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MAXIMIZING CAR OWNERSHIP UNDER CONSTRAINTS OF ENVIRONMENT SUSTAINABILITY IN A CITY

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Abstract: This study aims to optimize road traffic flow and forecast the maximum car ownership accommodated in a city to satisfy environmental requirement and quantify road traffic capacity. A bi-level optimization model is established, where upper level is a maximum car ownership model, whose objective function is the total zonal car ownerships and the constraint is that traffic environment load on a link should not exceed the transportation environmental capacity, while lower level is a fixed demand user equilibrium assignment model, which simulates travelers' path choice behavior. To realize the feedback between the two levels and solve the optimization problems simultaneously, an optimal algorithm based on sensitivity analysis was developed, namely acquire derivative function of link volume and traffic demand with respect to zonal car ownership, and feedback the function into upper level program. Finally, we verify the bi-level model and the algorithm with a case study.

Key Words: Car Ownership, Bi-level Programming, Environment Capacity, and Sensitivity Analysis

1 BACKGROUND

Increment of car ownership promotes economic growth, intensifies urban spaces, and causes some urban problems. In Britain, at least three-quarter of CO, a quarter of CO_2 and more than a half of NO_x are induced by road traffic (D.Stead, 1999). In some main large cities of China, about 60% of CO, 50% of NO_x and 30% of HC are from vehicles emission. In a city, about 30.2% noise is from car traffic, while only 26.9% noise from industry (Li, et al, 2001). Emission and noise from car traffic have become one of the main pollution sources. It destroyed living and ecological condition of human beings, and became a restriction factor and a bottle-neck for sustainable development.

Urban transportation environment load is mainly affected by two factors, vehicle population and engine emission level. The amount of vehicle emission is affected by many factors, while most of them could be controlled in a certain degree. However, the number of cars determines the pollution level in a city objectively. Generally, each 1000 cars can emit 50~100kg nitrogen oxide and 200~400kg carbon hydride every day, besides carbon oxide, formaldehyde and granule substances etc. (Meng, 1994). As the increment of car ownership, road traffic environment load will be aggravated. Therefore, forecasting car ownership and its trend accurately in a city is important to institute transportation planning schemes and reduce urban transport environment pollutions.

In fact, vehicle population in a city is affected by car-buyers' subjective factors and other objective factors, its increasing range is not unlimited. When traffic environment load reaches the upper limit of the capacity, namely transportation environmental capacity, authorities would take some measures to improve urban environmental quality, such as control on car buying. Thus, environment capacity should be considered as one of the major factors for the capacity of transportation system and as a base for urban planners and managers to control transportation demand.

How to sever the traffic demand caused by the increasing car population and control car population within the permitted range is important to sustainable development of economy and society. This paper, from the view of environment management, attempts to predict the maximum car ownership accommodated in a city with a restriction of transport environmental capacity. A bi-level programming model is used to establish the forecasting model. The upper level is a maximum car ownership model, in which objective function is the total zonal car ownerships and the constraint is that traffic environment load on a link should not exceed the corresponding environment capacity. The lower level is a fixed demand user equilibrium assignment model, which optimizes travelers' path choice behavior. Moreover, we develop a sensitivity analysis based optimization algorithm to solve the proposed model, and verify it with the actual data of Dalian city.

2 LITERATURE REVIEWS

2.1 Bi-level Programming Models

Bi-level programming problem is a branch of multi-level programming problems, which reflects synchronously associated decision-making behaviors between upper and lower levels (Gao, 2000). Decision-makers in upper level often refer to government and other authorities. They will improve the service level of transportation infrastructures by means of adding roads and widening existed links to satisfy traffic demand. Users in lower level determine their own travel behaviors with the provided transportation infrastructures. Travelers can select different travel modes and determine whether to make a trip. Normally, upper and lower problems have different objective functions, but the decisive variables of two levels integrated each other.

During transportation planning, it is necessary to combine planning or optimization problems with traffic assignment model, thus the bi-level optimal problems are formed. Recently, bi-level programming is used in transportation fields more and more widely. Yang and Yagar

(Yang, 1995) resolved the traffic assignment and signal control problems in saturated road networks. In their research, the lower-level problem represents a fixed demand user equilibrium assignment model involving queuing and congestion. The upper-level is an optimization problem about signal control, taking account of drivers' route choice behavior in response to signal split changes. Shan and Gao (Gao, 2004), based on bi-level programming theories, present the transit equilibrium network design problems, in which the upper model is a normal transit network design problem, which minimizes total system impendence and total expenses caused by frequency settings. The lower level is a transit equilibrium assignment model, which present users' route choice behavior. In addition, Yang and Yager (1995), Wong and Yang (1997) predicted the O-D demand with bi-level programming models. Yang and Lam (1996), Yang and Bell (1997) studied on road pricing problems under the condition of highway congestion with bi-level programming models (Gao, 2000).

In addition, Tam and Lam (Tam *et al*, 2000), based on bi-level programming models, presented the relationship between road capacity and car ownership, in which the upper level is maximum car ownership model, and the lower level is an equilibrium trip distribution /assignment problem. Moreover, Tam and Lam (Tam *et al*, 2004), from the view of both travel demand and road network provision, presented the conception of equilibrium car ownership, and gave some analysis on Hong Kong as an example. These studies are bases for the use of bi-level programming in transportation fields.

At present, there are varieties of algorithms for bi-level programming problems, but most of them are designed to concrete ones, and lack generalizations. As the complexity of road network and the verbosity of data, the existed methods could not be used directly. Till the year 1988, Tobin and Friesz firstly applied the sensitivity analysis methods of variational inequalities on urban transportation network planning, and proposed the restricted sensitivity analysis method for urban transportation equilibrium network (Sheffid *et al*, 1988), and proved the consistency between the result from restricted equilibrium methods and original problems, which provide a base for the use of bi-level programming in transportation. Yang (Yang, 1997) proposed the sensitivity analysis based elastic-demand network equilibrium problem, which put forth a method to calculate the derivatives of the equilibrium link flows with respect to perturbation parameters, and gave a simple applications on road network design and congestion pricing.

2.2 Transportation Environmental Capacity and Road Traffic Capacity

Transport environmental capacity is defined as the ability of a city (region or country) to accommodate the traffic environment load in the condition of living condition, ecological condition and natural resource are not destroyed (Wang *et al*, 2003). As the share of pollutants from road traffic in the city increment, many researchers have proposed to set an upper limit for environment load for traffic sector, namely road traffic environmental capacity, which can be taken as a control indicator for traffic pollutant and will guide the city to establish an environment friendly transport system. Judging whether the traffic environment load is under its corresponding capacity can provide a scientific decision base for the authority to plan urban transport, environment and development.

However, environment system is very complicated since it involves various activities. The effects of different time, different place and different economic behaviors have different environment capacities, which make environment capacity quantification difficult. Kyoto Protocol has specified that the emitted pollutants of six kinds of greenhouse gases from all developed countries in 2010 should be reduced by 5.2% compared with the emission level in 1990. Although the regulated value is lack of theoretical base, this instruction provides an upper limit for environment capacity. Based on Kyoto Protocol, maximum pollutants emitted by a country is limited, and then every city also has a maximum indicator. However, how to decide this maximum capacity is out of this study, we just suppose there is an environment capacity in a city and the weight of traffic section is given. Thus traffic capacity in a city can be estimated. Our model takes the estimated environment capacity as a constraint to forecast the total zonal car ownership.

3 MAXIMUM CAR OWNERSHIP FORECAST MODEL

3.1 Model Structure

Here, the optimization problem is divided into two levels, where upper level is to maximize total zonal car ownership and lower level is to solve user equilibrium traffic assignment problem, the domain constraint is traffic environment capacity. The model structure in upper level is as follows:

$$Max: \sum_{i \in I} u_i \tag{1}$$

$$\text{S.T.} \quad \boldsymbol{D}(\boldsymbol{E_c}) + \boldsymbol{D}(\boldsymbol{E_b}) \le \boldsymbol{E_a}, \quad \boldsymbol{a} \in \boldsymbol{A} \tag{2}$$

$$\boldsymbol{E_b} = \boldsymbol{g_b}(\boldsymbol{v_b}) \tag{3}$$

$$\boldsymbol{E_c} = \boldsymbol{g_c}(\boldsymbol{v_c}) \tag{4}$$

$$\boldsymbol{u_i} \ge 0, \boldsymbol{i} \in \boldsymbol{I} \tag{5}$$

Here, $\boldsymbol{u_i}$ - car ownership of zone $\boldsymbol{i}$;

$\boldsymbol{E_c}$ - car emission on links;

$\boldsymbol{E_b}$ - bus emission on links;

$\boldsymbol{D_a}(\bullet)$ - diffusion function along links;

$\boldsymbol{E_a}$ - ceiling permitted pollutants concentration of link $\boldsymbol{a}$, namely the environment capacity of link $\boldsymbol{a}$, which is related to link characters;

$\boldsymbol{I}$ - the set of zones;

$\boldsymbol{A}$ - the set of links;

$\boldsymbol{v_b}$ - car flow (pcu/h) on link $\boldsymbol{a}$;

v_c - bus flow (pcu/h) on link a;

$g_b(v_b)$ - function of bus emission on link a;

$g_c(v_c)$ - function of car emission on link a;

As road traffic environment load is determined by road condition, traffic volume and flow characters, when taking the whole city as study area, the pollutants in (2), (3), and (4) can be calculated with data of road network configuration, assigned traffic flow and environment emission/diffusion models. Then, lower level optimization problem is developed as user equilibrium assignment problem, which is formulated as follows:

$$\text{Min}: \sum_a \int_0^{v_a} C_a(x)dx \tag{6}$$

$$\text{S.T.} \quad \sum f_r = t_{ij}, \quad i \in I, \quad j \in J \tag{7}$$

$$v_a = \sum_{r \in R} f_r \delta_{ar}, \quad a \in A \tag{8}$$

$$f_r, t_{ij} \geq 0, \quad r \in R, \quad i \in I, \quad j \in J \tag{9}$$

Here, C_a - free-flow travel time (h) of link $a \in A$;

f_r - the flow on path r;

t_{ij} - trips between O-D pair (i, j);

δ - associated matrix of link/path;

3.2 Modal Split Model and Environment Load Model

Assuming only car and bus modes are available (the case in most China's cities), then because modal splits of car and bus between zones are decided by the utilities of car trip and bus trip, and car ownership, we use formula (10), (11) to calculate the car trips and bus trips respectively.

$$t_{ij}^c = T_{ij} P_{cij} \omega_i / \beta_c, if \sum_i P_{cij} \leq \omega_i, or, t_{ij}^c = T_{ij} P_{cij} / \beta_c, if \sum_i P_{cij} > \omega_i \tag{10}$$

$$t_{ij}^b = T_{ij}(P_{bij} + P_{cij}(1-\omega_i)) / \beta_b, if \sum_i P_{cij} \leq \omega_i, or, t_{ij}^b = T_{ij} P_{bij} / \beta_b, if \sum_i P_{cij} > \omega_i \tag{11}$$

Here, t_{ij}^c - car trips between O-D pair (i, j);

t_{ij}^b - bus trips between O-D pair (i, j);

T_{ij} - traveler numbers between O-D pair (i, j);

P_c - probability of choosing car mode;

P_b - probability of choosing bus mode;

β_c - carrying ratio of cars;

β_b - carrying ratio of buses;

$\boldsymbol{\omega_i}$ - $\boldsymbol{u_i}$ / population of zone $\boldsymbol{i}$;

The lower level problem is a user equilibrium assignment model, shown as formula (6) ~ (9), while the upper level problem is a car ownership maximum model, which maximizes total zonal car ownership and satisfy the condition of road traffic environment capacity as formula (1) ~ (5). Assuming only car and bus modes are available, because car trips between zones is decided by cars ownership, modal split and occupancy ratio, we use formula (10), (11) to calculate the trips happened by cars and buses respectively. It should be noted that travelers carried by bus composes of two kinds of persons if ω_i is less than P_{cij}, one is those who intend to use bus, another is those who intend to use cars but they have not cars.

The model combines both private and public traffic modes, integrates car ownership with choice probability of modal split to calculate the traffic demand for every OD pair. As traffic environment load is determined by road condition, traffic flow and its characters, while the concentrations of pollutants in streets concerns humans seriously, then we will allocate the traffic environment capacity of the whole city onto links of road network as follows.

$$\boldsymbol{E_a} = \boldsymbol{E}_0\boldsymbol{W_a} \qquad (12)$$

$$\boldsymbol{W_a} = \boldsymbol{h}(\boldsymbol{S_a}, \boldsymbol{V_a}, \boldsymbol{\xi_a}, \boldsymbol{L_a}) \qquad (13)$$

Here, $\boldsymbol{E}_0$ - traffic environment capacity of the whole city;

$\boldsymbol{W_a}$ - weight of traffic environment capacity on a link;

$\boldsymbol{h}$ - function of link traffic environment capacity;

$\boldsymbol{S_a}$ - designed link capacity;

$\boldsymbol{V_a}$ - vertical curve degree of a link;

$\boldsymbol{\xi_a}$ - horizontal curve coefficient of a link;

$\boldsymbol{L_a}$ - depth of the street canyon along a link;

The weight of link traffic environment capacity is stated as the function of designed capacity, vertical curve, horizontal curve and depth of the street canyon. Basically it is thought that link with larger designed capacity and no-depth street canyon permits more emission. Vertical and horizontal curves affect the emission of traffic, thus undulate and zigzag links will cause more emissions than flat and direct ones.

3.3 Model Algorithm

The above bi-level optimization problem could be illustrated by a game theory between administrators and followers. First, administrators attempt to control road traffic environment load to satisfy the environment capacity, then they adopt various policies or measures to control traffic emission, as the result car ownership is controlled indirectly. However, the managers could not determine the travel demand and route choice behavior of lower users. Under the condition of certain road network and car ownership, users can choose whether to make a trip or which path to select, thus, an optimal trip distribution could be obtained.

Problem in lower level is a standard user equilibrium assignment model. With the convex combination methods, we can get link flow, path flow, vehicle densities and running speeds of all links. Then we calculate the pollutants with emission factors and the traffic characteristics to determine whether it is satisfy the constraints or not. In addition, with sensitivity analysis based optimization algorithm we get the derivatives of the equilibrium link flows and path flows with respect to zonal car ownership, and feedback the derivative value and transportation environment load data into upper model. Through solving the upper linear optimization problem, zonal car ownership with road environment capacity as a restraint can be got. Furthermore, new OD pairs can be calculated by car ownership data and are assigned on the road network. This iteration will be repeated until convergence. Then the maximum car ownership accommodated in a city limited by environment capacity is found.

During the iteration, calculation of derivation of upper level decision variable (zonal car ownership) to lower level decision variable (link flow) is the key problem, which could be calculated by variational inequality sensitivity analysis based algorithm. Sensitivity analysis means the solution changes caused by the changes of variables. The purpose to analyze urban transportation network equilibrium problems with sensitivity analysis method is to find changes of equilibrium link flow in response to perturbation parameters. Therefore, it is necessary to calculate the derivatives of equilibrium link flow with respect to perturbation parameters. As road network equilibrium problem can be formulated as a form of variational inequality, the variational inequality based sensitivity analysis method helps the calculation of derivatives. But because of the network complexity and data volume, traditional sensitivity analysis method could not apply directly here. Tobin and Friesz (1988) proposed a sensitivity analysis based method for equilibrium flows in the restricted road network, which provide a foundation to calculate the derivatives of equilibrium link flow with respect to perturbation parameters.

Lower level problem is a fixed demand user equilibrium assignment model, in which the decision variable is link flow; the upper level problem is a maximum car ownership model, in which the decision variable is zonal car ownership. To calculate the derivatives of lower decision variable with respect to upper decision variable, rewrite the lower level problem as the form of variational inequality, namely find the equilibrium link flow $\boldsymbol{v}^* \in \boldsymbol{\Omega}$, for each $\boldsymbol{v} \in \boldsymbol{\Omega}$, there is:

$$\boldsymbol{c}(\boldsymbol{v}^*)^{\boldsymbol{T}}(\boldsymbol{v}-\boldsymbol{v}^*) \geq 0 \tag{14}$$

$$\boldsymbol{\Omega} = \{\boldsymbol{v} \mid \boldsymbol{v} = \boldsymbol{\Delta f}, \boldsymbol{\Lambda f} = \boldsymbol{q}, \boldsymbol{f} \geq 0\} \tag{15}$$

$\boldsymbol{c}(\boldsymbol{v})$ is link impedance vector, $\boldsymbol{f}$ is path flow, $\boldsymbol{q}$ is O-D matrix, $\boldsymbol{\Delta}$ is link/path incidence matrix. The sufficient and necessary condition that variational inequality has solutions is:

$$\boldsymbol{c}'(\boldsymbol{v}^*) - \boldsymbol{\pi} - \boldsymbol{\Lambda}^{\boldsymbol{T}}\boldsymbol{\mu} = 0 \tag{16}$$

$$\boldsymbol{\pi}_{\boldsymbol{k}} \boldsymbol{f}_{\boldsymbol{k}}^{*\boldsymbol{ij}} = 0\,,\quad \boldsymbol{i} \in \boldsymbol{I}\,,\quad \boldsymbol{j} \in \boldsymbol{J}\,,\quad \boldsymbol{k} \in \boldsymbol{K}_{\boldsymbol{ij}} \tag{17}$$

$$\boldsymbol{\Lambda f}^{*} - \boldsymbol{q} = 0 \tag{18}$$

$$\boldsymbol{\pi} \geq 0 \tag{19}$$

Obviously, perturbation urban traffic equilibrium network problem can be changed to a variational inequality problem, namely find the equilibrium link flow $\boldsymbol{v}^{*} \in \boldsymbol{\Omega}$, for each $\boldsymbol{v} \in \boldsymbol{\Omega}$, there is:

$$\boldsymbol{c}(\boldsymbol{v}^{*}, \boldsymbol{\varepsilon})^{\boldsymbol{T}} (\boldsymbol{v} - \boldsymbol{v}^{*}) \geq 0 \tag{20}$$

$$\boldsymbol{\Omega}(\boldsymbol{\varepsilon}) = \{\boldsymbol{v} \mid \boldsymbol{v} = \boldsymbol{\Delta f}, \boldsymbol{\Lambda f} = \boldsymbol{q}(\boldsymbol{\varepsilon}), \boldsymbol{f} \geq 0\} \tag{21}$$

In the restricted transportation equilibrium network, path flow has to be positive, namely just paths with flows are considered. Because the derivatives of equilibrium link flow with respect to perturbation parameters is completely same as the result of origin problems (Tobin, 1988), the nonbinding constraints in matrix should be ridded in actual calculation process. Then, the form of simplified equations is given by:

$$\boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0) - \boldsymbol{\Lambda}^{0\boldsymbol{T}} \boldsymbol{\mu} = 0 \tag{22}$$

$$\boldsymbol{\Lambda}^{0} \boldsymbol{f}^{0*} - \boldsymbol{q}(0) = 0 \tag{23}$$

Here **0** represents the corresponding vectors or matrixes with decreased columns or rows. Meanwhile, the Jacobin Matrix of equation group with respect to $(\boldsymbol{f}^{0}, \boldsymbol{\mu})$ could be formulated:

$$\boldsymbol{J}_{\boldsymbol{f}^{0}, \boldsymbol{\mu}} = \begin{bmatrix} \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0) & -\boldsymbol{\Lambda}^{0\boldsymbol{T}} \\ \boldsymbol{\Lambda}^{0} & 0 \end{bmatrix} \tag{24}$$

Here, assume that

$$[\boldsymbol{J}_{\boldsymbol{f}^{0}, \boldsymbol{\mu}}]^{-1} = \begin{bmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} \\ \boldsymbol{B}_{21} & \boldsymbol{B}_{22} \end{bmatrix} \tag{25}$$

The following equation can be easily obtained

$$\boldsymbol{B}_{22} = [\boldsymbol{\Lambda}^{0} \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} \boldsymbol{\Lambda}^{0\boldsymbol{T}}]^{-1} \tag{26}$$

$$\boldsymbol{B}_{12} = \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} \boldsymbol{\Lambda}^{0\boldsymbol{T}} [\boldsymbol{\Lambda}^{0} \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} \boldsymbol{\Lambda}^{0\boldsymbol{T}}]^{-1} = \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} \boldsymbol{\Lambda}^{0\boldsymbol{T}} \boldsymbol{B}_{22} \tag{27}$$

$$\boldsymbol{B}_{21} = -[\Lambda^{0} \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} \Lambda^{0\boldsymbol{T}}]^{-1} \Lambda^{0} \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} = -\boldsymbol{B}_{22} \boldsymbol{\Lambda}^{0} \nabla_{\boldsymbol{f}^{0}} \boldsymbol{c}^{'0}(\boldsymbol{f}^{*}, 0)^{-1} \tag{28}$$

$$\boldsymbol{B}_{11} = \nabla_{\boldsymbol{f}^0}\boldsymbol{c}'^0(\boldsymbol{f}^*,0)^{-1}\{\boldsymbol{I} - \boldsymbol{\Lambda}^{0\boldsymbol{T}}[\boldsymbol{\Lambda}^0\nabla_{\boldsymbol{f}^0}\boldsymbol{c}'^0(\boldsymbol{f}^*,0)^{-1}\boldsymbol{\Lambda}^{0\boldsymbol{T}}]^{-1}\boldsymbol{\Lambda}^0\nabla_{\boldsymbol{f}^0}\boldsymbol{c}'^0(\boldsymbol{f}^*,0)^{-1}\}$$

$$= \nabla_{\boldsymbol{f}^0}\boldsymbol{c}'^0(\boldsymbol{f}^*,0)^{-1}[\boldsymbol{I} + \boldsymbol{\Lambda}^{0\boldsymbol{T}}\boldsymbol{B}_{21}] \tag{29}$$

In addition,

$$\begin{bmatrix} \nabla_{\varepsilon}\boldsymbol{f}^0 \\ \nabla_{\varepsilon}\boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} \\ \boldsymbol{B}_{21} & \boldsymbol{B}_{22} \end{bmatrix} \begin{bmatrix} -\nabla_{\varepsilon}\boldsymbol{c}'^0(\boldsymbol{f}^*,0) \\ \nabla_{\varepsilon}\boldsymbol{q}(0) \end{bmatrix} \tag{30}$$

From the incidence relations between link and path, we obtained:

$$\boldsymbol{v}^0 = \Delta^0\boldsymbol{f}^0 \tag{31}$$

$$\boldsymbol{c}'^0(\boldsymbol{f}^*,0) = \Delta^{0\boldsymbol{T}}\boldsymbol{c}^0(\boldsymbol{v}^*,0) \tag{32}$$

$$\nabla_{\varepsilon}\boldsymbol{v}^0 = \Delta^0\nabla_{\varepsilon}\boldsymbol{f}^0 \tag{33}$$

$$\nabla_{\varepsilon}\boldsymbol{c}'^0(\boldsymbol{f}^*,0) = \Delta^{0\boldsymbol{T}}\nabla_{\varepsilon}\boldsymbol{c}^0(\boldsymbol{v}^*,0) \tag{34}$$

$$\nabla_{\boldsymbol{f}^0}\boldsymbol{c}'^0(\boldsymbol{f}^*,0) = \Delta^{0\boldsymbol{T}}\nabla_{\boldsymbol{v}^0}\boldsymbol{c}^0(\boldsymbol{v}^*,0)\Delta^0 \tag{35}$$

$\boldsymbol{c}^0(\boldsymbol{v}^*,0)$ is the vector of restricted link impendence function, $\boldsymbol{\Delta}^0$ is the corresponding matrix $\boldsymbol{\Delta}$, which cut the columns or rows. Thus it can be seen that the derivatives of equilibrium link flow with respect to perturbation parameters can be calculated from formula $\boldsymbol{B}_{11}, \boldsymbol{B}_{12}$, (33) ~ (35) and (30). Here the perturbation parameter in upper level problem means car ownership.

In lower level problem, user trips satisfy UE rule, but are not restrained by other conditions. Consequently, when the increment of car ownership reaches certain degree, the traffic environment load on some links will exceed the corresponding capacity. Then we think the travel time on the link becomes infinite, and the environment load equals the corresponding capacity. When all link environment loads reach the limit, the maximum car ownership permitted by environment capacity could be obtained.

To calculate the maximum car ownership in upper level, it needs to put the derivative information of equilibrium link flow with respect to car ownership and environment load caused by traffic into upper level, and make upper level model be a linear optimization problem. Then the simplex method or any other optimization algorithm could be used to solve the linear programming problem to get urban maximum car ownership. The realization steps of above sensitivity analysis based algorithm could be formulated as follow:

Step 0: Determine a set of initial car ownership $\boldsymbol{u}^{(\boldsymbol{k})}$, modal splits $\boldsymbol{P}_{\boldsymbol{b}}^{(\boldsymbol{k})}, \boldsymbol{P}_{\boldsymbol{c}}^{(\boldsymbol{k})}$, occupancy ratio $\boldsymbol{\beta}_{\boldsymbol{b}}, \boldsymbol{\beta}_{\boldsymbol{c}}$, number of travelers between zones $\boldsymbol{T}_{\boldsymbol{ij}}$, etc, and set $\boldsymbol{k} = 0$;

Step 1: Calculate travel demand $\boldsymbol{t}_{\boldsymbol{b}}^{(\boldsymbol{k})}, \boldsymbol{t}_{\boldsymbol{c}}^{(\boldsymbol{k})}$ of car and bus respectively between zone $\boldsymbol{i}$, $\boldsymbol{j}$;

Step 2: Solve user equilibrium assignment problem in lower level to get link flow $\boldsymbol{v}^{(\boldsymbol{k})}$, and calculate the corresponding environment load;

Step 3: Calculate $\nabla_{\boldsymbol{u}}\boldsymbol{v}^{(\boldsymbol{k})}$, the derivatives of link flow with respect to car ownership by the sensitivity analysis based algorithm;

Step 4: Take derivatives information and traffic environment load into upper model to solve upper linear programming problem to obtain a group of new car ownership $\boldsymbol{u}^{(\boldsymbol{k}+1)}$;

Step 5: If $\left|\boldsymbol{u}_{\boldsymbol{i}}^{(\boldsymbol{k}+1)} - \boldsymbol{u}_{\boldsymbol{i}}^{(\boldsymbol{k})}\right| \le \boldsymbol{\omega}$ ($\forall \boldsymbol{i} \in \boldsymbol{I}$) stop calculation, otherwise, let $\boldsymbol{k} = \boldsymbol{k} + 1$, return to step 2.

4. CASE STUDY

Taking Dalian city as an example, we test our model. Dalian is a mountain city, the main trip modes are car and bus, there is almost no bicycle in use. Fig. 1 shows Dalian's road network and zonal central IDs. There are 33 zones, 895 links and 544 nodes. During the traffic assignment link impedance function adopts BPR function, the concrete form is,

$$\boldsymbol{c_a}(\boldsymbol{v_a}) = \boldsymbol{C_a}\left\{1.0 + 0.15\left(\frac{\boldsymbol{v_a}}{\boldsymbol{S_a}}\right)^4\right\}$$

Although many pollutants are emitted from road traffic, here $\boldsymbol{CO}$ is considered as one of the main pollution sources because of it big share. To simulate the above forecast model, only the emitted $\boldsymbol{CO}$ is taken into account. Emission factor of $\boldsymbol{CO}$ from existing study is used as follows (Yang, 2003):

$$\boldsymbol{R_a} = 11.14272\frac{\boldsymbol{e}^{0.047772\boldsymbol{h_a}}}{3280.8\boldsymbol{h_a}}$$

Here, $\boldsymbol{R_a}$ = emission factor on links (g/vehicle ·m), $\boldsymbol{h_a}$ = average speed on link $\boldsymbol{a}$.

Based on environment and spatial data of Dalian city, we validate the coefficients of formulas to calculate traffic environment capacity on links, which is given by

$$\boldsymbol{W_a} = 2.3\boldsymbol{S_a} + 0.51\boldsymbol{V_a} + 0.73\boldsymbol{\xi_a} + 1.13\boldsymbol{L_a}$$

We input all data of road network and zones into a GIS database, and match zonal central IDs with a node layer in road network. These data are essential for traffic assignment with Frank-Wolfe method. OD demand is fixed and modal split changes according to zonal car ownership. To calculate the derivative of link flow with respect to car ownership, it needs to deal with lot of link, node and zone data. Here, we represent the data with matrix style and integrate Visual Basic with Matlab to carry out matrix calculation. At last with MapBasic we illustrate the calculated results in GIS platform (MapInfo). During the calculation, it is found that after 1187 iterations, output got to convergence. Table 1 shows a part of the calculated results.

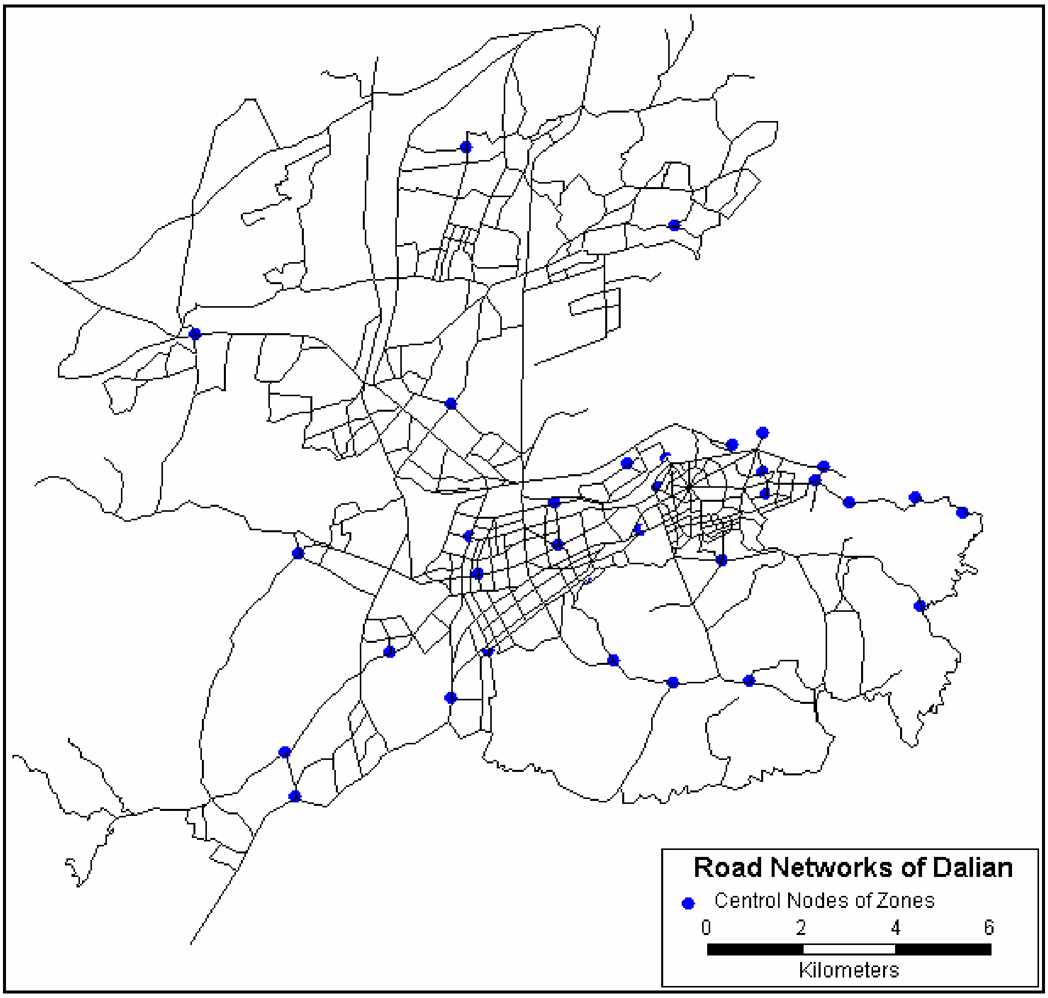

Fig. 1 Road Networks of Dalian

Table 1 Result of Zonal Car Ownership

Zone	Current Car Ownership	Zonal Population	Car Ownership in Iterations and Probability of Car Trips							
			5	P(5)	50	P(50)	200	P(200)	1187	P(1187)
1	2,103	50,460	2,269	0.0674	2,291	0.0681	2,844	0.0845	4,769	0.1418
2	1,428	34,273	1,646	0.0720	1,669	0.0730	2,184	0.0956	4,806	0.2103
3	1,031	24,752	1,259	0.0763	1,282	0.0777	1,858	0.1126	5,091	0.3085
4	4,205	100,919	4,538	0.0675	4,583	0.0681	5,687	0.0845	18,348	0.2727
5	2,063	49,504	2,518	0.0763	2,565	0.0777	3,716	0.1126	10,182	0.3085
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
31	1,365	32,768	2,134	0.0976	2,143	0.0980	2,387	0.1093	6,449	0.2952
32	2,731	65,535	4,267	0.0977	4,285	0.0981	4,774	0.1093	12,898	0.2952
33	2,856	68,545	3,292	0.0720	3,338	0.0730	4,368	0.0956	9,948	0.2177
Total	72,493	1,739,786	89,825	0.0774	90,771	0.0783	113,655	0.0980	296,412	0.2556

It can be seen from above results that with the increment of trip demands, urban car ownership increase gradually, which lead to the aggravation of road traffic environment pollution. When the emission pollutions on all links reach the control criterion of road traffic environment capacity, car ownership in a city reaches the maximum. With reference to this information, authority may take some management measures to control car increment and to improve urban environment and ecology, and then sustainable development may be realized.

5. SUMMARY

This study attempts to get to know the maximum car ownership of a city, which can satisfy the environment constraint, with bi-level programming model. By considering the requirement of sustainability and traffic demand, a bi-level model to forecast the maximum urban car ownership is developed. The upper problem is the car ownership model, which aims to maximum total of zonal car ownership under the condition of the environment load from traffic less than a certain level. Problem in the lower level is a user equilibrium assignment model, which simulate travelers' route choice behaviors and predict link traffic flows, cars densities and running velocities. Car ownership and transportation environment load are connecting variables between the two levels. In car ownership model, we calculated the modal splits between bus and car for zones, and get the car and bus trip OD for the whole city. Then traffic flows and traffic environment load on links can be calculated with output of assignment model. Since traffic environment load controls car numbers in the upper level, models in two levels depends each other.

To realize the feedback between the two levels and solve two optimization models, a sensitivity analysis based optimal algorithm is developed, in which we get the derivatives of link flow and road traffic demand with respect to zonal car ownership from lower level, and input this derivative data into the upper model. Through iterated feedback between the two layers, maximum car ownership subject to environment capacity is calculated. A case study is carried out for examining the model. The result shows that as the increment of trip demand and car ownership, environment load from car traffic aggravates continuously. When traffic environment loads on links reach their relevant ceiling limits, maximum car ownership is decided. Then based on the results, authorities can adopt some policies, regulations or measures to control car ownership and reduce pollutions from road traffic to realize a sustainable development.

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