

## THE OPTIMAL ROAD PRICING FOR A CORRIDOR WITH A NEW ELEVATED ROAD PROJECT

Chih-Peng CHU  
Department of Business Management  
National Dong Hwa University  
1, Sec. 2, Da-Hsueh Rd., Shou-Feng,  
Hualien, 97401, Taiwan  
Tel: +866-3-8633029  
Fax: +886-3-8633010  
E-mail: [chpchu@mail.ndhu.edu.tw](mailto:chpchu@mail.ndhu.edu.tw).

Jyh-Fa TSAI  
Department of Economics  
Soochow University  
56, Sec. 1, Kuei-Yang St.,  
Taipei, Taiwan.  
Tel: +886-2-2311-1531 Ext. 3449  
Fax: +886-2-2382-2001  
E-mail: [tsaij@scu.edu.tw](mailto:tsaij@scu.edu.tw).

**Abstract:** This paper explores the optimal road pricing policy when a new elevated road is constructed in a corridor that suffers from traffic congestion before the elevated road is built. Due to the extra construction/maintenance expenditure for this new elevated road and the need to lessen congestion situation, the government decides to apply a road-pricing mechanism on the transportation system. Three pricing policies are discussed in this paper: (a) charging the travelers who live after the entrance of the new elevated road by a flat toll; (b) charging the travelers who live after the entrance of the new elevated road but the new elevated road users and original road users would pay different toll; and (c) charging all the travelers with different congestion toll depending on which path they use. The numerical result shows the existence of the social optimal corresponding toll. The best pricing policy among these three alternatives is also discussed.

Key Words: Road pricing policy, Optimal toll, Monocentric city traffic, Elevated road

### 1. INTRODUCTION

This paper explores the optimal road pricing policy for a linear monocentric city while the distribution of residents is uniformly located along this corridor and each individual makes trip to the Central Business District (CBD) at the end-point of the corridor. Trip demand is elastic and dependent on the cost of car trip that incorporates congestion cost. To reduce the traffic congestion, the government decides to build an elevated road and also considers applying a road-pricing policy on it. There are two reasons for this road-pricing application: the first one is due to the extra construction/maintenance expenditure for this elevated road; and the second one is to improve the congestion situation for the road system. That is, the road pricing is introduced to help the government to maximize social welfare when the new elevated road is built.

The road pricing issue has been a main stream of transportation economics for the past decades. The principle of road pricing has been switched from the first-best pricing principle to second-best pricing due to the obstacle of the implementation for the first-best pricing policy. This concept of second-best road pricing was first introduced by Marchand (1968) and recently followed by most of researchers (For example: Liu & McDonald (1999), McDonald (1995), Verhoef (1995), Yang & Lam (1996), Yang & Bell (1997)).

The other studies related to our paper are those on the issue of highway investment. Mohring and Harwitz (1962) start the issue of optimal pricing and investment decision for highway. Mohring (1970) extends the study of Mohring and Harwitz (1962) to congestion toll. Kraus *et al.* (1976) provide rough estimates of the welfare losses in providing freeway transportation while the peak and off-peak demands as well as increasing returns to scale are considered. Keeler and Small (1977) deal the optimal pricing and investment policies for urban road with the joint decision of optimal toll, capacity, and service of level. Williams *et al.* (2001) focus on analysis of the benefits of highway investment for different pricing schemes. Mun *et al.* (2003) discuss the optimal cordon pricing for monocentric city and show the optimal location for cordon pricing as well as the optimal pricing. They find that the trips from locations in the fringe of the urban are under-priced and those outside the urban cordon pricing are over-priced under the optimal cordon pricing. Wang *et al.* (2004) also adopt the assumption of linear monocentric city with uniformly distributed population to discuss the highway investment benefits under alternative pricing regimes.

However, these previous studies do not consider that situation with a bypass road added to a linear monocentric city. Therefore, this paper focuses on the road pricing issue on a highway with bypass road investment when the location of the elevated road is given. Specifically, three policies are discussed: (a) charging the travelers who live after the entrance of the new elevated road only and the users paying a flat toll before the entrance of the new elevated road; (b) charging the travelers who live after the entrance of the new elevated road only but the new elevated road users and original road users would pay different tolls; and (c) charging all road users with different congestion toll.

## 2. MODEL

The distribution of residents is uniformly located along a corridor and each individual makes trip to the CBD of the corridor (Figure 1). Trip demand is a function of the cost of car trips that incorporates the cost with congestion. To simplify the model, we assume that the corridor is a one-way road. All trips are assumed heading to CBD only.

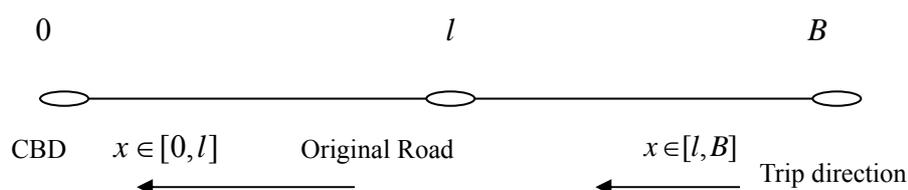


Figure 1. The Original Road System for Monocentric City

Let  $x$  be the location where the distance from the CBD is  $x$ , and  $q(x)$  be the trip demand of a resident located at  $x$ . Then the inverse trip demand is given as<sup>1</sup>

$$p(q(x)) = a - bq(x) \tag{1}$$

where  $a, b$  are positive constants and  $p(q(x))$  represents the private benefit of a trip.

We assume the road width is a constant for all locations and the cost for driving the unit distance around location  $x$  is an increasing function of the traffic volume there,  $Q(x)$ , which is denoted by  $t(Q(x))$ . Then, the cost for a trip from  $x$  to the CBD,  $c(x)$ , is given by

<sup>1</sup> The demand function can be referred to Mun *et al.* (2003).

$$c(x) = \int_0^x t(Q(y)) dy \tag{2}$$

where  $Q(x) = \int_x^B q(y) dy$  . (3)

Note that the starting points of trips are distributed uniformly and with the same destination, the CBD.

For further analysis, we assume that the function of trip cost is specified as follows:

$$t(Q(y)) = f + wQ(y) \tag{4}$$

where  $f$  is the free flow travel time and  $w$  is the marginal cost with respect to traffic volume.

### 2.1 Adding the Elevated Road and Toll Booths

In order to lessen the traffic congestion, the government builds a new elevated road parallel to the original road from a planned location,  $l$ , to CBD. Furthermore, the costs of having this elevated road are considered. The policy of tolling on the new road is also employed. There are three location settings for the tolling booths: (a) locating one booth on the original road close to CBD. The toll setting is denoted as  $\tau_1$ ; (b) locating one booth on the original road right after the entrance of elevated road. The toll setting is denoted as  $\tau_2$ ; and (c) locating one booth right after the entrance of the new elevated road. The toll setting is denoted as  $\tau_3$ . These location settings of the tolling booths and the tolls are shown in Figure 2.

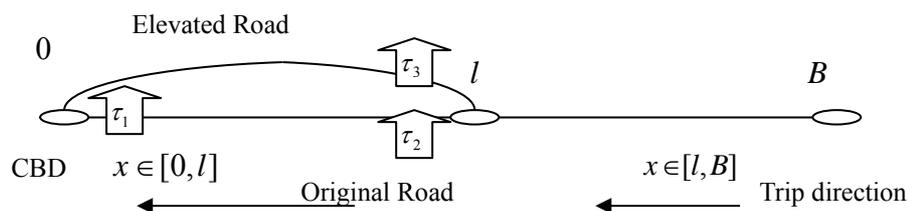


Figure 2. New Elevated Road with Entrance Location at  $l$

Therefore there are three pricing schemes for this new system: <sup>2</sup>

Scheme I: Charging the travelers of  $x \in [l, B]$  with a flat toll

In this scheme, all the travelers who live after the entrance of the new elevated road will be charged by a flat toll just before passing the location,  $l$ . That is, the tolls are the same for the travelers using the original road and those using the elevated road. In mathematics, the constraints  $\tau_1 = 0$  and  $\tau_2 = \tau_3$  are needed.

Scheme II: Charging the travelers of  $x \in [l, B]$  with different tolls

In this scheme, the tolling range of residents is the same as Scheme I. However, they will be charged by a toll just after passing the location,  $l$ . That is, the new elevated road users and original road users will be charged by different tolls. In mathematics, the constraint  $\tau_1 = 0$  is needed. This also means that  $\tau_2 \neq \tau_3$ .

<sup>2</sup> The situation of setting  $\tau_2 = 0$  and  $\tau_1 = \tau_3$  is regarded as the parking pricing. Therefore, we ignore this situation in our paper.

**Scheme III: Charging all the travelers**

In this scheme, all the travelers of  $x \in [0, l]$  and  $x \in [l, B]$  have to be charged for congestion toll. In mathematics, the constraint  $\tau_2 = 0$  is needed. This also means that  $\tau_1 \neq 0$  and  $\tau_3 \neq 0$ .

With this new elevated road, the residents in the section  $[l, B]$  will have two paths to choose, as shown in Figure 2. We introduce the trip demand by setting residents into two sections. Let  $q_1(x)$  be the trip demand of a resident located at  $x \in [0, l]$ , and  $q_2(x)$  be the trip demand of a resident located at  $x \in [l, B]$ . Note that the residents living between CBD and the entrance of elevated road,  $x \in [0, l]$ , will not be able to use this elevated road. Suppose that the construction/maintenance cost of building this elevated road,  $k(l)$ , is a function of the length of this road,  $l$ .

**2.2 The equilibrium trip generated at  $x \in [0, l]$**

Assume the cost of trip for the residents who live within this section traveling to CBD is  $c_{0l}(x, \tau_1)$ . The private marginal benefit (the price) faced by each individual who makes a trip at location  $x$  is  $p(q(x)) = a - bq(x)$ . Let  $t[Q(x)]$  be the cost of driving the unit distance around  $x$  as mentioned in Equation (2). Then, the equilibrium number of trips generated at each location in section  $[0, l]$  is obtained (refer to Appendix A) as

$$q_1^*(x) = \lambda_1 e^{\alpha x} + \lambda_2 e^{-\alpha x}, \quad x \in [0, l] \tag{5}$$

where  $\alpha = \sqrt{\frac{w}{b}}$ , and  $\lambda_1, \lambda_2$  are unknown constants to be determined.

**2.3 The equilibrium trip generated at  $x \in [l, B]$**

Let travelers living at the point  $x \in [l, B]$  bear a cost traveling from the location  $x$  to the entrance of the new elevated road is  $c_{lB}(x)$ . Also let the travel cost of using the original road from the entrance to CBD be denoted as  $c_{0l}(l, \tau_2)$ . The cost of using this new elevated road to CBD be denoted as  $c_{el}(l, \tau_3)$ . Since the travelers in this section have two alternatives to select, the cost of travel fro each alternative will be

$$c_{elB}(l, \tau_3) = c_{lB}(x) + c_{el}(l, \tau_3) \quad \text{for using new elevated road, and}$$

$$c_{0lB}(l, \tau_2) = c_{lB}(x) + c_{0l}(l, \tau_2) \quad \text{for using the original road.}$$

For the residents who live at the point,  $x \in [l, B]$ , the private marginal benefit of each individual making a trip at location  $x$  equals minimum private cost of these two alternatives. That is,  $p(q_2(x)) = \min\{c_{0lB}(x, \tau_2), c_{elB}(x, \tau_3)\}$ , for  $x = [l, B]$ .

In equilibrium, we have  $p(q_2(x)) = c_{0lB}(x, \tau_2) = c_{elB}(x, \tau_3)$  if both paths are used by the residents in the section,  $x \in [l, B]$ . Therefore, the equilibrium number of trips generated at each location in section  $[l, B]$  is obtained as follows (refer to Appendix A).

$$q_2^*(x) = \lambda_3 e^{\alpha x} + \lambda_4 e^{-\alpha x}, \quad x \in [l, B] \tag{6}$$

where  $\alpha = \sqrt{\frac{w}{b}}$ , and  $\lambda_3, \lambda_4$  are unknown constants to be determined.

### 3. ROAD PRICING POLICIES AND OPTIMAL TOLL

To find the equilibrium traffic flows for the whole system, there are five unknowns ( $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ , and  $\gamma$ ) to be determined.<sup>3</sup> If  $\gamma > 0$  then the original road and elevated road are both used and this means the costs of using these two routes are the same. Otherwise, the trip behavior does not follow Wardrop First Principle. Three different boundary conditions are used to solve these five unknowns: (A) the CBD point boundary; (B) the end-point boundary; and (C) the location of the entrance of the elevated road. The details of the relationship for these conditions are described as follows:

Condition A: The CBD point boundary

For the boundary condition at  $x = 0$  with  $c_{0l}(0) = 0$ , we have

$$p(q_1(0)) = c_{0l}(0) = 0 \tag{7}$$

$$p' \cdot \frac{dq_1(0)}{dx} - c'(0) = 0. \tag{8}$$

Condition B: The end-point boundary

For the end-point,  $x = B$ , the boundary condition is

$$p' \cdot \frac{dq_2(B)}{dx} - c'(B) = 0. \tag{9}$$

Condition C: The location of entrance of elevated road

For the boundary at  $x = l$ , the cost of using elevated road is  $c_{elB}(l)$ . The cost of using the original road is  $c_{0lB}(l)$ . The private marginal benefit is  $p(q_2(l))$ . Therefore, if  $0 < \gamma < 1$ , then

$$p(q_l(l)) = c_{0lB}(l), \text{ for } x = l, \tag{10}$$

$$c_{elB}(l) = c_{0lB}(l), \text{ for } x = l. \tag{11}$$

From conditions (7) to (11), the unknown parameters ( $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \gamma$ ) are determined. However, the optimal solution for these five unknowns is so complex in description.

The main purpose of his paper is to solve the location of booths and its corresponding optimal toll to maximize the social welfare. Therefore, the three pricing schemes stated in Section 2.1 will be discussed. In each scheme, the government is treated as the social planner to pursue the maximization of social welfare by determining the optimal road tolls.

Social welfare could be presented as the net benefit of the road system in which investment and road pricing are applied. Therefore, the social welfare for this transportation system under a set of toll policy is the consumer surplus in both locations minus the cost of providing the new elevated road. In detail, the social surplus at certain location  $x$  (in the location  $[0, l]$ ) is consumers' willingness-to-pay,  $\int_0^{q_1^*(x)} p(q) dq$ , minus the travel costs (including the travel time cost and tolls),  $c(x)q_1^*(x)$ . The objective function, social welfare, is described in Equation (12).<sup>4</sup>

<sup>3</sup> Besides the four unknowns in Equation (5) and (6),  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ , the parameter,  $\gamma$ , needs also to be determined.

<sup>4</sup> The social surplus setting can be referred to Mun *et al.* (2003).

$$\text{Max}_{\tau_1, \tau_2, \tau_3} \int_0^l \left[ \int_0^{q_1^*(x)} p(q) dq - c(x) q_1^*(x) \right] dx + \int_l^B \left[ \int_0^{q_2^*(x)} p(q) dq - c(x) q_2^*(x) \right] dx - k(l) \quad (12)$$

The first two terms of Equation (12) are the consumer surplus for the demand in the section  $[0, l]$  and  $[l, B]$ , respectively. The third term is the construction cost of building this new elevated road,  $k(l)$ .

#### 4. SIMULATION RESULTS

Due to the complexity of our model, the optimal road pricing and the optimal policy are hard to be described in closed-form. Therefore, in this section a numerical analysis is employed to provide insights of theoretical model and help us to obtain the optimal results.

We assume the total length of corridor is about 15km, that is,  $B = 15$ . The entrance of elevated road is set at 5 km far from CBD, that is,  $l = 5$ . The speed of free-flow is 50 km/hr. It implies that  $f = 1.2$  minutes. The parameter of travel cost  $w$  is set to approach the calibrated parameter in Mun's model.<sup>5</sup> This parameter is calibrated as 0.52 minutes in Mun's model and we assume it as 0.5 minutes to simplify the calculating process. The free-flow travel time for a 15 km corridor is 18 minutes (at the speed of 50 km/hr). The longest travel time among the trip to CBD is assumed to be 40 minutes, that is,  $a = 40$  minutes. The largest number of trip per person is 0.4 in this case,  $b = 100$  is then obtained by the linear function assumption (from Equation 1). The construction cost distributed to each kilometer per hour is regarded the maintenance cost and only counted for the pavement recovery cost. We assume the maintenance cost for an elevated road is about US\$2181.82 per kilometer per year.<sup>6</sup> The cost for one kilometer per hour is then about US\$ 0.3736. We also assume that the value of time is US\$ 7.5 per hour.<sup>7</sup> The cost of this elevated road is then about 3 minutes.<sup>8</sup> Therefore, we set  $k$ , the construction cost of the elevated road shared to per hour per kilometer, in this case to be 3 minutes of value.

The simulation results of the optimal tolls and maximized social welfare for all the pricing policies are shown in Table 1. In appendix B, we show the selected detail simulation charts for Scheme I, Scheme II, and Scheme III. From Table 1, we know if the government decides not to charge the travelers then the social welfare is about 6.81. This is the case of Benchmark. Under this scheme, the value of these parameters are ( $\lambda_1 = -0.0093612$ ,  $\lambda_2 = 0.3009361$ ,  $\lambda_3 = 0.0288361$ ,  $\lambda_4 = 0.2405547$ ,  $\gamma = 0.3358768$ ).

For the Scheme I, that is, if the government charges the all travelers live after the entrance of the new elevated road with a flat toll right before the entrance of the new elevated road (the

<sup>5</sup> Mun et. al (2003) calibrated their parameters by using the data from a survey for Keihanshin Area in 1990. In our paper, instead of calibrating from a survey, we set our parameters similar to Mun's. Note that this cost is measured by the value of time in minutes.

<sup>6</sup> The cost of rigid pavements and flexible pavement of highway per lane-mile are US\$12,800 with durability around 5.3 years and US\$ 24,820 with durability varying around 6.4 years (Small and Winston, 1988). Base on their study, we assume the maintenance cost is US\$12000 per lane-kilometer for the durability of 5.5 years. We also assume that 365 days for one year and this maintenance cost are shared to sixteen hours in one day (excluding the midnight 8 hours), then the cost for per hour per kilometer is the about US\$ 0.3736.

<sup>7</sup> According to Tretvik's study (Johansson and Mattsson, 1995), the average value across the sample per vehicle per person is about \$52 NOK/hour (convert to US dollar is around US\$7.5 per hour).

<sup>8</sup> The process is: US\$ 0.3736 per hour per kilometer divided by US\$7.5 = 2.988  $\cong$  3minutes.

situation of Scheme I). Then the optimal toll is US\$1.1875 and the maximized social welfare is 11.25.<sup>9</sup> Under this scheme, the value of these parameters are ( $\lambda_1 = -0.0006884$ ,  $\lambda_2 = 0.3006884$ ,  $\lambda_3 = 0.0286032$ ,  $\lambda_4 = 0.2386116$ ,  $\gamma = 0.3344578$ ).

On the other hand, the government can choose to charge the same group of users in Scheme I but let them pay different tolls for taking different path (the situation of Scheme II). That is, the new elevated road users and original road users would pay different tolls. The simulation result shows that the optimal toll for travelers who taking elevated road is US\$1.625 and for those who taking the original road is US\$0.3. The social welfare reaches the maximum as 12.26. Under this scheme, the value of these parameters are ( $\lambda_1 = -0.0086730$ ,  $\lambda_2 = 0.3086790$ ,  $\lambda_3 = 0.0276570$ ,  $\lambda_4 = 0.2307191$ ,  $\gamma = 0.4843567$ ).

If the government wants to charge all the travelers regardless where they live, then this is the situation of Scheme III. The simulation result shows the optimal toll for travelers who taking elevated road is US\$1 and for those who taking the original road is US\$0.1875. The maximized social welfare is 11.58. Under this scheme, the value of these parameters are ( $\lambda_1 = -0.0128292$ ,  $\lambda_2 = 0.3128292$ ,  $\lambda_3 = 0.0276570$ ,  $\lambda_4 = 0.2307186$ ,  $\gamma = 0.3401003$ ).

To compare the maximized social welfares in these four situations, we find that if the social welfare is the most important consideration, then the government should apply the pricing policy in Scheme II, to charge the travelers who live after the entrance of the new elevated road with different tolls. The toll level should be set at US\$1.625 for elevated road users and US\$0.3 for original road users. However, the toll difference between the users of the elevated road (US\$1.625) and the original road users (\$0.3) is quite large. Scheme III requires that all travelers pay for their direct and indirect external benefit of this new elevated road (the residents who live in area  $[0, l]$  have the indirect external benefit because the elevated road lessen the traffic congestion) and it would be an acceptable policy because of the equality consideration<sup>10</sup>.

Table 1. Optimal Tolls and Social Welfare for Different Pricing Policies

		$\tau_1$	$\tau_2$	$\tau_3$	Social Welfare
		US\$			
Benchmark	No charge	0	0	0	6.81
Scheme I	The travelers pay a flat toll before the entrance of the new elevated road.	0	1.1875	1.1875	11.25
Scheme II	The travelers pay toll right after the entrance of the new elevated road but the tolling amount depends on which path being used.	0	0.3	1.625	12.26
Scheme III	All the travelers have to pay different congestion toll depending on what path being used.	0.1875	0	1	11.58

<sup>9</sup> Note, the toll value shows in the figures in Appendix B is in the unit of minutes. The converting rate is US\$ 0.125 per minute.

<sup>10</sup> Thanks for one of EASTS anonymous referees pointed out the implementation issue.

## 5. CONCLUSIONS AND REMARKS

This paper presents a road pricing model for a monocentric city with a new elevated road project. The process of finding the optimal pricing policy and its corresponding optimal toll are demonstrated. The simulations of numeric examples provide more insights of the travel behaviors in this monocentric city transportation system. The simulation results suggest that when we only consider the social welfare the optimal road pricing policy is to apply the pricing policy of charging the travelers who live after the entrance of the new elevated road only and setting different tolls for elevated road and original road users. If we consider the equality issue, Scheme III (all the travelers have to pay different congestion toll depending on what path being used) would also be a considerable policy. This result depends on the parameter settings. However, it demonstrates the existence of the social optimal road pricing policy and the feasibility of our model.

## REFERENCES

- D'Ouille E.L. and McDonald, J.F. (1990) Optimal road capacity with a suboptimal congestion toll, **Journal of Urban Economics**, Vol. 28, 34-49.
- Gronau R. (1994) Optimal road capacity with a suboptimal congestion toll, **Journal of Urban Economics**, Vol. 36, 1-7.
- Keeler, T.E. and Small, K.A. (1977) Optimal peak-load pricing, investment and service levels on urban expressways, **Journal of Political Economy**, Vol. 85, No.1, 1-25.
- Kraus, M., Moring, H. and Pinfold, T. (1976) The welfare costs of nonoptimum pricing and investment policies for freeway transportation, **The American Economic Review**, Vol. 66, No. 4, 532-547.
- Morhring, H. (1970) The peak load problem with increasing returns and pricing constraints, **American Economic Review**, Vol. 60, No. 4, 693-705.
- Mun, S.I., Konishi, K.J. and Yoshikawa, K. (2003) Optimal cordon pricing, **Journal of Urban Economics**, Vol. 54, 21-38.
- Newbery, D. M. G. (1990) Pricing and congestion: Economic principles relevant to pricing roads, **Oxford Review of Economic Policy, Special Issues on Transport**, Vol. 6, No. 2, 22-38.
- Peterson, E.R. (2002) A highway corridor planning model: QROAD, **Transportation Research Part A**, Vol. 36, 107-125.
- Small, K.A., Winston, C.M. and Evans, C.A. (1989) **Road Work: A New Highway Pricing and Investment Policy**, Washington, DC: The Brookings Institution.
- Small, K.A. and Winston, C. (1988) Optimal Highway Durability, **American Economic Review**, Vol. 78, No. 3, 560-569.
- Tretvik, T. (1995) Willingness to pay for time savings: The Trondheim toll road experiences. From **Road Pricing: Theory, Empirical Assessment and Policy**, Chapter 5, edited by Johansson, B. and Mattsson, L. G., Kluwer Academic Publishers.
- Wang, J.Y.T, Yang, H., and Lindsey, R. (2004) Locating and pricing park-and-ride facilities in a linear monocentric City with Deterministic Mode Choice, **Transportation Research Part B**, Vol. 38, 709-731.
- Williams, H.C.W.L., Vliet, D.V., Parathira, C. and Kim, K.S. (2001) Highway investment benefits under alternative pricing regimes, **Journal of Transport Economics and Policy**, Vol. 35, No. 2, 257-284.
- Wilson J.D. (1983) Optimal road capacity in the presence of unpriced congestion, **Journal of**

**Urban Economics, Vol. 13, 337-357.**

**APPENDIX A:**

This appendix shows the detail of formatting process of the travel flow for the residents distributed on the corridor.

Let  $q_1(x)$  be the trip demand of the residents located at  $x \in [0, l]$ , and  $q_2(x)$  be the trip demand of the residents located at  $x \in [l, B]$ . The trip cost for the residents who live within this section traveling to CBD is  $c_{0l}(x, \tau_1)$ . The private marginal benefit (price) faced by each individual who makes a trip at location  $x$  is  $p(q(x)) = a - bq(x)$ . Let  $t[Q(x)]$  be the cost for driving the unit distance around  $x$  as mentioned in Equation (2).

(a) For the residents who live before the elevated road entrance,  $x \in [0, l]$

Since we assume the residents within this region will not turn back to take the elevated road, the accumulated travel flow at location  $x$  is a combination of two groups of users in these two sections.

$$Q_{0l}(x) = \int_x^l q_1(y)dy + \int_l^B \gamma q_2(y)dy \tag{A.1}$$

where  $\gamma$  is the ratio of the trip generated from the section  $[l, B]$ . The first term of RHS in Equation (5) is the accumulated trips by the residents who live before the entrance of elevated road. The second term represents the accumulated trip by the residents who live after the entrance of elevated road and do not take the elevated road (that is, they still use the original road to CBD).

The cost of trip for the residents that live within this section traveling to CBD is as follows.

$$c_{0l}(x) = \int_0^x t[Q_{0l}(y)]dy + \tau_1 \tag{A.2}$$

where  $Q_{0l}(x)$  is the accumulated trip at the point  $x \in [0, l]$  and  $t[Q(x)]$  is the cost for driving the unit distance around  $x$  as mentioned in Equation (2).  $\tau_1$  is the road toll when it is applied. That is, if there exists a tolling booth on the original road near CBD, the toll level is  $\tau_1$ .

In equilibrium, the private marginal benefit of each individual who makes a trip from location  $x$  to CBD equals his private cost. That is

$$p(q_1(x)) = c_{0l}(x), \text{ for all } x, 0 \leq x < l \tag{A.3}$$

Therefore, from Equations (1), (A.1)~(A.3), we can then analyze the traffic flow as:

$$a - bq_1(x) = \int_0^x t[Q_{0l}(y)]dy = xf + w \int_0^x [\int_x^l q_1(y)dy + \int_l^B \gamma q_2(y)dy]dx + \tau_1 \tag{A.4}$$

Differentiating both sides of Equation (A.4) with respect to  $x$ , we have

$$-bq_1'(x) = f + wQ_{0l}(x) = f + w[\int_x^l q_1(y)dy + \int_l^B \gamma q_2(y)dy] \tag{A.5}$$

Differentiating Equation (A.5) with respect to  $x$  once more yields

$$-bq_1''(x) = -wq_1(x), \quad x \in [0, l] \tag{A.6}$$

The equilibrium number of trip generated at each location in section  $[0, l]$  is obtained by solving the differential Equation (A.6), as follows:

$$q_1^*(x) = \lambda_1 e^{\alpha x} + \lambda_2 e^{-\alpha x}, \quad x \in [0, l] \tag{A.7}$$

where  $\alpha = \sqrt{\frac{w}{b}}$ , and  $\lambda_1, \lambda_2$  are unknown constants to be determined by boundary conditions.

(b) For the residents who live after the elevated road entrance,  $x \in [l, B]$

The accumulated traffic flow for the location in this section is

$$Q_{lB}(x) = \int_x^B q_2(y)dy \tag{A.8}$$

where  $q_2(x)$  is the trip demand of a resident located at  $x \in [l, B]$  as described in previous subsection. The road users have two alternative roads to choose to the CBD: the original road and the elevated road. We formulate the travel cost for section  $x \in [l, B]$  first as follows:

$$c_{lB}(x) = \int_l^x t[Q_{lB}(y)]dy \tag{A.9}$$

where  $Q_{lB}(x)$  is accumulated trip for the point  $x \in [l, B]$  and  $t[Q(x)]$  is the cost for driving the unit distance around  $x$ . The travel cost of using the original road from the entrance can be presented as

$$c_{0l}(l, \tau_2) = \int_0^l t[Q_{0l}(y)]dy + \tau_2 \tag{A.10}$$

where  $\tau_2$  is the road toll when it is applied. That is, if there exists a toll booth on the original road near the entrance of the new elevated road, the tolling level is  $\tau_2$ .

Therefore, this cost includes the trip cost from its location to entrance of elevated road and that from the entrance to CBD using the original road:

$$c_{0lB}(x, \tau_2) = c_{0l}(l, \tau_2) + c_{lB}(x) \tag{A.11}$$

Note that the cost in (A.11) will be identical to that is Equation (A.2) when  $x = l$ .

On the other hand, the second alternative for the trips to CBD is to use the elevated road. The cost of using this new elevated road can be presented as

$$c_{el}(l, \tau_3) = \int_0^l t[Q_{el}(y)]dy + \tau_3 = l \cdot t[(1 - \gamma)Q_{lB}(l)] + \tau_3 \tag{A.12}$$

where  $Q_{el}(x) = (1 - \gamma)Q_{lB}(x) = \int_l^B (1 - \gamma)q_2(y)dy$  represents the accumulated trip at point  $x \in [0, l]$ . Because there are no residents living in this new elevated road, this equation can be further simplified.  $\tau_3$  is the road toll when it can be applied. That is, if there exists a toll booth on the elevated road, the tolling level is  $\tau_3$ .

Therefore, the cost of using this new road for those travelers live at the point  $x \in [l, B]$  can be presented as follows.

$$c_{elB}(x, \tau_3) = c_{el}(l, \tau_3) + c_{lB}(x) \tag{A.13}$$

This amount of traffic flow at the location on elevated road only accumulates by the residents who live within the section  $[l, B]$  and select the new elevated road.  $Q_{lB}(x)$  denotes the trips as in Equation (A.8). The first term of RHS in Equation (A.13) represents the cost of trip starting from location  $l$  to CBD by taking the new elevated road. The second term of RHS represents the cost of trip from their locations to the entrance of elevated road.

Since there are two alternatives for the residents who live in this section, in equilibrium the private marginal benefit of each individual making a trip from location  $x$  to CBD equals minimum private cost of these two alternatives. That is,

$$p(q_2(x)) = \min \{c_{0lB}(x, \tau_2), c_{elB}(x, \tau_3)\}, \text{ for } x = [l, B]. \tag{A.14}$$

If both road alternatives have been used by the residents in the section,  $x \in [l, B]$ , we then have  $p(q_2(x)) = c_{0lB}(x, \tau_2) = c_{elB}(x, \tau_3)$ . That is

$$a - bq_2(x) = \int_0^l t[Q_{el}(x)]dy + \int_l^x t[Q_{lB}(y)]dy + \tau_3 \tag{A.15}$$

Substituting  $p(q_2(x)) = c_{elB}(x, \tau_3)$  into Equation (A.15) and differentiating it with respect to  $x$ , we have

$$-bq_2'(x) = t[Q_{lB}(x)] = f + w \left[ \int_x^B q_2(y)dy \right] \tag{A.16}$$

Differentiating Equation (A.16) with respect to  $x$  again yields

$$-bq_2''(x) = -wq_2(x), \quad x \in [l, B]. \tag{A.17}$$

The equilibrium number of trip generated at each location in section  $[l, B]$  is obtained by solving the differential Equation (A.17), as follows:

$$q_2^*(x) = \lambda_3 e^{\alpha x} + \lambda_4 e^{-\alpha x}, \quad x \in [l, B] \tag{A.18}$$

where  $\alpha = \sqrt{\frac{w}{b}}$ , and  $\lambda_3, \lambda_4$  are unknown constants to be determined by boundary conditions.

## APPENDIX B

In this appendix, we present the selected simulation results for the three schemes.

Scheme I: The simulation of Scheme I is to set  $\tau_1 = 0$ , and change  $\tau_2 = \tau_3$  from 0 to 20 to show the trend of impact on social welfare (Figure B1).

The road pricing impact on social welfare for case a

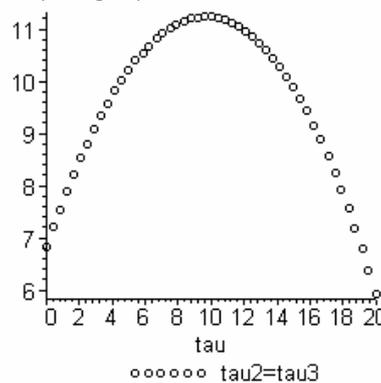


Figure B1. Social welfare simulation for Scheme I

Scheme II: We show three simulations for Scheme II. In these simulations, let  $\tau_1 = 0$ , and change  $\tau_2$  from 0 to 6 to show the trend of social welfare given a certain value for  $\tau_3$  in each simulation (Figure B2).

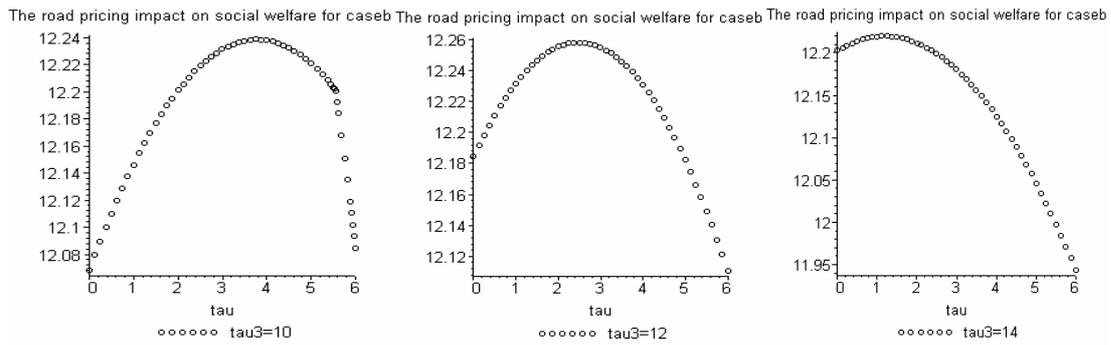


Figure B2. Three social welfare simulations for Scheme II

Scheme III: We show three simulations for Scheme III. In these simulations, let  $\tau_2 = 0$  and change  $\tau_1$  from 0 to 5 to show the trend of social welfare given a certain value for  $\tau_3$  in each simulation (Figure B3).

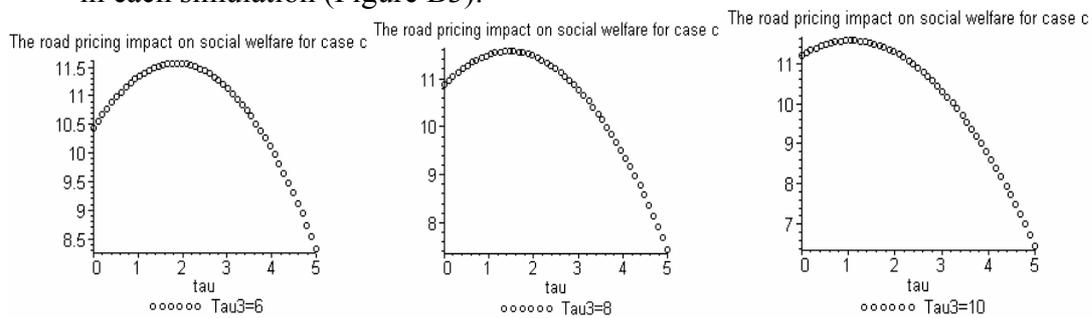


Figure B3. Three social welfare simulations for Scheme III