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DELAY RECOVERY IN AIR TRANSPORT HUB-SPOKES NETWORK SYSTEM CONCERNING HUB-TO-HUB SCHEDULE CONFLICT

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Abstract: This paper addresses the methodology of reduction of air traffic delay based on integer programming problem. It is assumed that each hub is used as the primary hub airport for the independent. In this paper, we propose the bi-level structure for optimizing the delay recovery: the upper level is the Aviation Operation Center (AOC), which is controlling the hub-to-hub schedule: the lower level is controlled by the Local Hub Operators (LHO), which handle both hub-to-hub and hub-to-spokes schedules. Finally, numerical examples are carried out to show how the proposed method works.

Key Words: Air Transport Market, Delay Recovery, Hub-Spokes Network

1. INTROUCTION

Recovery of the irregular event such as delay and cancellation is one of the most significant issues of the contemporary air transport management. This issue also becomes more important than ever in Asian air transport market because of heavy traffic at Asian gateway airports. It is forecasted that the short stage traffic, business trip for example, will increase a lot in intra-Asian air transport market, so that if we do not have the suitable method of handling air traffic delay, a big loss will be loaded by not only airlines but also air passengers. In particular, the flight schedule under hub-spokes service network may be vulnerable to irregular events, that is, delay/cancellation.

Schedule/reschedule problem is a very popular research field of OR/transport engineering, so that many research outcomes can be found in this field. Let us focus the major literatures dealing with schedule/reschedule problem in air transport research filed in the last decade.

As for the scheduling problem, Barnhart et al proposed a very systematic scheduling optimization method by applying the linear programming technology, column generation (Barnhart and Boland et al, 1998; Barnhart and Johnson et al, 1998). Barnhart deals with aircraft rotation and aircrew patch, simultaneously. Barnhart's optimization scheme works well and it can handle a very big scheduling problem (Barnhart et al, 1998b). Basically, the nature of the rescheduling problem is the same as the scheduling problem has, but its workability, which means "easy to compute", is more important because operators should determine the optimal schedule as fast as possible. Arguello et al developed a basic tool for delay/cancellation recovery problem (Arguello et al, 1998). Arguello proposed the "time-band approximation" to reduce the computation time. Arguello's model deals with aircraft rotation, so that it ignores the passengers' disutility and flight crews' patch schedule. Clarke and Rosenberger et al addressed more general rescheduling problem. Clarke proposed a very complex hybrid model consisting of flight sequence, passenger demand covering, crew availability, and slots and gate allocation (Clarke, 1997). Clarke's model adopts the heuristic optimization based on branch-and-bound. Rosenberger also developed the heuristic approach for the optimization (Rosenberger et al, 2003). Thengvall et al proposed another solution algorithm based on bundle algorithm for solving a very complex schedule recovery problem (Thengvall et al, 2003).

However, these articles address the optimization by a "single" operator, which means one operator controls all flight schedules. However, in the actual market, the local hub operators aim to optimize the benefit of their "direct" customers, that is, airlines operating direct flights as well as direct passengers. For example, San Francisco International Airport (SFO) wants to reduce the direct customers' disutility by adjusting the flight schedule as well as Chicago O'Hare International Airport (ORD) also reschedules the flights in order to minimize their customers' cost due to the irregular events. These local optimizations cause the "conflict" in rescheduling between airport operators.

This is an example in the U.S., but we may face the similar situation in Asia-based air transport market. Since the intra-Asia transport market grows a lot, a very complicated flight schedule is carried out and it is easily expected that due to the airport congestion the schedule delay will become more serious than current. Moreover, a number of code-share flights including alliance flights increases, so that the conflict between alliance members may occur in intra-Asia short haul air transport markets. Thus, one may say that the delay/cancellation recovery concerning the conflict between local operators becomes one of the most important roles in Asia-based air transport market. The key factor is how to deal with the conflict.

In the present paper, the methodology of delay/cancellation recovery concerning the conflict between local operators is discussed. In particular, the idea of equilibrium between local operators is introduced in order to adjust the conflict; the mathematical formulation and the algorithm of obtaining the equilibrium schedule are proposed.

The present paper consists of four bodies; in Chapter 2, the mathematical modeling and its solution procedure are discussed; particularly, the idea of bi-level programming plays the key role of this formulation; in Chapter 3, some numerical examples are computed and the performance of the model is evaluated; in Chapter 4, we show some remarks of this research and finally concluded.

2. THE MODEL

In this chapter, the mathematical modeling of delay/cancellation recovery problem concerning the conflict between local operators is discussed.

2.1 Framework/Conditions/Assumptions

(a) Airlines

When the airlines should adjust their flight schedules for reducing the negative impact from

the irregular events, the airlines should pay attention to two terms shown below:
1) Concern the sequence of aircraft allocation.
2) Passengers' disutility and capacity limitation of aircraft.

The first term is the operational constraint. This constraint makes the rescheduling problem in the air transport network very complicated and complex; when the airline determine the flight schedules, they also determine the sequence of aircraft movement/rotation; one flight delay/cancellation affects the entire aircraft movement/rotation sequence, so that the rescheduling also means the re-determination of aircraft movement/rotation sequence after the irregular event. Luckily, some existing researches propose how to reduce this complexity (Arguello et al, 1998) and we follow their ideas. In the following section, the procedure of reducing complexity is explained.

The second term is more problematic. The first aim of the airline to adjust the flight schedule is to reduce the additional cost due to the irregular event, which should involve the passengers' disutility. In other words, the airline should transport the passengers affected by the irregular event as in time as possible. However, each aircraft has a capacity limitation and active seat reservation, the airline should concern the desirable passenger allocation. For the actual use, we should deal with this rescheduling problem involving the second term constraint.

For refusing too-much complexity, we assume the following conditions:
1) The aircraft should move after minimum duration for the maintenance at least.
2) All airlines operate the same size aircraft with same seat configuration in the market.
3) Each airline should recover the schedule before the next day's first flight.
4) Each airline should transport the entire passengers until the next day's first flight.
5) The schedule problem that we deal with is regarded as deterministic, so that each flight's status of seat reservation is predetermined.

Assumption 1) is the constraint of the minimum duration for aircraft dispatch and it is necessary to reflect the situation of actual aircraft rotation. Assumptions 2) to 5) are introduced for reduction of complexity. In the future, we should relax these strict constraints for the actual use. Of course, some of these assumptions can be relaxed; for example, 4) can be relaxed, but this relaxation requires the transversality conditions for the optimization. Thus, we adopt the above assumptions for simple understanding of the model behavior.

(b) LHOs and AOC

Let us deal with the hub-spokes air transport network with a few hubs; in this network, it is assumed that each hub airport has its own dominated airline; in other words, each airline has the unique hub airport. In this paper, we call these hubs "local hubs," henceforth.

As mentioned above, we assume that each airline has its own hub airport, and at the hub airport the airline enjoys the priority on making flight schedules and adjusting the schedule, that is, rescheduling. Therefore, henceforth we regard a local hub operator (**LHO**) as a representative subject of its dominating airline, so that we discuss a LHO's behavior.

The LHO is assumed that the flight schedule of its dominated airline is adjusted as a first priority. Hence, we assume the myopic behavior of each LHO. Actually, many major airports have their own dominated airlines, so that this assumption is acceptable. Under this condition, due to the myopic behavior of each LHO, conflict on schedule/reschedule comes up: one airline's first best flight schedule may push other airlines' first best flight schedules away. Thus, if we do not eliminate this conflict, or reduce this conflict at least, we cannot design the effective flight schedule/reschedule for the whole system any more.

In order to eliminate the conflict, we accept to introduce the conflict coordinator, which we call the "Airline Operations Center-AOC." Many articles address the function of the AOC (Grandeau, Clarke and Mathaisel, 1998), so that we follow the functions of the AOC as existing articles address. The main function of the AOC is to design the whole network under no irregular events, but in emergency it should modify the schedules in order to optimize the whole network. However, it also should satisfy the LHOs' requirement as much as possible, so that the AOC should design the network concerning the equilibrium behavior of each hub operator.

On the other hand, if the AOC should design the entire schedule, it may be too much complicated and tough to handle. We propose to reform the decision-making area of AOC in order to reduce the problem size as follows:

1) The AOC can modify the hub-to-hub flight schedules. The hub-to-spoke flight schedules are controlled by LHOs.
2) The AOC aims to adjust the performance of entire network, but the performance can be improved indirectly.

One may say these conditions are very unique and idealistic; actually, in the real world, LHO does not follow the AOC's decision. However, this indirect control sometimes becomes pragmatic way because the outbound information from the AOC is declined. As a result, the constraints for the LHOs given by the AOC are quite reduced and they can design the rescheduled diagram as freely as they want. Therefore, these two conditions work as the main part of the modeling framework.

2.2 Formulation

Before the detailed discussion of mathematical formulation, let us introduce another important condition. Following Arguello's approach (Arguello, 1998), the following model accepts the "approximation of time-band," which means that the flight schedules are designed in time-band, not the actual discrete time. If we accept this approach, the cost of computation is quite reduced. Therefore, the following formulation is described in approximated time-band.

(a) LHO's Behavior

The aim of the LHO is to minimize both of the dominated hub airlines' and direct passengers' additional cost due to the irregular event by adjusting flight schedule based on their own hub airport. In the following formulation, let r and s be the origin and the destination, respectively. Let the passenger's path be k for pre-reschedule path flow and k' for rescheduled path flow. The problem that the LHO confronts is formulated as below:

[Optimization Problem: LHO]

Object: $$\min Z^n(g_l^{t,a}, u_{kk'}^{rs}; G^*) = \sum_l c_l \sum_t g_l^{t,a} - \sum_l c_l \sum_t f_l^t + \sum_{rs}\sum_k \sum_{k'} (delay)_{kk'}^{rs} u_{kk'}^{rs} D, \quad (1)$$

Subject to

$$\sum_l \sum_t^{t-1} g_l^{t,a} \delta_l^{t,i} \geq \sum_l \sum_t^{t} g_l^{t,d} \delta_l^{t,i}, for \quad \forall i \in I, \quad (2)$$

$$-\sum_l \sum_{t=0}^{T} g_l^{t,a} \delta_l^{t,i} + \sum_l \sum_{t-0}^{T-1} g_l^{t,d} \delta_l^{t,i} = h_i, for \quad \forall i \in I, \quad (3)$$

$$g_l^{t+R_l,a} = g_l^{t+R_l,d}, for \quad \forall l \in \Theta, \quad (4)$$

$$u_l^t \leq seat \cdot g_l^t, \quad (5)$$

$$g_l^{t,a} = \{0,1\}, \quad for \quad \forall l \in \Theta, t = \{0, T-1\}, \quad (6)$$

$$\sum_{k'} \sum_k u^{rs,k} = X_{rs}. \quad (7)$$

where

$g_l^{t,a}$ and $g_l^{t,d}$ are the control variables with dichotomous values. It indicates the reschedule flight sequence; the former obtains 1 when a departure is scheduled on link l in time period t and obtains zero otherwise; the latter obtains 1 when an arrival is scheduled on link l in time period t.

$u_{kk'}^{rs}$ is the amount of passengers' flow of OD pair rs, which is reallocated from k to k'. The construction of path flow is as below:

$$u_l^t = \sum_{rs} \sum_k u_k^{rs} \delta_k^{rs,l,t}, \quad (8)$$

$u_{l,}^t$ is the link flow of flight (l,t), and $\delta_k^{rs,l,t}$ is the dichotomous variable that obtains 1 when path flow rsk uses flight (l,t) and obtain zero otherwise.

c_l is the predetermined operational cost per flight depending on the link l.

f_l^t is the original flight schedule and the value is predetermined.

$(delay)^{rs}{}_{kk'}$ indicates the actual additional time for the rs passengers occurring by reallocation from k to k'. The construction is as below:

$$(delay)_{kk}^{rs} = R_{k'}^{rs} - R_k^{rs}, \quad (9)$$

where $R^{rs}{}_k$ and $R^{rs}{}_{k'}$ mean the original total travel time and rescheduled total travel time, respectively. $R^{rs}{}_{k'}$ involves the idling time.

D is the average delay cost per person loaded by a passenger. G^* means the optimal behavior of the AOC and G^* is given as the hub-to-hub flight schedule. R_l means the line haul time in link l.

Equation (1) is the object function. Constraint (2) means the constraint of the aircraft movement; the total number of departure flight at time period t cannot surpass the total number of parking aircrafts. Constraint (3) is the terminal constraint of aircraft movement balance at time period T. Constraint (4) means the sequence of aircraft movement. Constraint (5) is the aircraft's capacity constraint. Constraint (6) means that the control variables are dichotomous. Constraint (7) means the OD preservation constraint.

(b) AOC's Behavior
The AOC aims to improve the network performance. In the current research, we assume that the network performance is evaluated as the connecting passengers' convenience. The AOC aims to reduce the connecting passengers' disutility due to the delay/cancellation by controlling the hub-to-hub flight schedule. The AOC can control the hub-to-hub flight schedule directly, while the AOC cannot deal with the hub-to-spokes flight schedule directly. Thus, the AOC should make the hub-to-hub schedule effective for the indirect control of hub-to-spokes schedule.

The problem that the AOC faces is formulated as below:

[Optimization Problem: AOC]

Object: $$\min Z^{AOC}(g_l^{t,a} \in \Pi^{HUB}) = \sum_{rs}\sum_{k}\sum_{k'}(delay)_{kk'}^{rs} u_{kk'}^{rs} \delta_{kk'}^{rs,CONNECT} D, \qquad (10)$$

Subject to
$$u_{kk'}^{rs,CONNECT} = \arg\{\min(Z^n : n = 1, \cdots, N)\}, for \quad \forall rs \in \Omega, kk' \in \Xi \qquad (11)$$
$$g_l^{t,a} = \{0,1\}, for \quad \forall l, t \qquad (12)$$

$\delta^{rs}{}_{kk'}{}^{CONNECT}$ is a dichotomous variable that obtain 1 when path pair *kk*' of OD *rs* uses a connecting flight and obtain zero otherwise. Ω is a set of OD pair, and Ξ is a set of passenger's path. Π^{HUB} is a set of strategies that the AOC has.

Since the AOC controls the LHOs' behavior by determining the optimal hub-to-hub flight schedule as well as passengers' behavior, this problem is a class of bi-level programming problem. However, a problem classified as bi-level programming problem has a very serious complexity, what we should do is to reduce the complexity introducing some ideas of decomposition.

2.3 PROBLEM DECOMPOSITION AND ALGORITHM

As mentioned above, this problem has a serious complexity, so that we try to decompose this problem to more simple sub-problems. First, let us concern the LHOs; the LHOs control the flight schedules using their hubs, they allocate the passengers under their rescheduled flight diagram; thus, their procedure for optimization of rescheduled flight diagram is as follow: in the first step, they determine the rescheduled flight diagram, and in the second step, they check the feasibility of the allocation; in the third step, they allocate the passengers to the flights; in the final step, they search more desirable rescheduled diagram; therefore, their optimization problem can be expressed as Dynamic Programming with Linear Programming.

The sub-problem of each LHO is a dynamic problem with zero-one knapsack problem; they check the aircraft movement sequence, and determine the adjustment of the flight diagram. However, in case of optimal behavior of the single hub operator, Barnhart proposes the systematic solution algorithm based on linear programming, which is column generation (Barnhart et al, 1998). Let the solution of the optimization problem of each LHO be PROCEDURE 1. In the PROCEDURE 1, the reallocation of passengers affected by the irregular event is also examined. The passengers' reallocation problem is a simple linear programming problem, so that when this reallocation problem is involved to the legacy reschedule problem, the solution procedure can handle both passengers' and aircraft reallocation problems simultaneously with easy extension.

After the PROCEDURE 1, some conflicts may be found. In this model, conflict is defined as:

"The same aircraft is allocated to different rescheduled flight diagrams of hub-to-hub flight by operators."

The AOC should reduce these conflicts by determining the hub-to-hub flight; that is, the AOC evaluate which flight schedule is better for the reduction of connecting passengers' disutility. After one conflict adjustment, each LHO makes new rescheduled diagram again and another conflicts appear; then, the AOC's conflict adjustment is carried out again. Let this procedure be called PROCEDURE 2.

For optimization, the total algorithm consisting of PROCEDURE 1 and PROCEDURE 2 is:

[Algorithm: based on Column Generation Method (Barnhart et al, 1998)]
STEP 0: Generate the initial feasible solution.
STEP 1: [PROCEDURE 1]
STEP 1.1: Set the rescheduled flight diagram by linear programming with column generation. The passengers' reallocation is carried out in this STEP. Then, go to STEP 1.2.
STEP 1.2: If all rescheduled diagrams determined by each LHO are obtained, go to STEP 2, otherwise return to STEP 1.1.
STEP 2:
STEP 2.1: If all the feasible branches listed as conflicts are checked, computation is terminated; otherwise go to STEP 2.2.
STEP 2.2: Find the conflicts and select the conflict appearing in the earliest time-band. Then, go to STEP 2.3.
STEP 2.3: [PROCEDURE 2] Compare the effectiveness of the reduction of connecting passengers' disutility given by each operator's local optimal solution and determine the better flight schedule. The rest of evaluated flights are stored as "branches."
STEP 2.4: Go to STEP 1 and generate the next rescheduled diagrams.

The proposed algorithm has a very complex branch-and-bound scheme in STEP 2. If we need a very accurate solution, we should apply the very serious branch-and-bound; however, it is very time consuming. If we can accept the approximated solution, the branch-and-bound with some relaxation approaches can be adopted; for example, greedy method is one of the feasible approaches. Selection of the approach depends on the network size.

3. NUMERICAL COMPUTATION

In this chapter, we show how the model works and evaluate the efficiency and workability. In addition, the following computation deals with the simplest case; two hubs; the model can be applied to the general case; more than two hubs network with some extensions. Unfortunately, the developed model can be applied to the hub-spokes network with single assignment; one spoke connects to one hub; so, we cannot apply this model to the multi-assignment hub-spokes network.

3.1 Conditions

(a) Network

Figure 1 depicts the network shape of the numerical example. There are 2 hubs and each hub has 2 spoke airports. For the instance, we adopt the symmetry shape network. Airport 3 and 4 connect to the hub airport1 and airport 5 and 6 to the hub airport 2, respectively. The hub 1 can operate leg [1] to [6], [11] and [12]; the hub 2 can operate leg [1], [2], [7] to [10], [13] and [14]. Thus, the conflict appears on both leg [1] and [2].

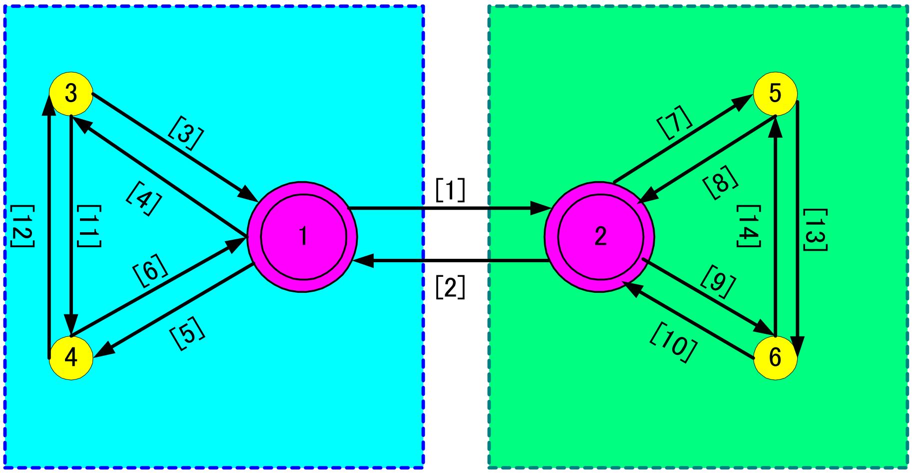

Figure 1 Network Shape of the Numerical Example

(b) Flight Diagram, Passengers Allocation and Aircraft Rotation

Since the proposed model is developed based on approximated time band, we set 20 time periods a day. The original flight diagram is shown in Figure 2. Aircraft 1 to 6's movements are drawn in blue, green, purple, red, pink and broken black lines, respectively.

The original aircraft rotation is given as Table 1. In Table 1, each number in column means the number of the airport; unsigned is "departure", negative is "arrival" and "s" means "parking"; "0" means that the aircraft is on flight. In this computation, it is assumed that each carrier has three aircrafts. Each aircraft starts operation at t=1 and ends its operation at t=16; after that period, each aircraft parks for the next first flight.

Table 1 Aircraft Rotation

t / *aircraft*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	s3	s3	3	−1	1	−4	4	−1	1	−3	3	−1	1	0	0	−2	s2	s2	s2	s2
2	s4	s4	4	−1	1	0	0	−2	2	−5	5	−2	2	0	0	−1	s1	s1	s1	s1
3	1	0	0	−2	2	−5	5	−2	2	0	0	−1	1	−4	s4	s4	s4	s4	s4	s4
4	2	0	0	−1	1	−3	3	−1	1	0	0	−2	2	−6	s6	s6	s6	s6	s6	s6
5	s5	s5	5	−2	2	−6	6	−2	2	−6	6	−2	2	−5	s5	s5	s5	s5	s5	s5
6	s6	s6	6	−2	2	0	0	−1	1	−4	4	−1	1	−3	s3	s3	s3	s3	s3	s3

The OD flow is shown in Table 2, and the OD flow per time-band is shown in Table 3; original seat occupancy on each flight is given as Table 4. In Table 3, the number shown in the top columns indicate the OD pair, for example 12 means Origin 1 to Destination 2. In the numerical example, we assume seat capacity is 375 seats per aircraft. The original flight schedule appearing in Table 4 is also described in Figure 2. Finally, we assume the cost per flight/parking is listed in Table 5. As for expressing the parking, we set link number 15 to 20 for expressing parking.

As for the monetary value of delay for passengers, we set D=100.

Table 2 OD Flow

r \ s	1	2	3	4	5	6
1		350	138	162	86	92
2	330		92	72	168	165
3	156	190		110	40	46
4	166	214	80		34	36
5	192	166	46	40		78
6	220	174	52	48	88	

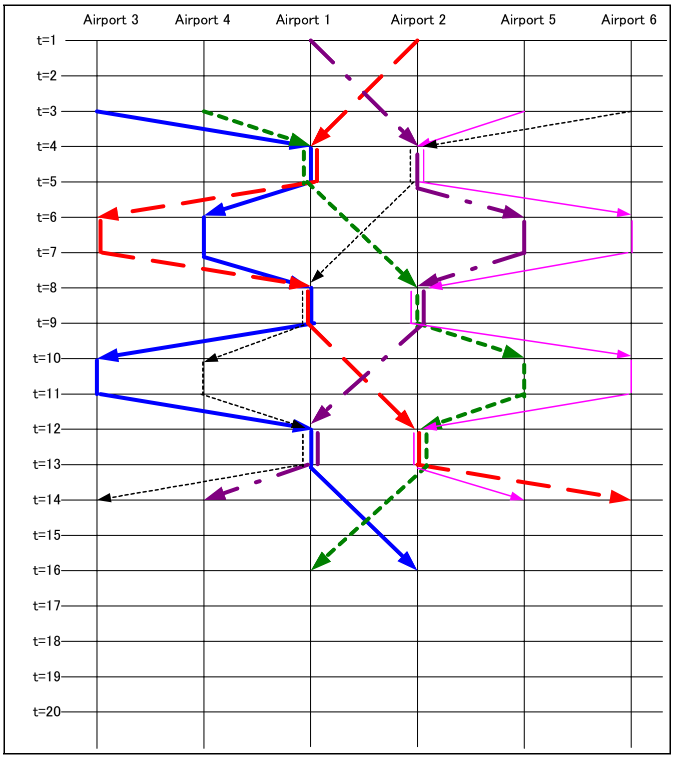

Figure 2 Original Flight Diagram

3.2 Computation

We examined several patterns of delay and cancellation, but due to the limitation of the space, we discuss the model performance under one typical scenario. The example event is:

[Example]
In time period t=5, the flight number 7, Aircraft number 2, drawn in green, from Airport 1 to 2, is delayed; its departure time is postponed to t=6 at least. The initial feasible schedule is given by postponing each schedule to the next time band, which is shown in Figure 3, affects to 3399 direct passengers and 1856 connecting passengers.

Each LHO starts his optimization from this initial condition. It is easy to understand that the conflict will happen at t=5 in Airport 1, which means Aircraft 1, 3 and 4 are the candidates of reliever flight of Aircraft 2 at t=5. Needless to say, the best choice for Airport 1's operator may not be the best choice for Airport2's operator. So, the adjustment by the AOC is required. The adjustment by the AOC gives the optimal rescheduled flight diagram; Figure 4 depicts the optimal schedule.

In Figure 4, Aircraft 1 (blue) is operated as a reliever flight from Airport 1 to Airport 2 between time band t=5 and t=8. Other flights are also adjusted; most of the flights are operated with "swapped" aircrafts. Aircraft 1 (blue) and 4 (red) are swapped each other at t=10, and Aircraft 2 (green) and 1 (blue) swap their flight each other at t=8.

These swapped flights comes from the AOC's adjustment on hub-to-hub schedule; in this example, flights at t=5 (operated by Aircraft 1 in the optimal schedule), at t= 10 (by Aircraft 4) and t=14 (by Aircraft 6) from Airport 1 to 2, flights at t=5, t=9 and t=13 from Airport 2 to 1 are adjusted by the AOC.

What is the big difference between Figure 3 and 4, the rescheduled diagram has longer "idling time" in some legs; at the Airport 1, between t=8 and t=10, Aircraft 3 (purple) and 4 (red) has two periods long idling time. These idling times are generated in order to adjust the connectivity at the hub airport 1 at period t=10. This result suggests that the efficient rescheduled diagram requires some redundancy in the network.

Let us confirm the efficiency of delay/cancellation recovery. These flight schedules are locally optimized for each LHO, so that these rescheduled diagrams assure the Pareto optimality between LHOs. Table 6 shows the comparison of results. It is obvious that both disutility of total passengers and connecting passengers are cut to more than 50points; the AOC's adjustment can reduce both disutility to half. Moreover, in this case, the additional flight does not occur, so that the total reduced cost is equal to the total amount of PAX's disutility. Needless to say, we may obtain the result that requires the additional flights for relive of the delay/cancellation in other cases; it depends on the situation. However, since the additional flights usually increase the operational cost, each LHO may refuse to use additional flights as much as possible.

For concluding this chapter, let us discuss the characteristics of the AOC's adjustment. As for the LHO, the Airport 1 should change the flight schedule more seriously than the Airport 2; the Airport 1 should give one period delay to all flights that use Airport 1, while the Airport 2 just swaps the aircraft rotation. This result suggests that each LHO independently determines his rescheduled diagram after the AOC's adjustment. Thus, the most import adjustment by the AOC is on the first flights using hubs just after irregular event; once serious adjustment by the AOC is enough. The same tendency is found in other cases, which have the same network shape.

Table 3 OD Flow per time band

t \ *rs*	11	12	13	14	15	16	21	22	23	24	25	26	31	32	33	34	35	36
1	0	94	0	0	42	56	92	0	38	46	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	60	62	0	34	22	20
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	82	52	66	20	18	78	0	24	14	48	54	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	46	66	0	36	18	26
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	90	38	44	24	18	82	0	30	12	62	63	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	50	62	0	40	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	84	48	52	0	0	78	0	0	0	58	48	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	350	138	162	86	92	330	0	92	72	168	165	156	190	0	110	40	46

t \ *rs*	41	42	43	44	45	46	51	52	53	54	55	56	61	62	63	64	65	66
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	66	70	28	0	18	16	62	58	22	20	0	30	72	62	32	26	34	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	48	74	16	0	16	20	64	60	24	20	0	18	72	52	20	22	30	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	52	70	36	0	0	0	66	48	0	0	0	30	76	60	0	0	24	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	166	214	80	0	34	36	192	166	46	40	0	78	220	174	52	48	88	0

Table 4 Seat Occupancy per Flight

fl. no	*dep.time*	*arr.time*	*line haul*	*dep.airport*	*arr.airport*	*pax*	*fl. no*	*dep.time*	*arr.time*	*line haul*	*dep.airport*	*arr.airport*	*pax*
1	1	4	3	1	2	192	17	9	12	3	1	2	352
2	1	4	3	2	1	176	18	9	12	3	2	1	348
3	3	4	1	3	1	198	19	9	10	1	1	3	132
4	3	4	1	4	1	198	20	9	10	1	1	4	140
5	3	4	1	5	2	192	21	9	10	1	2	5	152
6	3	4	1	6	2	226	22	9	10	1	2	6	135
7	5	8	3	1	2	328	23	11	12	1	3	1	152
8	5	8	3	2	1	350	24	11	12	1	4	1	158
9	5	6	1	1	3	118	25	11	12	1	5	2	144
10	5	6	1	1	4	146	26	11	12	1	6	2	160
11	5	6	1	2	5	124	27	13	16	3	1	2	216
12	5	6	1	2	6	140	28	13	16	3	2	1	220
13	7	8	1	3	1	192	29	13	14	1	1	3	158
14	7	8	1	4	1	174	30	13	14	1	1	4	148
15	7	8	1	5	2	186	31	13	14	1	2	5	140
16	7	8	1	6	2	196	32	13	14	1	2	6	142

Table 5 Operational Cost per Link

	link_no.	*dep.air poı*	*arr.airport*	*linehaul time*	*cost*
hub-to-hub or hub-to-spoke	1	1	2	3	35000
	2	2	1	3	35000
	3	1	3	1	25000
	4	3	1	1	25000
	5	1	4	1	25000
	6	4	1	1	25000
	7	2	5	1	20000
	8	5	2	1	20000
	9	2	6	1	20000
	10	6	2	1	20000
spoke-to-spo	11	3	4	1	10000
	12	4	3	1	10000
	13	5	6	1	10000
	14	6	5	1	10000
park at the arrival airport	15	1	1	1	1000
	16	2	2	1	1000
	17	3	3	1	1000
	18	4	4	1	1000
	19	5	5	1	1000
	20	6	6	1	1000

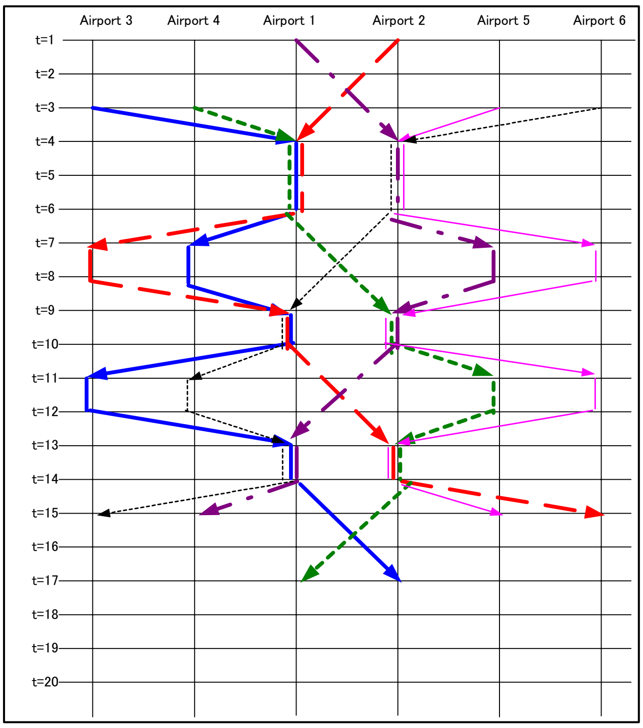

Figure 3 Initial Feasible Solution

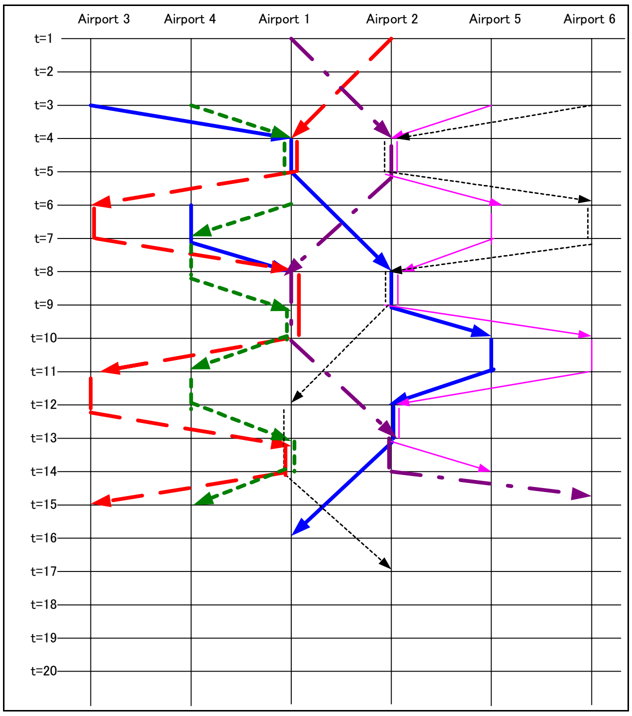

Figure 4 Optimal Schedule

Table 6 Cost Reduction by Rescheduled Diagram

Initial Condition		Optimized Rescheduled Diagram	
Total PAX	Connecting PAX	Total PAX	Connecting PAX
339900	185600	158400	77200

4. CONCLUING REMARKS

In the current paper, we propose the bi-level optimization model for reduction of the delay/cancellation affect in the hub-spokes shaped network system. The remarks that we obtain are as follows:

1) Mathematical formulation of rescheduling diagram problem under irregular event with hub-to-hub conflict is proposed. For reduction of the conflict, we adopt the following adjustment scheme; hub-to-hub conflict is adjusted by the AOC. As a result, the mathematical formulation belongs to a class of bi-level optimization with integer programming.
2) For the optimization, we modify the branch-and-bound with column generation approach for the proposed problem. Based on this approach, the solution algorithm for this bi-level programming problem is proposed.
3) The proposed system can reduce both of disutility of PAX and connecting PAX under the Pareto optimization between LHOs.
4) The AOC's serious adjustment is carried out once; the adjustment is on the first flights

using hubs just after irregular event.

These remarks show this approach works well for the delay reduction and hub-to-hub conflicts. However, since the proposed model is a prototype, we should overcome some difficulties for the actual use.

1) Column generation approach works well for the network design; however, the workability is verified under the "single assignment" hub-spokes system. It is concerned that the multi-assignment hub-spokes system is very tough to handle by the proposed approach.
2) As for the bi-level programming problem, even this linear-by-linear type bi-level programming is very complicated to handle; if the lower level problem has some nonlinear constraints, the problem is very difficult to solve.
3) The proposed model does not deal with uncertainty. The actual rescheduling problem does involve risks, more delays/cancellations after irregular events for example.

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