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A STUDY ON EMERGENCY EVACUATION AND RESCUE NETWORK RECONSTRCUCTION FOR NATURAL DISASTERS WITH MULTI-CLASS USERS TRAVEL BEHAVIOR CONSTRAINTS

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Abstract: In this research, we formulate the emergency evacuation and rescue network reconstruction problem as a bi-level programming network design model. The upper level problem is a decision of minimized system cost under limited resources. The lower level problem is one of the constraints for the upper level problem that considers users equilibrium route choice behaviors. We also consider the multi-class users' route choice behaviors in the lower level problem. In solving the problem, the variational inequality sensitivity analysis method, generalized inverse matrix method, diagonalization method, and gradient projection method are adopted to develop the solution algorithms for the bi-level model. The numerical test results indicated that the Stackelberg equilibrium solution of the emergency evacuation and rescue network reconstruction for natural disasters bi-level programming model do exist. More significantly, the optimal network reconstruction planning, disaster victim emergency evacuation and rescue route planning under multi-class user route choice conditions can be achieved.

Key Words: network design problem, bi-level programming model, multi-class users

1. INTRODUCTION

When the natural disasters such as typhoon or earthquake occur, often causes the transportation network to suffer the large-scale destruction. Formerly several typhoon and earthquake experiences, the destroyed network system is possible to affect the disaster area rescue and the emergency evacuation work execution. Therefore, when the natural disasters occur, how to rapidly restore the disaster area network system's basic capability is an important research issue. Specially, the disaster relief work should consider the "gold 72 hours" principle. The main objective of the present research is to provide a systematic framework in analyzing the network reconstruction plan after the network suffering the natural disasters destruction. For practical application concerns, the multi-class users travel choice constraints are considered in our network reconstruction model.

The emergency evacuation and rescue network reconstruction for natural disasters problem is a network design problem. We can formulate it as a bi-level programming model. The upper level problem is the decision of a minimized system cost under limited resources. The lower level problem is one of constraints for upper level problem that considers the multi-class users equilibrium route choice behaviors. The multi-class users equilibrium route choice problem is an asymmetric traffic assignment model. It is more complicated and difficult to solve in comparison with traditional standard network equilibrium problems. The asymmetric traffic assignment problem has not been formulated as any optimal programming forms, but we can establish it into a model as a variational inequality problem. Under the multi-class users route choice constraints, the bi-level problem regarding emergency evacuation and rescue networks reconstruction for natural disaster is difficult to solve. Because the solution set of this problem is non-convexity. The variational inequality sensitivity analysis method and the generalized inverse matrix are considered for solving the bi-level programming formulation. The gradient projection method and diagonalization method are also important components included in this research. Based on several numerical examples, the Stackelberg equilibrium solution of the emergency evacuation and rescue network reconstruction for natural disasters Bi-level programming model can be obtained. In the future, we expect the results of this research can be used in support of emergency evacuation and rescue network reconstruction decisions when large-scale network is destroyed due to natural disasters.

2. PROBLEM DESCRIPTION AND LITERATURE REVIEW

Transportation network reconstruction in many natural disaster issues is a main research topic. No matter what emergency evacuation and rescue strategies are implemented, the role of transportation network reconstruction is the kernel of natural disaster issues. How to maintain the emergency evacuation and rescue network basic capability, or reconstruct the destroyed transportation network systematically after natural disaster is significantly affecting the efficiency of emergency evacuation and rescue. In the emergency evacuation and rescue network reconstruction problem. For practical application considerations, the multi-class users conditions should be incorporated into the model. That is, we would develop an emergency evacuation and rescue network reconstruction model, which consider the multi-class users impact in the network is more realistic in modeling the proposed problem.

In the recent research, many studies of the literature usually focus on natural disaster emergency evacuation and rescue route planning, such as Lovas (1997) emphasized the importance of emergency evacuations wayfinding. Pidd *et al.* (1997) provided a spatial decision support system, combined the geographic information system and simulation model to aid the emergency evacuation planning. Tufekci (1995) developed a decision support system for hurricane emergency management. This software used simulation as well as several network optimization models in estimating the evacuation time and the traffic flow on given transportation network. But the natural disaster emergency evacuation and rescue network reconstruction problem is in reality the network design problem fields. Marcotte (1986) used bi-level programming model to formulate the network design problem. In actuality, the bi-level programming model is a non-cooperative Stackelberg game. He (2003) fomulated a bi-level natural disaster emergency evacuation and rescue network reconstruction model. He also developed an algorithm to solving the Stackelberg solution. But the model does not include the multi-class users route choice constraints. The present research is also a following research of He’s (2003).

In solution methods investigation, Tobin Friesz (1988) and Tobin (1986) applied the variational inequality sensitivity analysis method to solve network equilibrium problem. This method is also an efficient solution method for bi-level programming model. But this method must be assumed that the strict complementary slackness conditions are satisfied and that the extreme point is nondegenerate, it is shown that if the perturbed problem is restricted to only those routes that are positive in the extreme point, it can still solve the original perturbed problem for small perturbations. In practice, many of network equilibrium problems could not be satisfied the strict condition of nondegenerate. For the technical problem, Cho (1991) provided the generalized inverse approach to advance the variational inequality sensitivity analysis method. The generalized inverse method has no nondegenerate assumption and have been adopted in solving network design problem by Wang (1999).

The generalized inverse method used the network equilibrium solution characters that path flow solutions are not unique, link flow solution is unique, and path flow solutions can be through the path/link incidence matrix and transfer to link flow solution to avoid the solution nonunique problem. For more efficient finding the relationship between path flow solution and link flow solution. The path-based solution algorithms such as gradient projection method (Jayakrishnan, 1994) is more useful than link-based solution algorithms (such as Frank-Wolfe method).

The multi-class users route choice behaviors are also considered in our emergency evacuation and rescue network reconstruction model. It is an asymmetric traffic assignment problem. Dafermos(1980) developed a more general traffic assignment model. The model has been designed in order to handle situations where there is interaction between traffic on different links or between different user classes of transportation in the same link. And she established the variational inequality (VI) model to formulate the problems. Chen et al. (1995) developed new solution algorithms for asymmetric traffic assignment model and compared the performance of those solution algorithms which are diagonalization method, streamlined diagonalization method, hybrid method and streamlined hybrid method. But these solution algorithms are all link-based solution methods.

In view of the characteristics of the problem described above, we could formulate the emergency evacuation and rescue network reconstruction problem as a bi-level programming model. The upper level problem is a network reconstruction system optimal problem. Under the upper level system objective, we can assure that the network reconstruction planning would minimize the emergency evacuation and rescue travel time. The lower level problem is a multi-class user equilibrium route choice problem. It is also one of the constraints of upper level problem. Due to the change of the upper level problem (network reconstruction planning), it would derive the lower level problem (multi-class users evacuation behaviors) to change accordingly. The upper level problem can be treated as the leader player of the Stackelberg game, and the follower player is the lower level problem. In the solution method development, in order to transfer the path flow solution into link flow solution. The gradient projection method may be a good choice. In the next section, we will formulate the emergency evacuation and rescue network reconstruction bi-level programming model and demonstrate the corresponding solution algorithms.

3. MODEL FORMULATION

The emergency evacuation and rescue network reconstruction problem is to find the optimal transportation network recovery strategies after natural disaster. The so-called “network reconstruction” is recovery road capacities that is destroyed by natural disaster. The disaster victim emergency evacuation and rescue route choice behaviors would be impacted by different network reconstruction plans. On the other hand, the system performance of emergency evacuation and rescue would be impacted by disaster victim emergency evacuation and rescue route choice behaviors. Therefore, if we want to obtain the optimal transportation network recovery strategy, the disaster victim emergency evacuation and rescue route choice behaviors constraints should be considered. For practice consider, the factor of multi-class users route choice behavior should be added in the model, too. It would describe the emergency evacuation and rescue network reconstruction problem more reasonable. Therefore, we formulate the emergency evacuation and rescue network reconstruction bi-level model as follows:

$$\min \sum_{a}\sum_{i} f_{a_i} c_{a_i}\left(f_{a_i}, y_a\right), \quad \forall a, i \tag{1}$$

subject to

$$0 \le y_a \le 1, \quad \forall a \in m \tag{2}$$

$$\sum_{a} \Delta y_a CAP_a \le \Gamma, \quad \forall a \in m \tag{3}$$

$$\sum_{p\in r,s}\prod_{a\in p}\left(\mathrm{I}_a \bar{\delta}_{ap}^{rs}\right) \ge 1, \quad \forall p \in (r,s); \forall a \in p \tag{4}$$

$$\mathrm{I}_a = \begin{cases} 0, \text{if } y_a = 0 \\ 1, \text{if } y_a \ge 0 \end{cases} \tag{5}$$

User equilibrium constraint:

$$\mathbf{c}(\mathbf{f}^*, \mathbf{y})(\mathbf{f} - \mathbf{f}^*) \ge 0, \quad \forall \mathbf{f} \in \Omega_{\mathbf{y}} \tag{6}$$

The above model, (1) is the objective of upper level model that represents the requirement of the reconstruction network total system cost minimization. Where $c_{a_i}\left(f_{a_i}, y_a\right)$ is the travel cost function of link *a* in equation (1). The decision variable y_a is the ratio of link reserve capacities after destroying and original link capacities. The decision variable f_{a_i} represents the *ith* mode flows of link *a*. f_{a_i} is also the decision variable of the lower-level multi-class users network equilibrium model. As we know, different network structure or link capacities will derive different user's evacuation route choice behavior in a network. When the vector of upper-level decision variables $\mathbf{y}$ is changed, the vector of lower-level decision variables $\mathbf{f}$ will follow to change. We cannot exactly express this relationship as a complete functional form between $\mathbf{y}$ and $\mathbf{f}$. Therefore, the relationships of the vector of upper-level decision variables $\mathbf{y}$ and lower-level decision variables $\mathbf{f}$ are implicit function and it is expressed as $\mathbf{f}(\mathbf{y})$. Equation (2) defines the domain of decision variable y_a should between 0 and 1. That is, the maximum road recovery capacities only to the road original capacity. The *m* is the set of destroyed roads. Equation (3) represents the total capacities repair quantities must less or equal then system provided maximum repair quantities. Where the Γ is the maximum repair quantities that system can provide and it is a well-known constant. The Δy_a is the capacity repair quantities in link *a*. Equation (4) represents that there is at least one path between each O/D pair (r,s). The I_a is a indicator variable in equation (5), when $y_a = 0$ then $\mathrm{I}_a = 0$, represent road a has been destroyed and can't pass any flow. When $0 < y_a \le 1$ then $\mathrm{I}_a = 1$, represent road a might be destroyed but still can pass emergency evacuation and rescue traffic flow. The Equation (6) is a VI model which represents multi-class users network equilibrium route choice model. And it is the lower level model. The symbol Ω_y denotes the feasible region of the lower level model which associated constraints include flow conservation, nonnegativity and variable definition, as follows:

Flow conservation constraint:

$$\sum_{p_i} h_{p_i}^{rs} = \bar{q}_i^{rs}, \quad \forall p, r, s, i \tag{7}$$

Nonnegativity constraint:

$$h_{p_i}^{rs} \geq 0, \quad \forall p, r, s, i \tag{8}$$

Definitional constraints:

$$f_{a_i} = \sum_{rs} \sum_{p} h_{p_i}^{rs} \bar{\delta}_{a_i p_i}^{rs}, \quad \forall a, p, r, s, i \tag{9}$$

$$\bar{\delta}_{a_i p_i}^{rs} = \{0,1\} \tag{10}$$

4. SOLUTION ALGORITHMS

4.1 Variational Inequality Sensitivity Analysis

Analysis the above emergency evacuation and rescue network reconstruction bi-level programming model with multi-class users route choice behavior constraints. The feasible region delineated by expressions (2)~(10) is essentially nonconvex, because expressions (6) and the implicit function $\mathbf{f}(\mathbf{y})$ are nonlinear. (Where $\mathbf{y}$ is the vector of decision variables of upper-level problem and it is expressed the ratio of link reserve capacities after destroying and original link capacities. The $\mathbf{f}$ is the vector of lower-level problem's decision variables and represents the flows of link a.) Nonconvexity portends existence of local optima; hence, the global optimum is difficult to find, even with the most computationally efficient procedures. In solving this bi-level programming model, one can employ Variational Inequality Sensitivity Analysis method and generalized inverse approach.

Using the variational inequality sensitivity analysis, the corresponding implicit differentiation can be obtained. And the descent search direction of upper level objective function also can be calculated. Owing to multi-class users route choice behavior constraints will be considered in the bi-level model, the aforementioned diagonalization method is then applied with modifications for calculating the derivatives by the variational inequality sensitivity analysis. The diagonalization method is the most common to solve multi-class users route choice network equilibrium problem. The main idea of this method is to decompose the effects of different mode flows on a specified link travel time into two parts: the main effect and cross effect. The method through an iteration procedure, fixed the cross effects at the current level and solving the main effect. The above procedure is repeated for the updated problem solution until a convergence criterion is met. Where the method of solving the main effect, we can adopt gradient projection method.

Since many of network equilibrium problems may be not satisfied the strictly condition of nondegenerate. We adopt an auxiliary method that is called generalized inverse approach. This method can avoid the nondegenerate assumption and solving above bi-level programming problem. The relation theorems demonstration we can refer to Cho (1991), Tobin Friesz (1988) and Tobin (1986). In here, we provide a numerical test about multi-class users network equilibrium problem to explain that we can take the exactly descent search direction and solution from the methods of diagonalization method, gradient projection method, variational inequality sensitivity analysis and generalized inverse approach.

A simple network shown in Figure 1 is used for testing. The test network consists of 12 links and 69 nodes, in which nodes 1,2,5,6 represent both origins and destinations, and nodes 3, 4 are intermediates. The O/D demands are assumed in Table 4. The link travel time function is assumed as (11), (12). In order to express the asymmetric effect between different classes of users, we assume two different parameters 0.5 and 0.2 in equation (11) and (12). Moreover, it is also to emphasize the different effect of multi-class users emergency evacuation and rescue route choice behavior. Where the link free travel time in here assumed as 1. We further assume each link in this network that initial ratio of capacities to saturation flows is 0.5 and the saturation flow is 50 (Table 2). When the results converge to the multi-class users network equilibrium, the route flows and travel times are shown in Table 3. The multi-class users network equilibrium solution in Table 3 can be obtain by the diagonalization method and gradient projection method. In the results we can find that when multi-class users network equilibrium, given a specific O/D pair and mode, the travel costs of the paths being used are less than or equal to those of the paths are not used.

Next, we based on the multi-class users network equilibrium solution, put a small perturbation on the ratio of capacities to saturation flows. And compare the actual solution with the estimated solution that is calculated by variational inequality sensitivity analysis and generalized inverse approach. The results are summarized in Table 4. In results of Table 4 we can find that, when we put a small perturbation to capacity rate, the estimated solution is all most as same as actual solution. But it also can be found that if the perturbation was too big, the different between estimated solution and actual solution would be increased. After the numerical test, we demonstrate that the implicit differentiation will be calculated by variational inequality sensitivity analysis and generalized inverse approach. And the exactly descent search direction could be found. Therefore, after optimal step sizes designed, the Stackelberg solution of bi-level programming model would be solved.

Table 1. O/D Demands

O/D Pair	1-6	2-5
Vehicle Type A	80	70
Vehicle Type B	70	60

$$c_{aA}(\mathbf{f}) = c_{a_0}\left[1 + 0.15\left(\frac{f_{aA} + 0.2 f_{aB}}{CAP_a}\right)^4\right], \forall a \tag{11}$$

$$c_{aB}(\mathbf{f}) = c_{a_0}\left[1 + 0.15\left(\frac{0.5 f_{aA} + f_{aB}}{CAP_a}\right)^4\right], \forall a \tag{12}$$

Table 2. Parameters Setting for Each Link

Initial y_a	Saturation Flow Rate
0.5	50

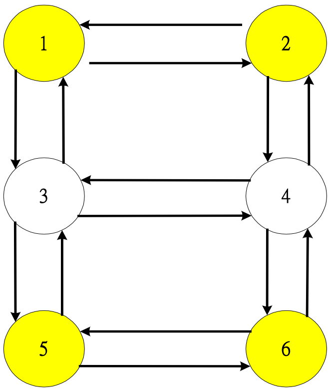

Figure 1. Test Network 1

Table 3. Path Flows and Travel Times

Vehicle Type		A		B	
Path No.	Route	Flow	Route Travel Time	Flow	Route Travel Time
1	1→2→4→6	27.41	7.43	23.94	6.02
2	1→3→5→6	27.32		24.07	
3	1→3→4→6	25.27		22.00	
4	2→1→3→5	22.45	7.40	18.98	6.01
5	2→4→3→5	25.18		21.91	
6	2→4→6→5	22.36		19.11	

Table 4. Comparison between Actual Solutions and Estimated Solutions

Link	$\varepsilon = 0$		$\varepsilon = 0.05$				$\varepsilon = 0.1$			
			Actual Sol.		Estimated Sol.		Actual Sol.		Estimated Sol	
	Vehicle A	Vehicle B	Vehicle A	Vehicle B	Vehicle A	Vehicle B	Vehicle A	Vehicle B	Vehicle A	Vehicle B
1→2	27.41	23.94	27.43	23.90	27.43	23.90	27.46	23.86	27.45	23.87
1→3	75.05	65.04	75.05	65.04	75.05	65.04	75.05	65.04	75.05	65.04
2→1	22.45	18.98	22.48	18.95	22.48	18.95	22.51	18.90	22.51	18.91
2→4	74.95	64.96	74.95	64.96	74.95	64.96	74.95	64.96	74.95	64.96
3→4	25.27	22.00	25.27	22.00	25.27	22.00	25.26	22.00	25.26	22.00
3→5	74.96	64.95	74.96	64.95	74.96	64.95	74.96	64.95	74.96	64.95
4→3	25.18	21.91	25.18	21.91	25.18	21.91	25.17	21.91	25.17	21.91
4→6	75.04	65.05	75.04	65.05	75.04	65.05	75.04	65.05	75.04	65.05
5→6	27.32	24.07	27.30	24.10	27.30	24.10	27.28	24.14	27.28	24.14
6→5	22.36	19.11	22.35	19.14	22.35	19.14	22.32	19.18	22.33	19.18

4.2 Computing Algorithm

After the test about variational inequality sensitivity analysis and generalized inverse approach, we design the solution algorithm for emergency evacuation and rescue network reconstruction bi-level programming model with multi-class users route choice behavior constraints as follows:

Step 1: Set the initial values of $\mathbf{y}^n$. Let n=1. The $\mathbf{y}^n$ is the ratio of link reserve capacities after destroyed divided link original capacities.

Step 2: Compute the link capacities based on $\mathbf{y}^n$.

Step 3: Using the diagonalization method and gradient projection method to solve the multi-class users network equilibrium variational inequality model as follows:

$$\mathbf{c}(\mathbf{f}^*,\mathbf{y})(\mathbf{f}-\mathbf{f}^*) \geq 0, \forall \mathbf{f} \in \Omega_{\mathrm{y}} \tag{13}$$

Where is the feasible solution areas under the conditions of equations (7)~(10).

Step 4: Applying the variational inequality sensitivity analysis and generalized inverse approach to calculate the implicit differentiation and obtain $\nabla_{\varepsilon}\mathbf{f}(\mathbf{0})$.

Step 5: Computing the link capacity modifies descent search direction as follows:

$$\mathbf{d}^n = -\nabla_\varepsilon(\mathbf{fc}(\mathbf{f},\mathbf{0})) \tag{14}$$

For example, the link cost functions (11), (12) in this research, the descent search direction can be derived as follows

$$\begin{aligned}
\mathbf{d}^n &= -\nabla_\varepsilon(\mathbf{fc}(\mathbf{f},\mathbf{0}))\\
&= -\nabla_\varepsilon\{(\mathbf{f}[\mathbf{c}_A(\mathbf{f},\mathbf{0})+\mathbf{c}_B(\mathbf{f},\mathbf{0})])\}\\
&= -[\nabla_\varepsilon(\mathbf{fc}_A(\mathbf{f},\mathbf{0}))+\nabla_\varepsilon(\mathbf{fc}_B(\mathbf{f},\mathbf{0}))]\\
&= -\{\mathbf{c}_0\nabla_\varepsilon\mathbf{f}_A(0)+0.15\mathbf{c}_0\nabla_\varepsilon\mathbf{f}_A(0)(\mathbf{f}_A+0.2\mathbf{f}_B)^4(\mathbf{Sy^n})^{-4}+\\
&\quad 0.6\mathbf{c}_0\mathbf{f}_A(\mathbf{f}_A+0.2\mathbf{f}_B)^3[\mathbf{y^n}(\nabla_\varepsilon\mathbf{f}_A(0)+0.2\nabla_\varepsilon\mathbf{f}_B(0))-(\mathbf{f}_A+0.2\mathbf{f}_B)]\mathbf{S}^{-4}(\mathbf{y^n})^{-5}+\\
&\quad \mathbf{c}_0\nabla_\varepsilon\mathbf{f}_B(0)+0.15\mathbf{c}_0\nabla_\varepsilon\mathbf{f}_B(0)(0.5\mathbf{f}_A+\mathbf{f}_B)^4(\mathbf{Sy^n})^{-4}+\\
&\quad 0.6\mathbf{c}_0\mathbf{f}_B(0.5\mathbf{f}_A+\mathbf{f}_B)^3[\mathbf{y^n}(0.5\nabla_\varepsilon\mathbf{f}_A(0)+\nabla_\varepsilon\mathbf{f}_B(0))-(0.5\mathbf{f}_A+\mathbf{f}_B)]\mathbf{S}^{-4}(\mathbf{y^n})^{-5}\}
\end{aligned} \tag{15}$$

Step 6: Update $\mathbf{y}^n$, where $\mathbf{y}^{n+1} = \mathbf{y}^n + \frac{1}{n+1}\mathbf{d}^n, \Delta\mathbf{y} = \mathbf{y}^{n+1} - \mathbf{y}^n, 0 \le \Delta\mathbf{y} \le 1$.

Step 7: Revising the link repair capability to satisfy the maximum repair capability constraint as follows:

$$\sum_a \Delta y_a \times CAP_a \le \Gamma, \forall a \in m \tag{16}$$

Step 8: Convergence Test. If $\{y_a^n\} \approx \{y_a^{n+1}\}$, stop. Otherwise, set n=n+1 and go to Step 2.

Applied the solution step as above, we can obtain the link repair rate $\mathbf{y}$. And it can be used in support of emergency evacuation and rescue network reconstruction decisions when large-scale network destroyed under natural disasters.

In the step3 of the above solution algorithm, using the diagonalization method and gradient projection method to solve the multi-class users network equilibrium variational inequality model can be enhanced as follows:

Transfer the variational inequality model to the optimal sub-problem as follows:

$$\min z(\mathbf{x}) = \sum_a \sum_i \int_0^{f_{a_i}} c_{a_i}(f_{a_1},\ldots,\omega,\ldots,f_{a_i})d\omega \tag{17}$$

Combine the diagonalization method and gradient projection method to solve above optimal sub-problem. The details of the algorithm are summarized as follows:

Step 1: Set $n = 0$, $l = 1$, calculate the *ith* mode shortest paths based on the free-flow link travel times $\{c_{a_0}\}$ of each OD pair *rs*, and generate a set of paths with path flow between *rs* of $h_p^{rs} = \bar{q}^{rs}, \forall r,s$, define the path flows set as $\{h_p^{rs}\}^{n+1}$, set $i = 1$.

Step 2: Set $i = i$+1, fixed the cross effects at the current level and solving the main effect.

Step 3: Solve the master problem:

Step 3.1: Set $n = n+1$, calculate the link travel times $\{c_{a_i}\}^{(n)}$ based on the *ith* mode route flows $\{h_{p_i}^{rs}\}^{(n)}$ between O/D pair (r,s).

Step 3.2: Calculate the shortest paths based on the prevailing link travel times $\{c_a^n(x)\}$, and label it as the first solution $\hat{p}^{rs}$ in the solution set $\{p_p^{rs}\}^n$.

Step 4: Update the path flows $\{h_{p_i}^{rs}\}^{(n+1)}$ and link flows $f_{a_i}^{(n+1)}$ in equation (18)~(21).

$$h_{p_i}^{rs(n+1)} = \max\left\{0, \left(h_{p_i}^{rs(n+1)} + \alpha_{p_i}^{rs(n+1)} d_{p_i}^{rs(n)}\right)\right\}, \forall r \in R, s \in S, p_i \ne \hat{p}_i \tag{18}$$

$$h_{\hat{p}_i}^{rs^{(n+1)}} = \overline{q}_i^{rs} - \sum_{p_i \neq \hat{p}_i} h_{p_i}^{rs^{(n+1)}}, \forall r \in R, s \in S \tag{19}$$

$$d_{p_i}^{rs^{(n)}} = \left(c_{p_j}^{rs\,(n)} - c_{\hat{p}_j}^{rs\,(n)}\right) + \left(c_{p_i}^{rs(n)} - c_{\hat{p}_i}^{rs(n)}\right), p_i \neq \hat{p}_i, p_j \neq \hat{p}_j \tag{20}$$

$$\alpha_{p_i}^{rs} = \frac{\lambda}{\sum_a c'_{a_i} \overline{\delta}_{a_i p_i}^{rs} + \sum_a c'_{a_i} \overline{\delta}_{a_i \hat{p}_i}^{rs} - \sum_{a_i \in p_i \cap \hat{p}_i} 2c'_{a_i}}, \forall r \in R, s \in S, p_i \neq \hat{p}_i, 0 < \lambda \leq 1 \tag{21}$$

Step 5: Mode convergence check, compare the difference of route cost between the same O/D pair and using the same mode. If the percentage difference is greater than a pre-specified value θ, then return to Step 3, otherwise check whether or not all of modes have been passed Step5, then go to Step 6, else set *n*=0, go to Step 2.

Step 6: Network equilibrium convergence check, compare the difference of link flows between two successive traffic assignment results, if the percentage difference is less than a pre-specified value θ, then STOP, otherwise set *i*=0, *l*=*l*+1,return to Step 2.

5. NUMERICAL TESTS

Based the model framework and solution algorithms described in the previous section, the present section conduct various numerical analysis to demonstrate the feasibility of the proposed framework and approach in formulating optimal network reconstruction planning and design in case of natural disasters.

5.1 Data Input

The test network consists of a simplified freeway/expressway system as shown in figure 2. The link lengths are referring a digital map released by the government office. The free flow travel times are obtained indirectly from the transformation of corresponding speed limits on the observed links. In terms of the link capacities, it was set by various link capacities depending on the geometric characteristics of each link. The vehicle loadings are classified as two types, namely types A and B. The FHWA form of link cost function was employed and modified. In order to express the asymmetric impact between different class of users, that we assumed and added two different parameters 0.5 and 0.2 to the link cost functions, and to emphasize the effect of multi-class users emergency evacuation and rescue route choice behaviors. The cost function formulations are shown in equations (11) and (12). The computing environment is at the PC under Pentium III-500 platform, and the solution algorithms were implementing by applying Borland C^{++} Version 5.02 language.

5.2 Scenario Designs

In the design of test scenarios, it is supposed that a large-scale natural disaster causing different degrees of destruction of various links in the observed network. The road repair and rescue unit has 1,000 man powers with 10 units of repairing capacity of each man. The rescue and repairing unit equips with two types of vehicles, one is bus (type A) and the other is minibus (type B). The OD demands of each type of vehicles are shown in table 5, and the network destruction was surveyed and demonstrated in table 6. The purposes of the numerical tests are to analyze the specific problem and obtain prioritized links needed to be repaired at the very first stage, and plan the optimal rescue and evacuation routes.

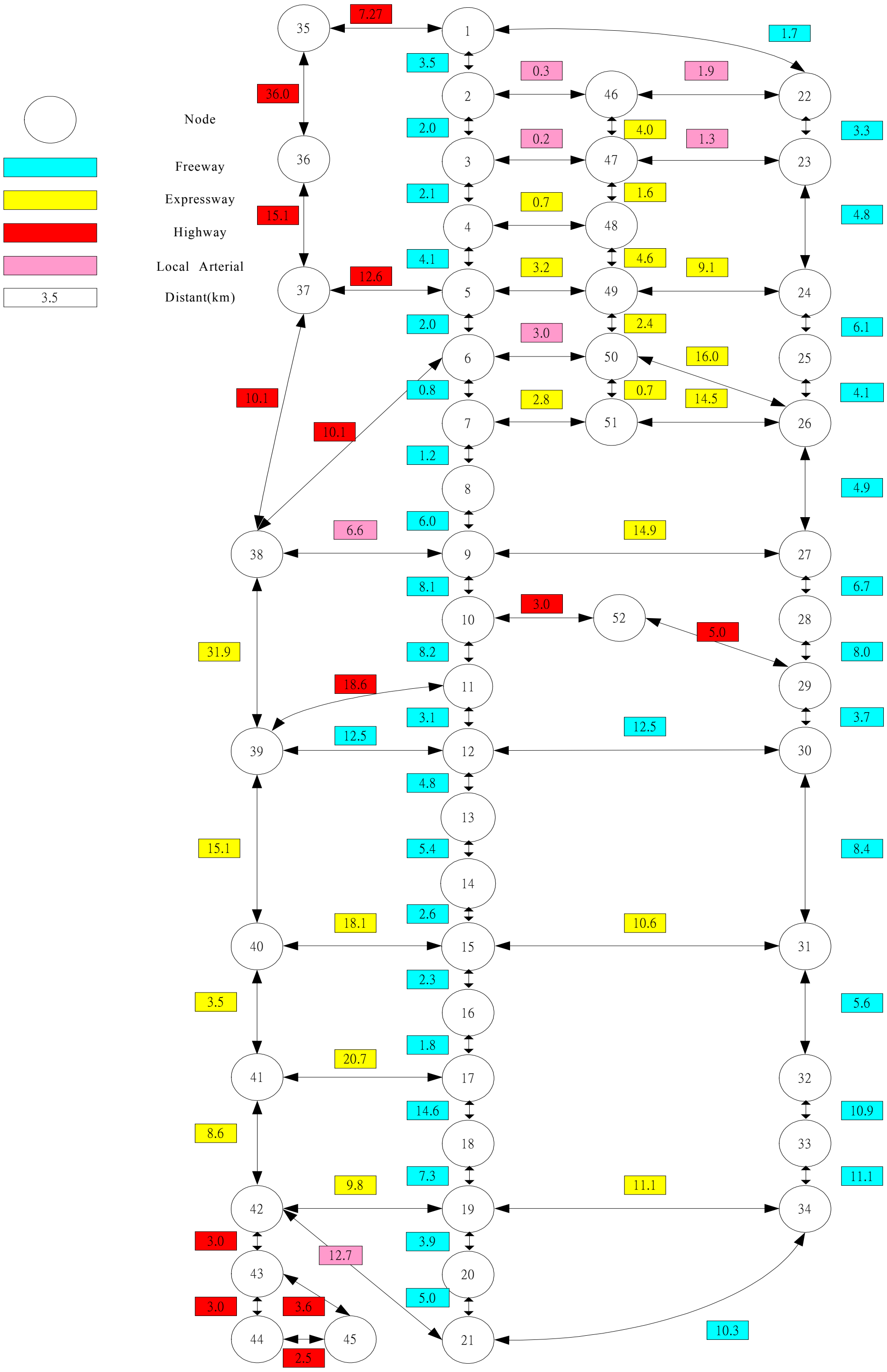

Figure. 2 Test Network 2

Table 5 OD Pair and Demands

O→D Pair		1→11	1→21	6→30	6→43	21→23
Demands	A	600	560	750	450	500
	B	750	650	550	750	600
O→D Pairs		30→35	30→42	35→26	39→23	39→34
Demands	A	750	500	680	480	760
	B	600	580	700	550	860

Table 6 Ratios of Link Capacities before and after Disaster

Link	1-2	1-22	1-35	2-1	2-3	3-2	3-47	4-5	5-4	5-37	6-50	7-8	8-7	9-10
Ratio of Capacities before and after disaster	0.3	0.9	0.5	0.7	0.6	0.6	0.5	0.5	0.5	0.8	0.9	0.4	0.4	0.6
Link	9-27	10- 9	10-52	11-12	12-11	12-30	13-14	14-13	15-31	15-40	16-17	17-16	17-18	17-41
Ratio of Capacities before and after disaster	0.6	0.6	0.7	0.8	0.8	0.9	0.5	0.5	0.8	0.3	0.7	0.7	0.6	0.5
Link	18-17	19-34	20-21	21-20	21-34	22- 1	22-23	23 22	23-24	24-23	25-26	26-25	26-50	27- 9
Ratio of Capacities before and after disaster	0.6	0.9	0.7	0.7	0.5	0.9	0.8	0.8	0.6	0.6	0.7	0.7	0.3	0.6
Link	27-28	28-27	29-30	30-12	30-29	30-31	31-15	31-30	31-32	32-31	34-19	34-21	35- 1	35-36
Ratio of Capacities before and after disaster	0.3	0.3	0.9	0.9	0.9	0.6	0.8	0.6	0.4	0.4	0.9	0.5	0.5	0.7
Link	36-35	36-37	37- 5	37-36	37-38	38-37	39-40	40-15	40-39	41-17	41-42	42-41	43-45	45-43
Ratio of Capacities before and after disaster	0.7	0.5	0.8	0.5	0.8	0.8	0.6	0.3	0.6	0.5	0.8	0.8	0.4	0.4
Link	47- 3	47-48	48-47	49-50	50- 6	50-26	50-49	50-51	51-26	51-50	52-10			
Ratio of Capacities before and after disaster	0.5	0.6	0.6	0.7	0.9	0.3	0.7	0.5	0.4	0.5	0.7			

5.3 Results Analysis

According the results shown in table 7 and table 8 in the appendix, we have found the following outcomes:

1. The information obtained in table 7 including used paths, traffic loadings, and path travel times is crucial for the rescue and repairing unit in preparing necessary for resources allocation, and estimated time required to call for assistance.
2. Under equilibrium conditions, the path travel times of the used paths are essentially the same, that is the costs of the paths being used for a specific OD pair are the same. The outcome is complying with Wardrop's first principle of equilibrium.
3. In the results of table 7, it is found that the systems cost is 572846.49 when the repairing capacity is not optimally allocated. After the reallocation of the resources obtained by the present research, the system cost is reduced to 537380.10. Therefore, the test result has demonstrated the proposed framework is capable of predicting better resources allocation and routing strategies, which in turn one can improve the effectiveness and efficiency of total emergency rescue mechanism.
4. The contents shown in table 8 in the appendix demonstrate the ratios of capacities and traffic flows on each link of vehicle types A and B after repairing capacity optimization. One of the important aspect of the present research is that for practical applications one can have a clear picture of degrees of repairing on each link by referring the ratios of capacities before and after disaster and arrange corresponding repairing capacities in reconstruct specific links. Moreover, one can monitor and control traffic flows on various links depending on the predicted traffic loadings on different links so that assure the rescue work might not be influenced by inefficient assignment of traffic flows.

Table 7 Path Flows of the Lower Level Problem

OD Pair	Vehicle Type	Used Path	Traffic Loads	Path Travel Time
1→11	A	1→2→3→4→5→6→7→8→9→10→11	600.00	31.62
	B	1→2→3→4→5→6→7→8→9→10→11	589.77	35.58
		1→22→46→2→3→4→5→6→7→8→9→10→11	160.23	
1→1	A	1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→19→20→21	43.24	64.62

		1→22→23→24→25→26→27→28→29→30→31→32→33→34→21	516.76	
	B	1→22→23→24→25→26→27→28→29→30→31→32→33→34→21	650.00	68.43
6→30	A	6→7→8→9→10→52→29→30	750.00	26.10
	B	6→7→8→9→10→52→29→30	550.00	28.82
6→43	A	6→7→8→9→10→11→12→13→14→15→16→17→41→42→43	234.18	59.93
		6→7→8→9→10→11→12→39→40→41→42→43	42.41	
		6→7→8→9→10→11→12→13→14→15→16→17→18→19→42→43	173.41	
	B	6→7→8→9→10→11→12→13→14→15→16→17→18→19→42→43	26.57	63.80
		6→7→8→9→10→11→12→13→14→15→16→17→41→42→43	70.71	
		6→38→39→40→41→42→43	652.71	
21→23	A	21→20→19→18→17→16→15→14→13→12→11→10→9→8→7→6→5→4→3→47→23	500.00	58.68
	B	21→34→33→32→31→30→29→28→27→26→25→24→23	277.10	60.48
		21→20→19→18→17→16→15→14→13→12→11→10→9→8→7→6→5→4→3→47→23	322.90	
30→35	A	30→29→28→27→26→25→24→23→22→1→35	651.89	37.34
		30→29→52→10→9→8→7→6→5→4→3→2→1→35	98.11	
	B	30→29→28→27→26→25→24→23→22→1→35	600.00	38.07
30→42	A	30→31→15→16→17→41→42	441.53	40.85
		30→31→15→16→17→18→19→42	58.47	
	B	30→31→15→16→17→18→19→42	199.72	42.96
		30→12→13→14→15→16→17→41→42	211.10	
		30→12→13→14→15→16→17→18→19→42	50.62	
		30→31→15→16→17→41→42	118.56	
35→26	A	35→1→22→23→24→25→26	680.00	21.24
	B	35→1→22→23→24→25→26	700.00	23.03
39→23	A	39→12→11→10→9→8→7→6→5→4→48→47→23	186.37	35.83
		39→12→11→10→9→8→7→6→5→4→3→47→23	293.63	
	B	39→12→11→10→9→8→7→6→5→4→48→47→23	368.63	38.08
		39→12→11→10→9→8→7→6→5→4→3→47→23	181.37	
39→34	A	39→40→41→42→21→34	727.98	42.02
		39→40→41→42→19→34	32.02	
	B	39→40→41→42→21→34	54.70	44.56
		39→12→30→31→32→33→34	352.11	
		39→12→13→14→15→16→17→18→19→34	453.19	
Initial Solution of the Objective Function at the Upper Level		572846.49		
Final Solution of the Objective Function at the Upper Level		537380.10		

6. CONCLUSIONS AND REMARK

The purpose of this research on how to reconstruct the emergency evacuation and rescue network reconstruction after the transportation network has been destroyed by natural disasters. Based on the network design and multi-class users route choice behavior constraints consider, the bi-level has been formulated. And we develop a solution algorithm that applies the variational inequality sensitivity analysis method, generalized inverse matrix method, diagonalization method and gradient projection method to solve the problem. After numerical test, we can derive the conclusions and remarks as follows:

1. The bi-level programming model is a non-convexity problem. But we adopt the variational inequality sensitivity analysis method, generalized inverse approach and obtain

the Stackelberg solution.

2. In practice consider, we involve the multi-class users route choice behavior constraints in out model. And use the diagonalization method and gradient projection method to solve the problem.
3. The outputs of the emergency evacuation and rescue network reconstruction bi-level programming model with multi-class users route choice behavior constraints include: the repair ratio suggestions for destroyed links, emergency evacuation and rescue routes planning, route travel cost and demands, the travel demands on each link and link travel time. These data are very importation in emergency evacuation and rescue mission.

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NOTATIONS

a	link number
$\mathbf{c}$	vector of link travel cost
c_a	travel cost of link a
$c_a(\cdot)$	travel cost function of link a
CAP_a	link capacity of link a
$\mathbf{d}$	vector of improved directions
d_p^{rs}	improved direction of the p^{th} path given OD pair rs
$\mathbf{f}$	vector of link traffic flow
f_a	traffic flow of link a
$\mathbf{h}$	vector of path flow
h_p^{rs}	path flow of p given OD pair rs
I_a	1：flow still can through link a 0：flow still can't through link a
i	mode number
l,n	Number
J	Jacobian matrix
m	set of destroyed links
$\mathbf{p}$	set of path
p	path p
$\hat{p}$	the least distance among paths set
$\bar{q}^{rs}$	traffic demands between OD pair rs
r	origin r
s	destination s
$\boldsymbol{S}$	saturation flow rate or link original capacity
$\mathbf{y}$	vector of link a reserve capacities after destroying and link original capacities
y_a	ratio of link a reserve capacities after destroying and link original capacities
Δy_a	the capacity repair quantities in link a
z	objective function
α	step size
Γ	The maximum total links capacity reconstruction capability
$\bar{\delta}_{ap}^{rs}$	indicator variable
ε	perturbation parameters
θ	the threshold value of convergence
λ	improved pace of the step size
Ω	feasible solution area
*	the optimal value

APPENDIX

Table 8 The Repair Ratio Suggestions for Destroyed Links

Link	Free Flow Travel Time	Link Travel Time		Link Flow		Link Original Capacity	Link Capacity before Repairing	Link Capacity after Repairing	Link Capacit Ratio befor Repairing	Link Capacity Ratio after Repairing
		Vehicle Type A	Vehicle Type B	Vehicle Type A	Vehicle Type B					
1→2	2.26	3.14	4.06	643.24	589.77	2000.00	600.00	600.00	0.30	0.30
1→22	1.20	1.26	1.42	1196.76	1510.23	2000.00	1800.00	2000.00	0.90	1.00
1→35	6.23	6.60	6.81	750.00	600.00	1500.00	750.00	1095.00	0.50	0.73
2→1	2.26	2.26	2.26	98.11	0.00	2000.00	1400.00	1400.00	0.70	0.70
2→3	1.29	1.33	1.41	643.24	750.00	2000.00	1200.00	1200.00	0.60	0.60
3→2	1.29	1.29	1.29	98.11	0.00	2000.00	1200.00	1200.00	0.60	0.60
3→47	0.24	0.61	0.62	793.63	504.27	1000.00	500.00	500.00	0.50	0.50
4→5	2.65	2.81	3.17	643.24	750.00	2000.00	1000.00	1000.00	0.50	0.50
5→4	2.65	3.57	4.14	1078.11	872.90	2000.00	1000.00	1020.00	0.50	0.51
5→37	10.80	10.80	10.80	0.00	0.00	1500.00	1200.00	1200.00	0.80	0.80
6→50	3.60	3.60	3.60	0.00	0.00	1000.00	900.00	900.00	0.90	0.90
7→8	0.77	1.11	1.26	1843.24	1397.29	3000.00	1200.00	1620.00	0.40	0.54
8→7	0.77	0.90	0.98	1078.11	872.90	3000.00	1200.00	1230.00	0.40	0.41
9→10	5.23	10.62	12.91	1843.24	1397.29	2000.00	1200.00	1320.00	0.60	0.66
9→27	12.77	12.77	12.77	0.00	0.00	1500.00	900.00	900.00	0.60	0.60
10→9	5.23	6.16	6.73	1078.11	872.90	2000.00	1200.00	1200.00	0.60	0.60
10→52	2.57	2.71	2.76	750.00	550.00	1500.00	1050.00	1095.00	0.70	0.73
11→12	2.00	2.00	2.00	493.24	97.29	2000.00	1600.00	1600.00	0.80	0.80
12→11	2.00	2.08	2.16	980.00	872.90	2000.00	1600.00	1600.00	0.80	0.80
12→30	8.06	8.06	8.06	0.00	352.11	2000.00	1800.00	1940.00	0.90	0.97
13→14	3.48	3.55	4.09	450.83	812.20	2000.00	1000.00	1000.00	0.50	0.50
14→13	3.48	3.53	3.54	500.00	322.90	2000.00	1000.00	1000.00	0.50	0.50
15→31	9.09	9.09	9.09	0.00	0.00	1500.00	1200.00	1200.00	0.80	0.80
15→40	15.51	15.51	15.51	0.00	0.00	1500.00	450.00	450.00	0.30	0.30
16→17	1.16	1.25	1.46	950.83	1130.48	2000.00	1400.00	1400.00	0.70	0.70
17→16	1.16	1.16	1.16	500.00	322.90	2000.00	1400.00	1400.00	0.70	0.70
17→18	9.42	9.44	9.81	275.12	730.11	2000.00	1200.00	1200.00	0.60	0.60
17→41	14.61	15.33	15.26	675.71	400.37	2000.00	1000.00	1000.00	0.50	0.50
18→17	9.42	9.49	9.49	500.00	322.90	2000.00	1200.00	1200.00	0.60	0.60
19→34	9.51	9.51	9.52	32.02	453.19	1500.00	1350.00	1500.00	0.90	1.00
20→21	3.23	3.23	3.23	43.24	0.00	2000.00	1400.00	1400.00	0.70	0.70
21→20	3.23	3.24	3.24	500.00	322.90	2000.00	1400.00	1400.00	0.70	0.70
21→34	7.27	7.45	7.37	727.98	331.81	2000.00	1000.00	1260.00	0.50	0.63
22→1	1.20	1.20	1.21	651.89	600.00	2000.00	1800.00	1900.00	0.90	0.95
22→23	2.33	2.43	2.65	1196.76	1350.00	2000.00	1600.00	2000.00	0.80	1.00
23→22	2.33	2.34	2.36	651.89	600.00	2000.00	1600.00	1780.00	0.80	0.89
23→24	3.39	3.54	3.87	1196.76	1350.00	2000.00	1200.00	1980.00	0.60	0.99
24→23	3.39	3.42	3.51	651.89	877.10	2000.00	1200.00	1740.00	0.60	0.87
25→26	2.89	3.02	3.29	1196.76	1350.00	2000.00	1400.00	1980.00	0.70	0.99
26→25	2.89	2.91	2.97	651.89	877.10	2000.00	1400.00	1820.00	0.70	0.91
26→50	13.71	13.71	13.71	0.00	0.00	1500.00	450.00	450.00	0.30	0.30
27→9	12.77	12.77	12.77	0.00	0.00	1500.00	900.00	900.00	0.60	0.60
27→28	4.73	4.77	4.90	516.76	650.00	2000.00	600.00	1300.00	0.30	0.65
28→27	4.73	4.76	4.86	651.89	877.10	2000.00	600.00	1860.00	0.30	0.93
29→30	2.61	2.74	2.89	1266.76	1200.00	2000.00	1800.00	2000.00	0.90	1.00
30→12	8.06	8.06	8.06	0.00	261.72	2000.00	1800.00	2000.00	0.90	1.00
30→29	2.61	2.63	2.67	750.00	877.10	2000.00	1800.00	2000.00	0.90	1.00
30→31	5.93	6.38	7.82	1016.76	1320.39	2000.00	1200.00	1520.00	0.60	0.76
31→15	9.09	9.16	9.16	500.00	318.28	1500.00	1200.00	1200.00	0.80	0.80

31→30	5.93	5.93	5.93	0.00	277.10	2000.00	1200.00	1620.00	0.60	0.81
31→32	3.95	3.98	4.24	516.76	1002.11	2000.00	800.00	1500.00	0.40	0.75
32→31	3.95	3.95	3.95	0.00	277.10	2000.00	800.00	1080.00	0.40	0.54
34→19	9.51	9.51	9.51	0.00	0.00	1500.00	1350.00	1350.00	0.90	0.90
34→21	7.27	7.30	7.39	516.76	650.00	2000.00	1000.00	1580.00	0.50	0.79
35→1	6.23	6.49	6.91	680.00	700.00	1500.00	750.00	1125.00	0.50	0.75
35→36	30.86	30.86	30.86	0.00	0.00	1500.00	1050.00	1050.00	0.70	0.70
36→35	30.86	30.86	30.86	0.00	0.00	1500.00	1050.00	1050.00	0.70	0.70
36→37	12.94	12.94	12.94	0.00	0.00	1500.00	750.00	750.00	0.50	0.50
37→5	10.80	10.80	10.80	0.00	0.00	1500.00	1200.00	1200.00	0.80	0.80
37→36	12.94	12.94	12.94	0.00	0.00	1500.00	750.00	750.00	0.50	0.50
37→38	8.66	8.66	8.66	0.00	0.00	1500.00	1200.00	1200.00	0.80	0.80
38→37	8.66	8.66	8.66	0.00	0.00	1500.00	1200.00	1200.00	0.80	0.80
39→40	12.94	15.29	17.41	802.41	707.42	1500.00	900.00	900.00	0.60	0.60
40→15	15.51	15.51	15.51	0.00	0.00	1500.00	450.00	450.00	0.30	0.30
40→39	12.94	12.94	12.94	0.00	0.00	1500.00	900.00	900.00	0.60	0.60
41→17	14.61	14.61	14.61	0.00	0.00	2000.00	1000.00	1000.00	0.50	0.50
41→42	6.07	7.23	7.69	1478.12	1107.78	2000.00	1600.00	1600.00	0.80	0.80
42→41	6.07	6.07	6.07	0.00	0.00	2000.00	1600.00	1600.00	0.80	0.80
43→45	3.09	3.09	3.09	0.00	0.00	1500.00	600.00	600.00	0.40	0.40
45→43	3.09	3.09	3.09	0.00	0.00	1500.00	600.00	600.00	0.40	0.40
47→3	0.24	0.24	0.24	0.00	0.00	1000.00	500.00	500.00	0.50	0.50
47→48	1.37	1.37	1.37	0.00	0.00	1500.00	900.00	900.00	0.60	0.60
48→47	1.37	1.37	1.38	186.37	368.63	1500.00	900.00	1005.00	0.60	0.67
49→50	2.06	2.06	2.06	0.00	0.00	1500.00	1050.00	1050.00	0.70	0.70
50→6	3.60	3.60	3.60	0.00	0.00	1000.00	900.00	900.00	0.90	0.90
50→26	13.71	13.71	13.71	0.00	0.00	1500.00	450.00	450.00	0.30	0.30
50→49	2.06	2.06	2.06	0.00	0.00	1500.00	1050.00	1050.00	0.70	0.70
50→51	0.60	0.60	0.60	0.00	0.00	1500.00	750.00	750.00	0.50	0.50
51→26	12.43	12.43	12.43	0.00	0.00	1500.00	600.00	600.00	0.40	0.40
51→50	0.60	0.60	0.60	0.00	0.00	1500.00	750.00	750.00	0.50	0.50
52→10	2.57	2.57	2.57	98.11	0.00	1500.00	1050.00	1050.00	0.70	0.70