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SHORT-TERM FLOW MANAGEMENT BASED ON DYNAMIC FLOW PROGRAMMING NETWORK

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Abstract: It is a practical exigent problem for short-term flow management when air traffic congestion happens in a control area, and it is also an important subject of academic researches. By far, the good research is to set up dynamic programming model of flow management aiming to obtain optimal benefits. Then a dynamic programming management model is put forward in this paper to meet the practical condition of control, with a farther research having been done. In order to obtain the optimal solution of the model, a fast method is presented to lessen calculation by the setting of a threshold according to the operational demand of practical control. The model can obtain the optimal solution quickly and is fit for real-time dynamic management. The correctness and validity of the model and method is verified by simulation results.

Key Words: air traffic control, dynamic flow management, network flow

1. INTRODUCTION

Since Odoni (1987) first put forward the problem of real-time allocation of flight to minimize congestion expenses, a lot of research results have been obtained about the air traffic flow management. Some researches had been done in the single-airport ground-holding strategy problem (Terrab M. and Odoni, 1993) , in multi-airport ground-holding strategy problem (Vranas P. et al. 1994), and then the ground-holding strategy was extended to the whole airport network. Some researches had also been done in the problem of optimal slot allocation for European air traffic management (Vranas P. 1997). What is more, a farther research had been done about the problem of optimal speed adjustment of flight and the rerouting problem to avoid bad weather conditions in the airport network (Stock-Patterson S. 1997).

In order to solve the problem that is how to manage network flow in a certain control area, the dynamic flow network model of short-term flow management in control area was put forward (Cheng peng, 2000). The model established dynamic programming model according to the principle of optimization benefits. Subsequently, the air traffic network flow management model, which concretely described the optimization problem of single flight, is presented (Cheng peng, 2001). Both models are for closed system. Besides, the amount of work to solve models was very heavy, so it will take much time to obtain the solution.

In this paper, the dynamic flow management model for an open system is given, which is more practical. Then the method solving the model is put forward by simplifying models and by reducing feasible solution, and with this method the optimal solution can be obtained quickly.

2. DYNAMIC FLOW MANAGEMENT MODEL

We construct air traffic flow management programming model according to the principle of optimal benefits. That is to say, in near future, when the capacity of control units has changed, the flight flow will be adjusted to minimize the loss caused by flow control (or minimum total waiting cost) (Cheng peng, 2000).

For this reason, consisted of routes of different heading and different flight levels in control area, the network is regarded as a tri-dimensional directed network $G(I, A)$, where I is a set of vertexes, and A is a set of directed arc. In the network, the airport, the transferring points, navaids are regarded as vertexes of the network; and routes, approach corridors are regarded as directed arcs. The flight flows along the directed arc between vertexes orderly, and is subjected to capacity. So the flow management of control area is a tri-dimensional dynamic flow network.

In the network, the airport and the transferring points chained with one-way route is served as a model of a pair of source node and sink node, that is to say, they are regarded as different source nodes and sink nodes. The navaids connected with bi-directional route is a single source node or a single sink node. Navaids are the connection point between the route segments, and they are served as a model of an undirected node connecting arcs neither a source node nor a sink node). Each route segment or corridor is served as a model of directed arc, and is expressed by the ordered pair of network nodes. The flight holding in air is looked as flowing on the self-loop of a undirected node, let us suppose that the transmission time of the self-loop equals unit time, and that flow equals maximal numbers of admitted holding flights; ground holding is served as a source node added a self-loop of the model, and the rate of flow is large enough.

In the control area, the flight path (or flight plan) of a flight is expressed as the set of directed arcs to connect a source node and a sink node. That is, it is a directed chain of network $G(I, A)$ and is also called the main path (main chain) of the flight. Usually, several alternative airports are prepared for the flight, so there are several feasible directed chains (the paths of flight). We can define the paths of flight as the sets of all feasible. The feasible chains are the alternative flight plan, except main chains.

According to the difference of flight plans, we can define the category of flow: the flows that have the same feasible chain belong to the same category. In this way the air traffic flows are multi-category dynamic network flows. In $G(I, A)$, the category of flows is expressed by k, $k \in \{1, 2, \ldots, K\}$, where K is the total number of category.

For the modeling, the hypothesis is as follows. 1) The minimal unit of air traffic flow is single flight. 2) All the arcs of air traffic network are directional and bi-directional routes are defined as directed arcs, which have the same nodes and opposite directions. 3) The flow transmission time of each arc is constant. The transmission of self-loop is unit time. 4) The capacity of arc (i, j) is $c_{i,j}(t)$, $t \in [0, T]$, T is the length of management time. 5) Each control area is divided into several sub-areas (for example, ATC sector), and the total flight flow has an up-limit, that is, the maximal workload of sector. The sub-area is expressed as $s \in \{1, 2, \ldots, S\}$, and the up-limit of workload in control sector s is expressed as W_s.

To express the dynamic model, the time interval [0,T] is subdivided into N+1 time $0=t_0, t_1, \ldots, t_N=T$ and the capacity of the arc (i, j) is noted as $c_{i,j}(t_n)$ $(n=0, 1, \ldots, N)$, which expresses the admitted entry flow up-limit of the arc (i, j) between time t_{n-1} and t_n.

Many flights outside the control area will fly into the control area, and flights inside the control area will fly out. So an entry transferring point along each route in the control area is regarded as a source node, where aircraft flying from the outside can be regarded as setting out and enters the control area. The transferring where the aircraft flies out of the Control area is regarded as sink node, and the flight flying out of the Control area can be regarded as the corresponding sink node. So, some of source nodes and sink nodes are considered when we solve the flow management problem of the network vertexes.

Let I_{dep} denote the sets of take-off airports, I_{arr} denote the sets of landing-airports, I_{tpi} denote the sets of points of handling over which enter flow, I_{tpo} denote the sets of points of handling over which go out of flow. The sets of the source nodes is $I_{stc}= I_{dep} \cup (I_{tpi} \subset I)$, and the sets of the sink nodes is $I_{\sin k}= I_{arr} \cup I_{tpo} \subset I$, $I_{stc} \cap I_{\sin k}=\phi$), and the sets of undirected nodes is $I_{nod}=I \backslash \{I_{stc} \cup I_{\sin k}\}$.

Let a_{ij} denote flight time during arc segment (i, j), r denote the oil cost coefficient of unit time, c_i^g denote the cost coefficient of ground holding of airport i.

$P_i^k(t)$ denote flow of k category traffic flow plan to be sent out from source node i at the moment t, $H_k(i, j)$ denote the set of Feasible air route segment of the k category traffic flow, $B(k)$ denote the set of alternative airport of the k category traffic flow.

Now, consider the case: some flights are flying at the arc and they need to evacuate in control area at initial moment 0. Let M be total number of flight, for every flight α_m ($m=1, \ldots, M$), the position of its can be expressed with order group ($\alpha_m, i, j, t_{\alpha_m}$), which means when flight α_m is

located at air route segment (i, j), whose flight time to j is t_{α_m}. Let K' is the total number of category of existing flights, k' ($\in\{1, \ldots, K'\}$) is the new category of traffic flow, of which the source node is j. This means send a unit of the $k' \in K'$ category traffic flow by source node j after the moment t_{α_m}. Construct the set:

$$L = \left\{ \left(j, t_{\alpha_m}, k' \right) \middle| j \in I_{nod}, t_{\alpha_m} \le \max\left\{ a_{i,j} \right\}, k' \in K' \right\}.$$

Let $H_{k'}(i, j)=\{(k', i, j)\}$ to be the set of flight path for k' category traffic flow, where (i, j) denote the route segments of feasible path for the k' category traffic flow.

While the network operates, some flights have the priority of taking off and landing. These can't be considered with benefit principle. This problem can be solved like this. Because the routes of these flights are specific, flight flow can be regarded as known. Suppose that there have U flights which have no priority, the flow of air route segment (i, j) at the moment t is $u_{ij}(t)$. There have V flights with priority that need to be evacuated, and the flow of air route segment (i, j) at the moment t is $v_{ij}(t)$.

After the preparation above, the air traffic management system can be regarded as an open system, then the model of air traffic management can be described with the following planning problem.

Define the traffic flow variable $f_{i,j}^{k}(t)$ as the actual air traffic entry amount of the category k on arc segment (i, j) during the moment t to (t +1), the evacuating variable $g_{i,j}^{k'}(t)$ as the actual air traffic entry amount of the category k' on route segment during the moment t to (t +1).

The objective function of programming model is divided into two parts: the first part is the expenses of the no-evacuating flow, another part is the expenses of evacuating network flow.

The expenses that the plane does not utilize the alternative airports are

$$z_1 = \sum_{t\in\{t_0,\cdots,t_N\};(k,i,j)\in H_k(i,j),i\notin B(k)} ra_{ij} f_{i,j}^{k}(t) + \sum_{t\in\{t_0,\cdots,t_N\};i\in I_{dep}\cup I_{tpi}} C_i(t) d_i(t) \tag{1}$$

Where, the first item is the expenses of fuel of all flight plans. The second is the expenses of both ground holding and air holding in control area. When $i \in I_{dep}$, $C_i(t) = c_i^g$, where $C_i(t)$ is the holding expenses of unit when delayed. And the flow of detention in the source node i is

$$d_i(t) = \sum_{0\le\tau\le T} \sum_{k} \left[p_i^k(\tau) - \sum_{(i,j)\in A} f_{i,j}^{k}(\tau) \right], \quad i \in I_{src}.$$

The expenses that the flight utilizes the alternative airport are

$$z_2 = \sum_{t\in\{t_0,\cdots,t_N\};\, i\in B(k),(k,i,j)\in H_k(i,j)} M_b f_{i,j}^k(t),$$

where M_b is the cost coefficient of the flight landing in the alternate airport for one time.

The second part is the flow expenses for evacuating flow:

$$z_3 = \sum_{\substack{t\in\{t_0,\cdots,t_N\};(i,t,k')\in L,\\ (k',i,j)\in H_{k'}(i,j)}} ra_{ij} g_{i,j}^{k'}(t) + \sum_{\substack{t\in\{t_0,\cdots,t_N\};i\in B(k'),\\ (k',i,j)\in H_{k'}(i,j)}} M_b g_{j,i}^{k'}(t) \tag{2}$$

So the objective function is:

$$Min\,(z_1 + z_2 + z_3)$$

The restraint conditions of the programming problem are as follows:

$$\sum_{(i,j)\in A,\, j\neq i} f_{i,j}^k(t) + f_{i,i}^k(t) - f_{i,i}^k(t-1) - P_i^k(t) = 0 \tag{3}$$

(Conservation equation of the dynamic flow of source node)

$$\sum_{t,k,i\in I_{\sin k}} \sum_{j;(j,i)\in A} -f_{j,i}^k(t) + \sum_{t,k',i\in I_{\sin k}} \sum_{j;(j,i)\in A} -g_{j,i}^{k'}(t) + \sum_{t,k} \sum_{i\in I_{\sin k}} \sum_{j(j,i)\in A} P_i^k(t) + N + U + V = 0 \tag{4}$$

(Conservation equation of sink node)

$$\sum_{(i,j)\in A} f_{i,j}^k(t) - \sum_{(i,j)\in A} f_{j,i}^k(t - a_{ji}) = 0 \tag{5}$$

(Conservation equation of the dynamic flow of medial node)

$$\sum_k f_{j,i}^k(t) + \sum_{k'} g_{j,i}^{k'}(t) \le c_{ij}(t) - u_{ij} - v_{ij}, \quad (i,j)\in A, \forall t \tag{6}$$

(Capacity restrained of air route segment)

$$\sum_k \sum_{i,j;(i,j)\in A_s} \sum_{\tau;t-a_{ij}<\tau\le t} f_{j,i}^k(\tau) + \sum_{k'} \sum_{i,j;(i,j)\in A_s} \sum_{\tau;t-a_{ij}<\tau\le t} g_{j,i}^{k'}(\tau) + \sum_{i,j;(i,j)\in A_s} \sum_{\tau;t-a_{ij}<\tau\le t} u_{i,j} \le W_s, \quad \forall s,t \tag{7}$$

(Controller load restrained)

$$\sum_{j;(i,j)\in A_{k'},(i,t,k')\in L} g_{i,j}^{k'}(t) = Q_i^{k'}(t), \tag{8}$$

(The conservation equation of dynamic flow for source node of evacuating flow, where $Q_i^{k'}(t)$ denotes Evacuating value of category k' which is sent out from the undirected node at the moment t.)

$$\sum_{j;(k',i,j)\in A_{k'}} g_{i,j}^{k'}(t) - \sum_{j;(k',j,i)\in A_{k'}} g_{i,j}^{k'}(t - a_{ji}) = 0, \quad i\in I_{nod}, \forall t, k' \tag{9}$$

(The conservation equation of medial node of evacuating flow)

So, the model can be described as:

$$Min\ (z_1 + z_2 + z_3)$$

$$s.t. \quad (3),(4),(5),(6),(7),(8),(9), \text{ and}$$

$$f_{j,i}^{k}(t), g_{j,i}^{k'}(t) \geq 0,\ f_{j,i}^{k}(t), g_{j,i}^{k'}(t) \in Z \tag{10}$$

3. THE METHOD OF SOLVING THE MODEL

First, according to the conditions given, the feasible solutions of the model constructed in this paper are finite, so the model must have solution.

Secondly, the calculating amount to solve programming model is very huge. For example, there are 50 flights in control area, and the category number of flight is 10, control sub-sectors count up to 5, 1-minute time interval within 3 hours. Then the variable of the model reaches more than 30000, and the restrained conditions are much more. So it takes much more time to solve and validate the problem using method of exhaustions.

It takes more than 300 seconds to solve the model described by Cheng peng (2000, 2001) , if iteration method is adopted to solve the programming model.. The principle solving is adopting revised simplex method as linear programming (LP), and branch-bound method as mixed linear programming (MLP).

We propose a new approach for solving the problem:

1) First step, we simplify the model, remove the volume with priority from the objective function and restrained conditions. Namely, let $f_{i,j}^{k}(t) - u_{i,j}^{k}(t)$, $g_{i,j}^{k'}(t) - v_{i,j}^{k'}(t)$ be the volume variable, and substitute the model to replace the corresponding items.

2) It is possible that the expenses of main path are more than alternative path, but we still choose the main path if the constraint condition is met. So, in the second step, we judge if the flow management main path is the feasible path chain. The method is that the main path can meet the restraint conditions.

3) Define the distance of Alternative plan to main plans of k category flow as

$$d_k = \sum \left[\left|(i, j)\right| - \left|(i, j)\right|' \right].$$

Then select d_k as a threshold value, and only consider the feasible chain in certain threshold value range. The method can reduce the number of feasible chain with an appropriate threshold.

4. EXAMPLES OF THE MODEL AND METHOD IN SOLVING PROBLEMS

The control area selected by Cheng peng (2000) is showed in figure 1, and the flow network is described as follows:

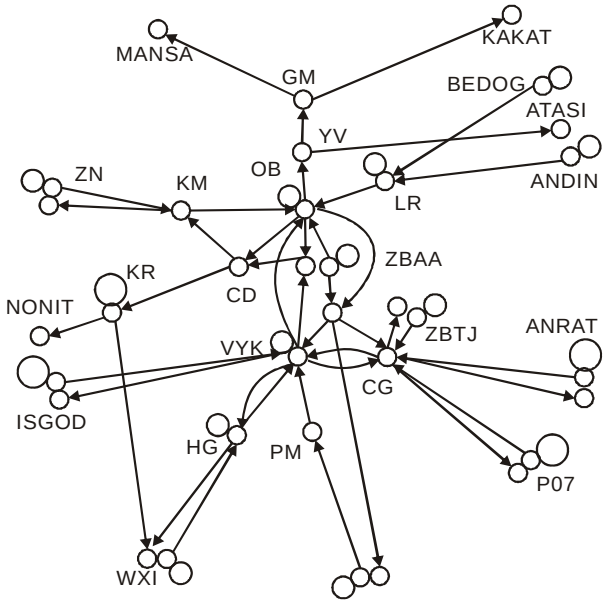

Figure 1. The network of the control area

In network, I_{stc}={WXI, EPGAM, P07, ANRAT, ANDIN, BEDOG, ZN, ISGOD}, *I*sin k= {ZBAA, ZBTJ}, and others are in I_{nod}, in which there are 6 holding points in air, the capacity is as follows：11, 21, 19, 14, 4, 4. The category number of flow is 8. The control area is divided into 5 control sector, and the maximal workload is 6, 4, 4, 4, and 4. Then set Mp=1000, Mb=100, $C_i(t)$=10, r=1. Select T=10800s , and sampling interval is 60s.

Because of the shortage of actual data, the values of parameters in model are similar to the data in [3]. If we select only 1or 2 feasible chains, the time of solving model is within 100s except a few cases, and the maximal solution time reach 182s. Some calculation results of solving time are given in table 1.

Table 1 :solving time comparison between the normal method and proposed method

order	solving time with general method	solving time with proposed method
1	340s	182s
2	158s	98s
3	120s	89s
4	90s	80s

From the results of simulation, the model proposed meets the area control requirements, and the method of solving model can reduce the solution time to meet dynamic flow management.

5. CONCLUSIONS

Within the control area, short-term flow management is the effective method to solve the bottleneck problem at the time of the traffic jam in the sky. By setting up dynamic programming model with optimized benefit principle, the air traffic flow problem may be settled preferably.

This paper provides open dynamic programming management model that can be used more frequently in worse conditions. In the dynamic flow management model, we consider the problems of flights that enter and fly off the control area and priority problem, besides the flights flow in control area.

Then, we provide method of fast solving model in order to be suitable for real-time dynamic management.

Through calculating the actual examples, the model proposed in this paper is suitable for the actual conditions, and the proposed algorithm can be applied correctly and effectively to real-time flow management.

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