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OPTIMAL CONSTANT PRICE POLICY FOR PERISHABLE ASSET YIELD MANAGEMENT

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Abstract: Perishable asset yield management has wide applications in airlines seat inventory control, rental car and hotel room reservations. In contrast to the research on optimal dynamic pricing that has taken place in the past decade, we examine the practical merit of a constant price policy. This means a single price is applied to an item throughout the sales cycle.. We present procedures to identify the optimal constant price for two different applications. In the first, there is full dilution and in the second there is no dilution (dilution is when a customer that is willing to pay a high price is satisfied at a low price). We also propose a more flexible constant price policy (hybrid method) in which prices can be changed at discrete intervals and we present a method for its optimal policy. Furthermore, this hybrid method is capable of incorporating the costs associated with price change explicitly.

Key Words: Yield management, constant price policy, perishable asset, dynamic programming

1. INTRODUCTION

Yield management with a fixed stock over a finite period of time has wide applications in airline seat inventory control, rental car and hotel room reservations where the value of the remaining stock perish at the end of the sales period. Weatherford and Bodily (1992) defined yield management as the optimal revenue management of perishable assets through price segmentation. The basic decision is whether to sell a unit of product at a certain price or to hold it for more profitable customers. In this process, an important task is to evaluate each incremental unit of product at different time and inventory level.

Yield management theory is both a necessity and a revenue enhancer in many industries. Take the airline industry as an example. There could be up to several hundred calls for reservations within a second during peak periods requiring the system to make decisions to accept or reject in real time. Yield management theory underpins this decision so that its total revenue is maximized. In fact, significant revenue improvements have been reported by American Airlines who claimed a five percent increase in revenue over a three year period due to the yield management system worth a total of $1.4 billion (Smith, *et al.* 1992).

While the literature is rich on dynamic pricing policies, in this paper, we derive optimal solution for a constant price policy in the context of airline yield management on a single flight. We develop a method for calculating the optimal price for an entire sales period and allow for cases where prices can be updated at a finite number of times to reflect changes in inventory.

1.1 Dynamic Pricing Policy Background

Yield management has been in development for more than two decades and has received tremendous attention in recent years. Early work was seen in Littlewood (1972) who proposed a *marginal seat revenue* principle and applied it to a two-class, single-leg problem. Belobaba (1987, 1989) generalized this concept to the *expected marginal seat revenue* principle (EMSR) in the presence of multiple fares and was extended by Brumelle and McGill (1990), Curry (1990), Robinson (1995), and Wollmer (1992). Gallego and Van Ryzin (1994) and later, Feng and Xiao (2000) assumed that the demand on each fare level after dilution is known. More specifically, Feng and Xiao (2000) developed an optimal pricing policy based on intensity control theory.

Liang (1999) showed that the optimal policy is a threshold policy using a dynamic programming technique. A booking request is satisfied when a price is paid higher than the bid price (shadow price) of an incremental seat. Liang (1999) made special use of the booking curve information and dynamically updates the bid price for each incremental seat whenever the time and the number of seats available change. By way of background, dilution takes place when a high fare demand purchases low fare product. Liang (1999) assumed that the demand on each fare level is known and did not consider the dilution effect. Assuming the future demand curve unchanging, both Feng and Xiao (2000) and Liang (1999) provide highly dynamic pricing schemes: prices being updated with time elapsing and available stock decreasing.

Other developments parallel the aforementioned research. Typical work was seen in Zhao and Zheng (1998a, b) who reached similar conclusion to Liang's (1999), but differed in that it considered diversion and no shows. A diversion means a booking is shifted between flights (more accurately, itineraries) and no show refers to a passenger's not showing up for departure. In a different method, Chatwin (1999) modeled the airline reservation process as a continuous birth and death process to accommodate the cancellation and no shows. Wang and Regan (2003) specifically addressed the yield management problem in anticipation of aircraft swapping. The authors extended Liang's (1999) result and confirmed that the threshold policy remains valid.

All the research so far, however, considers the policy that allows the capability of dynamically changing its bid prices (estimate of the value of each incremental unit of product). In many cases, it might not be practically feasible and leads to our study on the constant price policy.

1.2 Merit of a Constant Price Policy

A constant price policy is one that sets a fixed price over the sales period. In contrast, a dynamic pricing scheme updates the price dynamically when time elapses and demand unfolds. It basically adopts methodologies such as dynamic programming, intensity control theory, or Markavian theory. Taking a cue from Gallego and Van Ryzin (1994), a typical dynamic pricing process is compared to a constant price policy as shown in Figure 1. In this figure, the price spikes whenever a purchase takes place and then decreases as time elapses. The optimal price is always a function of time and stock available.

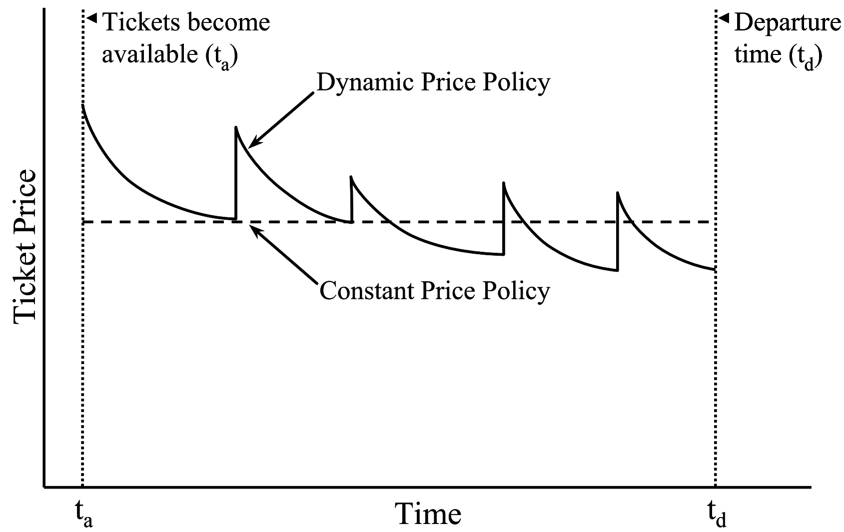

Figure 1. Dynamic Price Policy versus Constant Price Policy

In reality, a constant price scheme has three advantages. First, there is no requirement to update the price as time elapses and stock is sold. In many applications such as the fashion industry as studied by Gallego and Van Ryzin (1994), it is almost impossible to update the prices instantly after each unit of stock is sold or after a short time has elapsed. Therefore, the practicability is its primary advantage. Second, it is easier for travel agents to work with customers under a constant price scheme. For instance, the airline industry has traditionally allowed passengers with reservations to change to a ticket at a lower fare later. Third, a constant price scheme is more robust with group purchases. If one agrees that a group rate (the unit price paid for more than one item) should be no more than the rate for the last incremental item in the group under a dynamic pricing policy, she would have to agree that group booking would make suboptimal the optimal dynamic pricing policy studied in Feng and Xiao (2000) and Liang (1999).

1.3 Framing the Constant Price Policy Problem

The constant price policy problem examined in this paper continues on the theme developed in the influential paper by Gallego and Van Ryzin (1994) that examines the asymptotic property of a constant price policy. Their paper proved that a simple constant price policy is asymptotically optimal to the dynamic pricing problem when the remaining time is long and the remaining stock large, but did not discuss the optimal constant price solution. Our contributions include the following:

1. We present a solution procedure for the optimal constant price for two problems: one studied by Gallego and Van Ryzin (1994) and the other by Liang (1999),
2. We show that an optimal constant price policy possesses some similar properties to those of a highly dynamic pricing scheme, and
3. We further present a solution procedure to a hybrid constant price policy with limited number of price updates. This method explicitly incorporates the price changing cost.

In the following, we derive methods for the optimal constant price and present some analytical properties of this constant price policy. Finally, a solution procedure for a hybrid constant price policy with a limited number of price updates is presented.

2. OPTIMAL CONSTANT PRICE POLICY

The optimal constant price policy examined here is based on the optimal dynamic pricing policy problem studied in Feng and Xiao (2000) and Gallego and Van Ryzin (1994). It has applications in industries where customers in different classes pay the same labeled price such as the fashion industry. Feng and Xiao (2000) gave an optimal dynamic pricing policy while Gallego and Van Ryzin (1994) studied a simple heuristic constant price policy.

To derive optimal price policy, let $R_u(t, n)$ denote the expected revenue under a policy u at time t when there are n units of stock available. A pricing policy u is one that decides whether to sell a unit of stock at a time prior to the end of the time horizon by setting the profitable price level. If we denote the price at which a unit of stock is sold at time s by p_s, and the cumulative stock sold before time s by N_s, the expected revenue under policy u can be expressed as

$$R_u(t,n) = E_u\left[\int p_s dN_s\right] \tag{1}$$

with the boundary conditions

$$R_u(t,0) = 0 \qquad \forall t \geq 0 \tag{2}$$

$$R_u(0,n) = 0 \qquad \forall n \geq 0,1,2,... \tag{3}$$

An optimal pricing policy u^* is one defined as

$$R_{u^*}(t,n) = \sup_{u\in\xi} E_u\left[\int p_s dN_s\right] \tag{4}$$

where ξ is the set of all pricing policies.

A constant price policy u is one that sets a constant acceptable price throughout the time horizon (i.e. p_s = constant). Under a constant price policy u, a demand is satisfied if it pays the labeled constant price and if there is stock available. The stock is reduced by one unit when a demand is satisfied. The optimal constant price policy is one defined as

$$R_{p^*}(t,n) = \sup_{p_s\in P} E\left[p_s N_s\right] \tag{5}$$

If we denote by ψ the set of all constant price policies, obviously the set of constant price policy is a subset of all possible policies, that is, $\psi \subset \xi$. Therefore, it is natural to have

$$R_{u^*}(t,n) \geq R_{p^*}(t,n) \tag{6}$$

however, it is shown by Gallego and Van Ryzin (1994) that

$$\lim_{\substack{t\to+\infty \\ n\to+\infty}} R_{p^*}(t,n) = R_{u^*}(t,n) \tag{7}$$

and regretfully in that work, no optimal constant price policy is provided.

Calculating the optimal constant price policy is a two step process. First, the expected revenue is obtained at each constant price. Second, compare the revenues brought about at each constant price and determine the optimal one which brings the most revenue. In the following, we present details about both steps.

2.1 Expected Revenue at a Given Price with Dilution

Denote the sales season by [0, T]. The season begins with a given inventory of M items. It is assumed that no replenishment of inventory is possible after the sales have started. We assume a discrete set of predetermined prices $P = \{p_1, p_2, \ldots, p_k\}$ with $p_1 > p_2 > \ldots > p_k$. For each price, p_i, the demand is a Poisson process with a point intensity λ_i. Note that there is only one labeled price out of P that applies at any given time. It is assumed that all the demands pay the same labeled price. Our objective is to find a price out of P at which the items are sold until the end of the time horizon to maximize the total expected revenue.

In recursive form, the expected revenue with a particular constant price, P_i, can be expressed as

$$R^i(t,n) = \int_0^t d\tau \exp\left(-\int_\tau^t \lambda_i d\tau'\right)\left(p_i + R^i(\tau, n-1)\right)\lambda_i \tag{8}$$

Note that the term

$$\exp\left(-\int_\tau^t \lambda_i d\tau'\right) \tag{9}$$

is the probability of having no demand arrival by time τ and

$$\exp\left(-\int_\tau^t \lambda_i d\tau'\right)\lambda_i d\tau \tag{10}$$

represents the probability of having the first demand at time τ. Taking derivative of Equation 8 with respect to t yields

$$\dot{R}^i(t,n) = \lambda_i\left(p_i - R^i(t,n) + R^i(t,n-1)\right) \tag{11}$$

or alternatively,

$$\dot{R}^i(t,n) + \lambda_i R^i(t,n) = \lambda_i\left(p_i + R^i(t,n-1)\right) \tag{12}$$

Equation 12 is a first order differential equation that has the following form

$$\dot{y} + P(t)y = Q(t) \tag{13}$$

whose general solution can be expressed as (see, for example, Wang and Wang, 1981)

$$y = Ce^{-\int P(t)dt} + e^{-\int P(t)dt}\int Q(t)e^{\int P(t)dt}dt \tag{14}$$

Substituting the appropriate terms from Equation 12 yields the optimal pricing policy general solution

$$R^i(t,n) = C_n e^{-\lambda_i t} + e^{-\lambda_i t}\int\left[\lambda_i\left(p_i + R^i(t,n-1)\right)\right]e^{\lambda_i t}dt \tag{15}$$

The particular solution can be found by subjecting optimal pricing policy general solution shown in Equation 15 to the boundary conditions

$$R^i(t,0) = 0 \qquad \forall t \geq 0 \tag{16}$$

$$R^i(0,n) = 0 \qquad \forall n \geq 0,1,2,... \tag{17}$$

Setting n = 1 and applying the first boundary condition into the optimal pricing policy yields

$$\dot{R}^i(t,1) + \lambda_i R^i(t,1) = \lambda_i p_i \tag{18}$$

and leads to the general solution

$$R^i(t,1) = C_1 e^{-\lambda_i t} + p_i \tag{19}$$

Applying the second boundary condition and solving for C_1, the particular solution for n = 1 becomes

$$R^i(t,1) = p_i(1 - e^{-\lambda_i t}) \tag{20}$$

This result serves as the seed for recursively calculating the optimal pricing policy, $R^i(t, n)$, using Equation 15 and the boundary conditions in Equations 16 and 17. Similarly, the optimal revenue of having two units of stock available at time t is

$$\dot{R}^i(t,2) + \lambda_i R^i(t,2) = \lambda_i \left(p_i + R^i(t,1) \right) = 2 p_i \lambda_i - p_i \lambda e^{-\lambda_i t} \tag{21}$$

that leads to the following solution

$$R^i(t,2) = -2 p_i e^{-\lambda_i t} + 2 p_i - p_i \lambda_i t e^{-\lambda_i t} \tag{22}$$

In many applications such as the airline seat inventory control, there are a limited number of fare levels (typically between 10 and 20) making it very efficient to find the expected revenue for each fare level. The optimal constant price becomes

$$p^* \in \left\{ p_i : R^i(t,n) \geq R^j(t,n), \forall p_i, p_j \in P, j \neq i \right\} \tag{23}$$

and optimal revenue can be expressed as

$$R^{opt} = \max_i \left\{ R^i : p_i \in P \right\} \tag{24}$$

Proposition 1

The revenue function, $R^i(t, n)$, increases monotonically and is concave over time, and the marginal expected revenue decreases as the stock level increases.

Proof

We first prove with induction that the revenue function, $R^i(t, n)$, increases monotonically, that is.

$$\dot{R}^i(t,n) \geq 0 \tag{25}$$

To begin the induction, we assume that

$$\dot{R}^i(t,n-1) \geq 0 \tag{26}$$

which means

$$R^i(t,n-1) \geq R^i(\tau,n-1) \qquad t \geq \tau \tag{27}$$

From the expected revenue with a particular constant price shown in Equation 8, we have

$$\begin{aligned} R^i(t,n) &= \int_0^t d\tau \exp\left(-\int_\tau^t \lambda_i(\tau')d\tau'\right) p_i\lambda_i(\tau) + \int_0^t d\tau \exp\left(-\int_\tau^t \lambda_i(\tau')d\tau'\right) R^i(\tau,n-1)\lambda_i(\tau) \\ &\le \int_0^t d\tau \exp\left(-\int_\tau^t \lambda_i(\tau')d\tau'\right) p_i\lambda_i(\tau) + \int_0^t d\tau \exp\left(-\int_\tau^t \lambda_i(\tau')d\tau'\right) R^i(t,n-1)\lambda_i(\tau) \\ &\le p_i + R^i(t,n-1) \end{aligned} \tag{28}$$

Combining this with Equation 11, we have

$$\dot{R}(t,n) \ge 0 \tag{29}$$

The induction finishes with the fact that

$$\dot{R}^i(t,1) \ge 0 \tag{30}$$

thus finishing the proof for this part.

We then prove that the marginal expected revenue decreases as the stock level increases. This can be shown by proving

$$R^i(t,n+2) - R^i(t,n+1) \le R^i(t,n+1) - R^i(t,n) \qquad \forall n \ge 1 \tag{31}$$

The derivative of the optimal pricing policy for a single price shown in Equation 11 can also be applied to incremental values of n and written as follows

$$\dot{R}^i(t,n+1) = p_i\lambda_i + \lambda_i\left(R^i(t,n) - R^i(t,n+1)\right) \tag{32}$$

and

$$\dot{R}^i(t,n+2) = p_i\lambda_i + \lambda_i\left(R^i(t,n+1) - R^i(t,n+2)\right) \tag{33}$$

We start the induction by showing that

$$\dot{R}(t,n+2) \ge \dot{R}(t,n+1) \tag{34}$$

given

$$\dot{R}(t,n+1) \ge \dot{R}(t,n) \tag{35}$$

Based on the expected revenue with a particular constant price in Equation 8, we have

$$R^i(t,n+2) - R^i(t,n+1) = \int_0^t d\tau e^{-\int_\tau^t \lambda_i d\tau'} \left(R^i(\tau,n+1) - R^i(\tau,n)\right)\lambda_i \tag{36}$$

Partially integrating the right hand side yields

$$R^i(t,n+2) - R^i(t,n+1) = R^i(t,n+1) - R^i(t,n) - \int_0^t d\tau e^{-\int_\tau^t \lambda_i d\tau'} \left(\dot{R}^i(\tau,n+1) - \dot{R}^i(\tau,n)\right) \tag{37}$$

Since the last term is less than or equal to zero based on the assumption, we have the following result

$$R^i(t,n+2) - R^i(t,n+1) \le R^i(t,n+1) - R^i(t,n) \tag{38}$$

thus finishing the proof with the fact that

$$\dot{R}(t,1) \ge \dot{R}(t,0) \tag{39}$$

The concavity of the revenue function in time can be shown by taking the second derivative of expected revenue with a particular constant price as in Equation 8:

$$\ddot{R}^i(t,n) = \lambda_i\left(\dot{R}^i(t,n-1) - \dot{R}^i(t,n)\right) \tag{40}$$

With Equation 34, we conclude that

$$\ddot{R}^i(t,n) \le 0 \tag{41}$$

(End of proof)

2.2 Expected Revenue at a Given Price without Dilution

Having these results allows the model to be extended to an alternative problem defined in Liang (1999) where no dilution effect exists. For a sales season [0, T], there is a stock of M items for sale. There is a set of discrete prices $P = \{p_1, p_2, \ldots, p_k\}$. In this problem, we also assume that $p_i > p_{i+1}\ \forall i$. At any time during the time period, the demand is a Poisson process with the point intensity at each price level being denoted by $\mu_i(t)$. A demand at level i is satisfied when the paid price p_i is higher than or equal to an a priori constant control price. The control price is one of the discrete prices. The demand at each fare level pays its own fare even if a lower fare is available. The salvage value of the remaining stock at the end of the season is zero. The objective is to find the optimal constant control price over the sales period to maximize the total revenue.

This problem reflects a typical market segmentation practice in many service industries. The set of discrete prices corresponds to the set of fares in the airline industry in particular. Similar problem is studied in Liang (1999) for an optimal dynamic pricing policy. We derive the optimal constant price policy through the following recursion

$$R^j(t,n) = \sum_{i \le j} \int_0^t d\tau \exp\left(-\int_\tau^t \sum_{i \le j} \mu_i(\tau')d\tau' \right)\left(p_i + R^j(\tau, n-1) \right)\mu_i(\tau) \tag{42}$$

where j is an index for p_j used as the control price. Again, taking the derivative with respect to t, yields

$$\dot{R}^j(t,n) = \sum_{i \le j} \mu_i(t)\left(p_i - R^j(t,n) + R^j(t,n-1) \right) \tag{43}$$

Similarly, we can have the solution in the form

$$R^j(t,n) = Ce^{-\sum_{i \le j}\int_0^t \mu_i(\tau)d\tau} + e^{-\sum_{i \le j}\int_0^t \mu_i(\tau)d\tau} \int dt \left(\sum_{i \le j} \mu_i(t) p_i + \sum_{i \le j} \mu_i(t) R^j(t,n-1) \right) e^{\sum_{i \le j}\int_0^t \mu_i(\tau)d\tau} \tag{44}$$

Note that there exists a boundary condition

$$R^j(t,0) = R^j(0,n) = 0 \qquad \forall j, n = 0,1,2,\ldots \tag{45}$$

The integration is a recursive process from n = 1 to higher inventory level. The integration here is noticeably more complicated than in the problem studied first.

As a special case, if the demand intensity at each fare level is a constant with time, i.e. $\mu_i(t) = \mu_i$, we have the solution when n = 1, 2 as

$$R^j(t,1) = \frac{\sum_{i \le j} \mu_i p_i}{\sum_{i \le j} \mu_i}\left(1 - e^{-\sum_{i \le j} \mu_i t} \right) \tag{46}$$

and

$$R^j(t,2) = \frac{-2\sum_{i \le j} \mu_i p_i}{\sum_{i \le j} \mu_i} e^{-\sum_{i \le j} \mu_i t} + \frac{2\sum_{i \le j} \mu_i p_i}{\sum_{i \le j} \mu_i} - \sum_{i \le j} \mu_i p_i t e^{-\sum_{i \le j} \mu_i t} \tag{47}$$

2.3 A Hybrid Method with Limited Times of Price Adjustment

By developing a constant price policy, a retailer does not have to lock himself into to a fixed price and lose all the opportunities for re-optimization. One can always adjust the optimal constant price at times when the demand is realized unexpectedly.

We discuss two ways for price adjustment. In both cases, price adjustment takes place at *a priori* determined time points (t_n,…, t_2, t_1) with $t_n > t_{n-1}$. One method, called Method I, is to set the optimal constant price in anticipation of these price adjustments in the future while the other, Method II, does not anticipate such future price adjustment.

We outline a solution procedure in Method I. Be aware that price adjustment takes place at *a priori* determined time points (t_n,…, t_2, t_1) with $t_n > t_{n-1}$. For any starting capacity n with the optimal constant price p_i for the time interval [t_{k-1}, t_{k-2}], suppose the optimal revenue is $\underline{R}^{opt}(t_{k-1}, n)$. Using this as the salvage function, the revenue function for each constant price during the time interval [t_{k-1}, t_{k-2}] is obtainable. And therefore, we can decide the optimal constant price for each time period. Solution details are explained as follows for Problem 1.

The recursion is in the form

$$\begin{aligned} R^i(t_k,n) &= \int_{t_{k-1}}^{t_k} d\tau \exp\left(-\int_{\tau}^{t_k} \lambda_i(\tau')d\tau'\right)\left(p_i + R^i(\tau,n-1)\right)\lambda_i(\tau) \\ &+R^{opt}(t_{k-1},n)\exp\left(-\int_{t_{k-1}}^{t_k} \lambda_i(\tau')d\tau'\right) + c(t) \end{aligned} \tag{48}$$

The last term of the equation represents the salvage value and $c(t)$ is the price changing cost at time t. Note that $c(t)$ is a constant, exogenous to the model. Taking derivative to t_k, we have a differential equation similar to derivative of the expected revenue with a particular constant price shown in Equation 11 as

$$\dot{R}^i(t_k,n) = \lambda_i\left(p_i - R^i(t_k,n) + R^i(t_k,n-1)\right) \tag{49}$$

with a boundary condition $R(t_k, 0) = 0$ and can be solved in a similar manner as before. In fact, when the time intervals [t_k, t_{k-1}] $\forall k$ shrinks towards zero, the hybrid constant price policy reduces to the dynamic pricing policy as in Feng and Xiao (2000) under the condition that the price changing cost is ignored. Generally, by updating the constant prices at a set of predetermined time points, one can achieve a balance between the practicability advantage of a permanent constant price policy and a revenue advantage of a dynamic pricing policy.

Proposition 2
Repeatedly updating the optimal constant price over the sales period leads to revenue improvement.

Proof
Suppose at time t, $T > t > 0$, the optimal constant price is p_i as a function of the remaining time t and the available stock n for the entire interval [t, 0]. At another time $t' < t$, suppose the remaining capacity turns out to be $c \leq n$ with a corresponding probability $P(c)$. We denote by $p_m(t', c)$ the optimal constant price. We must have the following result

$$\sum_c P(c)R^l(t',c) \leq \sum_c P(c)R^m(t',c) \quad \forall t' < t \tag{50}$$

The reason is

$$R^l(t',c) \le R^m(t',c) \qquad \forall c \tag{51}$$

The reason is $R^l(t',c) \le R^m(t',c), \forall c,$ based on the definition of p_m. Similar proof can be given in the case of multiple price updates. Obviously, Method I leads to more revenue than Method II.
(End Proof)

Albeit simple, it answers a question raised in Cooper (2002). There is a clear difference between a heuristic constant price policy and an optimal constant price policy even if both are proved to be asymptotically optimal to the dynamic pricing problem as in Gallego and Van Ryzin (1994). Cooper (2002) proves the underlying mechanism why a simple constant price scheme provides asymptotically optimal solution. The author further gives a counter intuitive example where a repeated use of his simple algorithm leads to a worse result. We present the example as follows.

2.4 Example Application

Consider a single leg problem with two fare classes in the airline operation: Class 1 and Class 2, who pays p_1 = 10 and p_2 = 2, respectively. Suppose there are two seats remaining at two time units prior to departure. The demand processes for Class 1 and Class 2 are both homogeneous Poisson processes with $\mu_1 = \mu_2 = 1$.

Cooper (2002) showed that a simple fixed price policy resulting from a linear program is asymptotically optimal

$$\max_x \left\{ p \cdot x : Ax \le c, 0 \le x \le \mu \right\} \tag{52}$$

where x is the seat allocation to the fare buckets; c is the total number of seats available for sale on each flight; A is the incidence matrix; μ is the expected demand over the time horizon. (Note that all the variables are in vectors and the notation slightly changed for consistency purposes. For more details, please refer to Cooper, 2002.)

We refer to the time two units before departure as Time 2, and similarly one unit before departure as Time 1. We call the strategy to update the constant price based on new information as re-solving strategy. Based on that simple policy, Cooper allocated both seats for the high fare bucket at Time 2, which means a fixed bid price p = 10 is in effect. When it comes to Time 1, Cooper reapplied that simple algorithm and found that the solution is worse off by this re-solving strategy. The case is explained in the following. There are three scenarios from Time 2 to Time 1 referred to as first time period.

Scenario 1: There are two bookings accepted during the first time period for Class 1. Re-solving strategy leads to no improvement in the total revenue;

Scenario 2: There is no demand during the first time period. There are two seats available at Time 1. The probability for this scenario is e^{-1}. Under the re-solving strategy, Cooper had one seat allocated to each bucket which is equivalent to a fixed bid price at p = 2. The expected revenue during the second time period is $10(1-e^{-1})$ for Class 1 and $2(1-e^{-1})$ for Class 2 with a total 7.59. Without the re-solving strategy, both seats are allocated to the high bucket and the expected revenue for the second time period is $20 - 30\, e^{-1} = 8.96$.

Scenario 3: There is one booking accepted during the first time period. Then there is one seat left for sale during the second time period. In this case, re-solving strategy does not lead to any change in the pricing policy.

As seen, the difference happens in Scenario 2 where re-solving policy brings about less revenue by 1.37. In contrast, re-applying the optimal constant price policy does not lead to worse policies. Based on Equation (11) and (12), we have the following results. At Time 2, we check the expected revenue with the two different price levels. We find $R^1(2, 2) = 20 - 40e^{-2} = 14.59$ and $R^2(2, 2) = 12 - 36\, e^{-4} = 11.34$. Therefore, only bucket 1 should be open at Time 2 when there are two seats available. Apply the revenue function to Scenario 2. We have $R^1(1, 2) = 20 - 30e^{-1} = 8.96$ and $R^2(1, 2) = 12 - 24e^{-2} = 8.75$. It means that having bucket 1 open brings about slightly more expected revenue. Therefore re-applying the optimal constant price policy at Time 1 does not lead to a worse solution.

Note that at Time 1, the expected revenue we have here for opening both price classes is different from the calculation by Cooper (2002) because Cooper (2002) does not consider nesting effect (That is the reason his method corresponds to a *heuristic* constant price policy, -- the price is constant for just a period of one time unit.). Nesting effect means that a high fare passenger can take seats allocated to low fare demands when all the seats specially allocated to the high fare demand are used up.

Although this example is simple in nature, it is illuminating. It shows that although some constant price policy possesses asymptotically optimal property, repeatedly applying it could lead to a worse result. In contrast, optimal constant price policy always leads to revenue improvement when repeatedly applied. This result shows the advantage of having an optimal constant price scheme.

There are two observations worth mention. First, the optimal dynamic pricing policy as in Feng and Xiao (2000) and Liang (1999) ignores the cost of price update. This cost could be in the form of changing price tags or in the form of loss of customer goodwill for frequent price change. In contrast, a constant price policy does not have these costs. Even without taking into account these cost, the dynamic pricing policy just outperforms a simple heuristic constant price policy as examined in Gallego and Van Ryzin (1994) by about 2 to 3 percent. Together with other reasons as examined at the start of this paper, optimal constant price policy shows superiority over the dynamic pricing ones.

Second, in the particular setting of network yield management, generally a discrete allocation method is used. That method may correspond to a heuristic constant price policy when the bid price (shadow price) of the incremental seat on each flight (for details please see Talluri and Van Ryzin, 1998) is set valid and unchanged throughout the sales period. When seat allocation to the lowest fare class is used up, that class is closed. That does not represent an optimal constant price policy. Although it is possible to study for the optimal constant price policy under a network setting, as a starting point, we focus this research on an optimal constant price policy applicable to leg based system only. We hope following the techniques here, we will be able to study the optimal constant price policy under the network setting.

3. CONCLUSION

A constant price policy could be more robust in a competitive market or in a market full of anticipating consumer behaviors. Therefore, we focus our study on the optimal constant price policy. We present a solution procedure for an optimal constant price for two problems that are typically studied in literature with one assuming full dilution and the other assuming no dilution effect. Since Gallego and Van Ryzin (1994) has shown the asymptotically optimal property of the optimal constant price policy, no discussion is given here about the optimality of the optimal constant price policy as opposed to the dynamic pricing one. We briefly examine some properties of this solution scheme and find that, similar to a highly dynamic pricing scheme, the revenue function of a constant price policy increases and is concave in time and that the value of an incremental unit of stock diminishes with the stock level increasing. In addition, a hybrid solution procedure is presented with the optimal constant price being updated at a limited number of preset times. This hybrid method bridges the constant price policy and the dynamic pricing one. When the frequency of updates tends to be high, this solution scheme becomes the dynamic pricing one. We provide a compromise between a highly dynamic pricing policy and a permanent constant price policy by limiting the number of price updates to a predetermined set of times. In addition, the hybrid method is capable of incorporating the price changing cost into the model, which no other method is capable of to our best knowledge.

In this paper, we only examine a constant price policy in a leg based yield management system. This method may be relatively easily implemented in a leg-based yield management system which is being used by many major airlines in the world. However, we also notice that many airlines are in a transition into a network yield management systems. The concept of constant price policy in a network system could be both interesting and challenging. The reason is that the itineraries are interwoven. How to account for the network effect remains to be studied. We would leave this to our future research. Of course, study also needs to accommodate LCCs and overcapacity.

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