T_t - t_{c_1}\right)}{p_j} \;\; \forall i \in \{c,s\}, \forall j \in \{1,2\} \quad (42)$$

$$q_i^* = \frac{\chi_3 \cdot \omega_i \cdot \left(T_t - t_{c_1}\right)}{\pi_i} \; \forall i \in \{c,s\} \quad (43)$$

$$t_{l_i}^* = \chi_2 \cdot \left(T_t - t_{c_1}\right) \forall i \in \{c,s\} \quad (44)$$

2.6 Labor Equilibrium

This study assumes that labor demand matches labor supply. Hence, we consider relationship between wage rate and demand-supply equilibrium of labor. By substituting (42)-(44) to (39), an indirect utility function of each region is obtained. By using this indirect utility function, a probability of a household choosing each region is represented by using logit model.

$$s_i = \frac{\exp v_i^*}{\sum_{i \in \{c,s\}} \exp v_i^*} \; \forall i \in \{c,s\} \quad (45)$$

Next, we set wage rate of CBD as 1, $w_c = 1$. Amounts of labor inputs for CBD and SO are represented as functions of wage rate at SO, w_s. A labor demand ratio of each region is then represented as:

$$\hat{s}_i = \frac{\exp l_i^*}{\sum_{i \in \{c,s\}} \exp l_i^*} \; \forall i \in \{c,s\} \quad (46)$$

Equilibrium of labor market is represented as (47). (47) is formulated as a fixed-point problem with respect to wage rate at candidate location k, w_s.

$$s_i(\omega_s) = \hat{s}_i(\omega_s) \; \forall i \in \{c,s\} \quad (47)$$

2.7 Location Choice of a Private Firm

For representing location choice behavior of a private firm, a two-stage nested logit model shown in Figure 2 is adopted. K is a set of candidate locations and λ_1 and λ_2 are parameters defining nested structure. A firm is assumed to choose either introduce a SO or not, and to choose also the location of the SO at the same time if necessary.

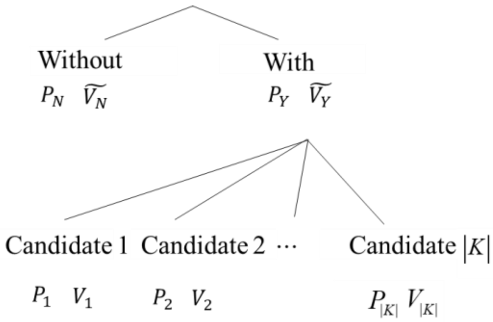

Figure 2. Structure of a nested logit model

A deterministic term of utility for introducing a SO and the utility for each candidate location are respectively formulated as:

$$\tilde{V}_m = \frac{1}{\lambda_1} \ln \sum_{k \in K} \exp(\lambda_1 \cdot V_k) \;\; \forall m \in \{Y, N\} \tag{48}$$

$$U_k = V_k + \varepsilon_k \quad \forall k \in K \tag{49}$$

Here, ε_k is a random term which a private firm chooses candidate location $k \in K$ and follows a Gumbel distribution in which mean and variance are 0 and λ_1, respectively. A proportion of a private firm introducing its SO is formulated as:

$$P_m = \frac{\exp(\lambda_2 \cdot \tilde{V}_m)}{\exp(\lambda_2 \cdot \tilde{V}_Y) + \exp(\lambda_2 \cdot \tilde{V}_N)} \quad \forall m \in \{Y, N\} \tag{50}$$

This study assumes that a private firm chooses location of SO based on (i) cost for producing a unit of good, and on (ii) utility of a household. The deterministic term of firm's utility obtained when choosing candidate location *k* as a SO is defined as:

$$V_k = \alpha \cdot c_k + \beta \cdot v_k \;\; \forall k \in K \tag{51}$$

The utility of a private firm and probability of choosing candidate location *k* are respectively defined as:

$$U_k = V_k + \varepsilon_k \quad \forall k \in K \tag{52}$$

$$P(k|Y) = \frac{\exp(\lambda_1 \cdot V_k)}{\sum_{k \in K} \exp(\lambda_1 \cdot V_k)} \quad \forall k \in K \tag{53}$$

The cost for producing a unit of goods is represented by the weighted mean with respect to the amount of goods.

$$c_k = \frac{x_{1,k}^*}{x_{1,k}^* + x_{2,k}^*} \cdot c_{1,k}^* + \frac{x_{2,k}^*}{x_{1,k}^* + x_{2,k}^*} \cdot c_{2,k}^* \; \forall k \in K \tag{54}$$

Here, $x_{1,k}$, $x_{2,k}$, $c_{1,k}$ and $c_{2,k}$ are consumptions and costs for producing good 1 and good 2, respectively, when SO is located in candidate location *k*. Similarly, utility of each household is represented by the weighted mean with respect to the amount of labor inputs.

$$v_k = \frac{l_{c,1,k}^* + l_{2,k}^*}{l_{c,1,k}^* + l_{2,k}^* + l_{s,1,k}^*} \cdot u_{c,k}^* + \frac{l_{s,1,k}^*}{l_{c,1,k}^* + l_{2,k}^* + l_{s,1,k}^*} \cdot u_{s,k}^* \quad \forall k \in K \tag{55}$$

Here, $l_{c,1,k}$, $l_{s,1,k}$, $l_{2,k}$, $u_{c,k}$ and $u_{s,k}$ are labor inputs and utilities for a household, respectively, when SO is located in candidate location *k*. Thus, probability of a private firm choosing candidate location *k* as a SO is represented as:

$$P_k = P_Y \cdot P_k(k|Y) \quad \forall k \in K \tag{56}$$

3. NUMERICAL CALCULATION

3.1 Parameter Settings

This chapter demonstrates our proposed model. We set parameters of proposed model shown as:

Table 2. Parameters

Notation	Definition	Value
σ	Substitute parameter of CES function	0.8
α_c	Distribution parameter of CES function	0.6
α_s	Distribution parameter of CES function	0.4
β_0	Parameter of Cobb-Douglas productive function	1
$\beta_{1,1}$	Labor elasticity for producing good 1	0.41
$\beta_{1,2}$	Capital elasticity for producing good 1	0.59
$\beta_{2,1}$	Labor elasticity for producing good 2	0.36
$\beta_{2,2}$	Capital elasticity for producing good 2	0.64
$\lambda_{1,1}$	Parameter of Cobb-Douglas utility function	0.25
$\lambda_{1,2}$	Parameter of Cobb-Douglas utility function	0.25
λ_2	Parameter of Cobb-Douglas utility function	0.25
λ_3	Parameter of Cobb-Douglas utility function	0.25
T_t	Disposal time	18
t_c	Commuting time from home to CBD	1.34
r_1	Price of capital for producing good 1	1
r_2	Price of capital for producing good 2	1
π_c	Land rent at CBD	3
w_c	Wage rate at CBD	1

3.3 Sensitivity Analysis with Respect to Subsidies from Government

We perform sensitivity analysis with respect to subsidies from government to a private firm. As simplicity, we assume that a figure has one candidate location for a SO. Figure 4 shows wage rate at SO. When subsidy proportion, φ increases from zero to one, wage rate also increases, but it has a peak around $\varphi = 0.95$. Figure 5 shows that the amount of labor input for producing goods. Labor input for good 2 has a peak around $\varphi = 0.95$.

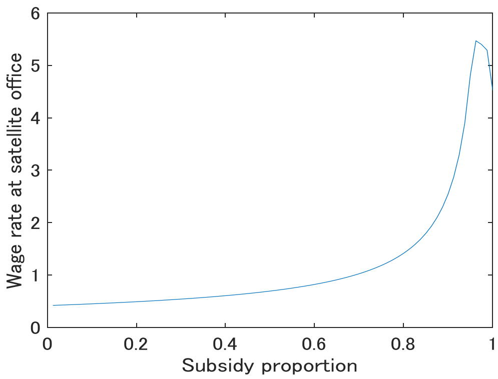

Figure 4. Wage rate at SO

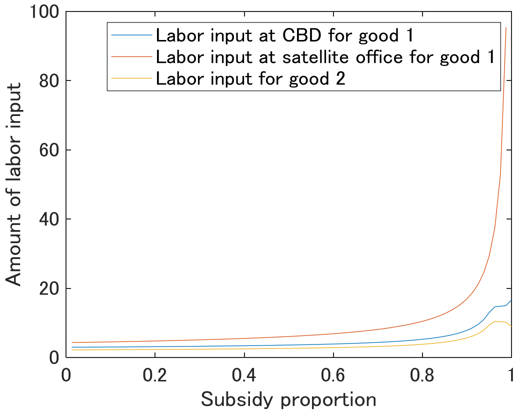

Figure 5. Labor inputs

Figure 6 shows the total subsidy which government gives to a private firm. The total subsidy increases exponentially around $\varphi \in \{0.8, 1\}$. Figure 7 shows costs for producing goods. The trend of curve is almost same as that of wage rate of Figure 4. Note that, in Figure7, costs for good 1 and for good 2 are same, so that two curves are overlapped.

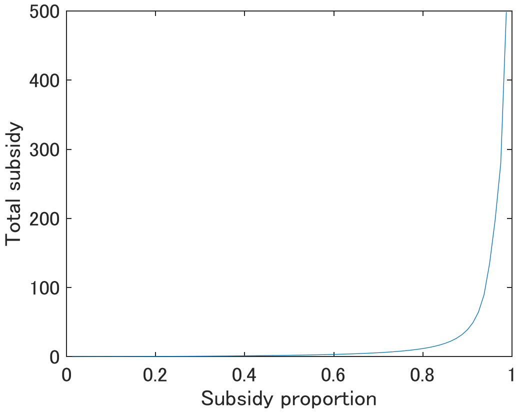

Figure 6. Total subsidy

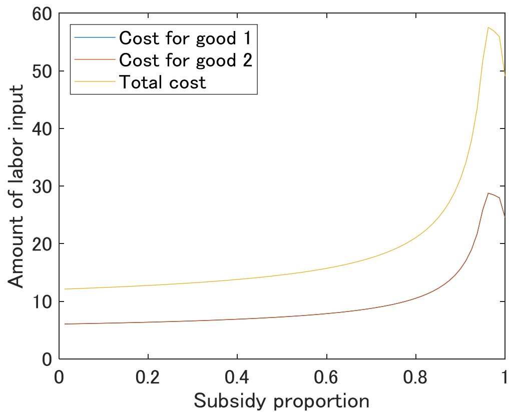

Figure 7. Costs for producing goods

Figure 8 shows utility curves of a household in CBD and that in SO, respectively. Figure 9 shows that location choice probability when a private firm can choose two cases; (i) with SO and (ii) without SO. The figure shows the probability of a private firm introducing its SO increases, when the subsidy proportion is between around 0.5 and 0.6. Note that we set weighting parameters, α and β in (51) as -5.58 and 2.55, respectively.

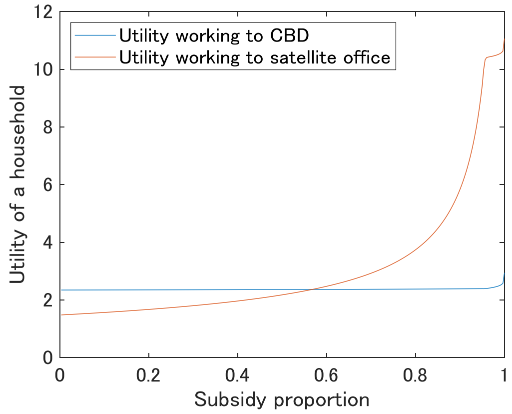

Figure 8. Utility of a household

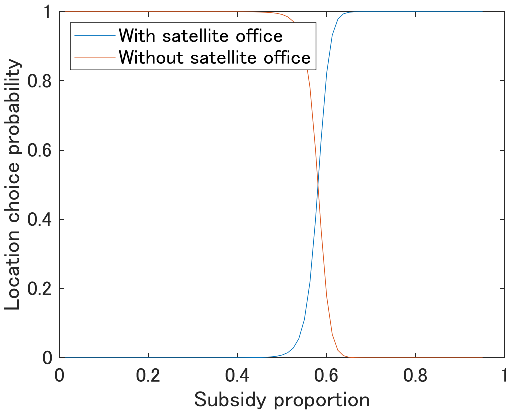

Figure 9. Location choice probability

4. CONCLUDING REMARKS

This study proposes a model for representing location choice behavior of a private firm which intends to introduce a SO out of CBD. When a private firm has some candidate locations for a SO, it is assumed to decide the location based on cost for producing a unit of goods and utility of a household. The decision-making process is formulated by a nested logit model in this article, thus the firm is assumed to decide whether or not it introduces its SO, and where to set a SO at the same time.

We perform sensitivity analysis with respect to subsidy for wage of workers at SO. The results of numerical calculations demonstrate that location choice probability of introducing a SO increases as subsidy proportion increases.

As the future tasks, the effect of disaster risk reduction at CBD should be considered. The earthquake risk under the capital of Tokyo is now expected and national and local governments propose to prepare a Business Continuity Plan (BCP) to a private firm. As part of BCP, telework and introduction of SO are expected. However, this study addressed only motivations of relocation from which a private firm can avoid commuting congestion and gain subsidies from local government. Thus, it is a major task to formulate location choice behavior of a private firm considering disaster risk reduction at CBD.

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