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INTEGRATING THE INVENTORY MANAGEMENT AND VEHICLE ROUTING PROBLEMS FOR CONGESTED URBAN LOGISTICS NETWORK

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Abstract: Consider a congested urban logistics network with one depot and many geographically dispersed retailers facing demands at constant and deterministic rate over a period of planning horizon, but the lead time is variable due to traffic congestion. All stock enters the logistics network through the depot and from where it is distributed to the retailers by a fleet of vehicles. In this paper, we propose a new class of strategies for giving the optimal inventory replenishments for each retailer while the efficient delivery design is taken into account such that the minimization of total inventory cost and transportation cost is achieved. A mathematical program is formulated for this combined problem and a new class of iterative solution strategies is developed. Numerical computations are conducted and the proposed strategies obtain better results in comparison with other alternative with reasonable computational efforts.

Key Words: Inventory routing problem, Inventory allocation, Vehicle routing problem, Urban logistics network, Mathematical program

1. INTRODUCTION

Consider an urban logistics network, the role of logistics management is changing. Companies are recognizing that the value for customers can be realized through a kind of integrated service of effective logistics management and product availability. For such an integrated kind of service, inventory allocation and vehicle routing are two important and closely interrelated decisions that arise in logistics management contexts, which have been investigated extensively as the inventory routing problem. (Golden, B. et al., 1984, Federgruen and Zipkin, 1984, Chien, T. et al., 1989, Dror, M. et al., 1985, Dror and Ball, 1987, Anily and Federgruen, 1990, Bard, J. et al., 1998, Chan, L.M.A. et al., 1998, Campbell, A. et al., 1998 and Jaillet, P. et al., 2002).

The inventory routing problem (IRP) addresses the coordination of inventory replenishment and transportation. The IRP refers to the situation where the inventory replenishment at a number of locations is controlled by a central manager with a fleet of vehicles. A good distribution strategy is developed for which the minimization of distribution cost is achieved while the demands for retailers are satisfied without stock-out occurrence during the period of planning horizon. Regarding an urban logistics network, traffic congestion is often neglected in analyzing the replenishment policies for the IRP and therefore constant and static demand rates are assumed to hold for retailers during planning horizon. (Chien, T. et al., 1989, Dror, M. et al., 1985, Chan, L.M.A. et al., 1998 and Campbell, A. et al., 1998) However, as it has been noted by Campbell, A. et al. (1998) that travel times and the corresponding costs are severely affected by traffic conditions of logistics network and thus for the sake of realistic situations, more attentions need to focus on the variable nature of the transportation times when modelling the combined problem.

In this paper, consider an urban logistics network with one depot and many geographically dispersed retailers facing external demands at constant and deterministic rate over a period of planning horizon with variable lead-times, where all stock enters the logistics network through the depot and from where it is distributed to the retailers by a fleet of vehicles. In this urban logistics network, it is supposed that all stock is kept at the retailers and no stock is kept at the depot. We propose a new class of strategies for giving the optimal inventory replenishments for each retailer while the efficient delivery design is taken into account. The objective of the proposed new class of inventory replenishment strategies is to pursue the minimization of total inventory cost and transportation cost over a period of planning horizon while variable lead-times are taken into account. A mathematical program is formulated for this combined problem where total inventory replenishment cost is expressed as the sum of inventory holding costs and procurement costs for all retailers provided stock-out at any retailer is not allowed.

Regarding the inventory holding costs, contrary to conventional design, e.g. the Economic Order Quantity (EOQ), we consider the lead-time a mapping of the result of the vehicle routings, which is difficultly expressed as a closed form due to the NP hard nature of vehicle routing problems, and thus the objective function at retailers is to minimize the total inventory management costs with respect to inventory replenishment quantities and vehicle routings. For the combined inventory replenishment and vehicle routings problem, it becomes a non-convex problem in our mathematical programming formulation in the following ways.

Firstly, for the inventory cost function it is supposed that the lead-time of on-route orders is dependent on the results of vehicle routings and thus the lead-time demand is not a deterministic item as it appeared in the classical Economic Order Quantity (EOQ) model, but an implicit form of the results of vehicle routings. Secondly, regarding the computation for vehicle routings, the results are strongly influenced by the results of inventory replenishments once they have been determined by the mathematical model. A new class of iterative solution strategies is developed in simultaneously solving the combined problem. Numerical computations have been conducted on a series of experimental scenarios. In comparisons with the previous method, for example, separately solving the two independent problems of optimal inventory replenishments and vehicle routings, where the lead-times are regarded as fixed values, the proposed new class of solution strategies has obtained better results either in decreasing the transportation costs or in decreasing the inventory management costs over the period of planning horizon with reasonable computational efforts.

The remainder of this paper is organized as follows. In next section, relevant literature in the field of inventory allocation and vehicle routings is briefly reviewed. Mathematical programs for combined problem of the inventory routing problem with variable lead-time are formulated in section 3 together with the solution procedures. A new class of implementation heuristics for the inventory routing problem is also addressed in this section. In section 4, numerical computations for the proposed new class of solution strategies for the inventory routing problem are conducted at two randomly generated instances of the problem. Computational results for demonstrating the effectiveness of the proposed heuristics are given as well. Good results are obtained especially when comparing with other alternative to solving the inventory and routing problem, e.g. separately solving the inventory allocation and vehicle routing problem with fixed lead times. Conclusions and further research opportunities will be marked in section 5.

2. LITERATURE REVIEW

Golden, B. et al.(1984) are the first ones of those who investigated the interrelated problem of inventory allocation and vehicle routing problems. For an energy-products company that distributes liquid propane to its customers, Golden, B. et al. proposed a simulation model SNEW to determine the set of which customers should be serviced, the corresponding amount to supply the selected customers and the way in how to route the vehicles to deliver the allocated amounts. As it was reported, the simulation experiments showed good results with improvement of 8.4% in production, reduced the stock-out by 50% and total cost by 23%.

Federgruen and Zipkin (1984) approached the inventory routing problem as a special case of vehicle routing problem for a single day period. They considered stochastic demands and non-linear inventory costs, and suggested a non-linear integer programming formulation for the inventory and routing problem. The non-linear integer programs has the property that for any assignment of customers to routes, the problem is decomposed into an inventory allocation problem and a number of Traveling Salesman Problem (TSP). Starting with an initial inventory allocation, Federgruen and Zipkin iteratively applied interchange heuristics for constructing a better set of traveling salesman tour and an optimization procedure for improving the inventory allocation. The algorithm procedure proposed by Federgruen and Zipkin terminates when no more improvement in the total inventory and routing costs is possible. The results show that about 6-7% savings in operating costs can be achieved by using the combined approach for the inventory allocation and vehicle routings when compared to the conventional separately solutions.

Chien, T. et al. (1989) also developed a single day model of the inventory and routing problem and proposed a mixed integer programming model, which attempts to find a less myopic solution by passing inventory information from one day to the next. A Lagrangian based procedure was proposed to generate the upper bounds and lower bounds for the feasible solutions to the inventory and routing problem and good results have shown the effectiveness of the proposed procedure.

For the inventory allocation and vehicle routing problems over a long time period, Dror and Ball (1987) proposed an approach to take into account what happens after the single day planning period. Dror and Ball gave a reduced procedure for which the long-term effect of the problem can be brought into a short-term period such that long-term delivery cost is minimized while no customer runs out of stock at any time over the planning horizon of interest. Dror and Ball also applied the solution heuristic developed by Dror, M. et al. (1985) over the short-term period to the long-term one and numerical results have been reported. Anily and Federgruen (1990), on the other hand, took a look at minimizing long run average transportation and inventory costs by determining long-term routing patterns. Anily and Federgruen analyzed fixed partition policies for the inventory routing problem with constant deterministic demand rates and an unlimited number of vehicles. The routing patterns are determined by using a modified circular partition scheme. After the customers are partitioned, customers within a partition are distributed into regions so as to make the demand of each region. A customer may appear in more than one region, but a certain percent of customer’s demand is allocated to each region. When one customer in a region gets a visit, all customers in the region are visited. A lower bound for the long run average cost is also determined by which the performance of the determined routing patterns can be evaluated. Following the fixed partition policy of Anily and Federgruen, Chan, L.M.A. et al. (1998) continually analyzed the zero-inventory ordering policies for the inventory routing problem and derived asymptotic worst-case bounds on performance evaluation for setting the replenishment policies to minimize long-term average costs of the IRP.

Bertazzi, L. et al. (2002) considered a deterministic model of the inventory and routing problem with a single capacitated vehicle over long-term period. Each customer has a specified minimum and maximum inventory level. Bertazzi, L. et al presented a heuristic to determine the vehicle route at each discrete time point, while an order-up-to inventory policy is supposed to adopt. Various objective functions from different levels of decision makers are considered and numerical computations on a set of randomly generated problem instances have been conducted.

3. THE INVENTORY AND ROUTING PROBLEM FORMULATION

In this section, a mathematical program is given for the inventory routing problem with variable lead-times. First, notation used throughout this paper is given below.

3.1 Notation

K : number of vehicles.
n : number of locations, index from 1 to n ; index 0 denotes the central depot.
Q : total amount of product available at the central depot.
b_k : capacity of vehicle k .
A : ordering cost.
u_i : retailer i demand rate.
h : inventory carrying cost.
x_{ijk} : 1 if vehicle k travels directly from location i to j ; 0 otherwise.
y_{ik} : 1 if delivery point i assigned to route k ; 0 otherwise.
w_i : amount delivered to location i .
τ_i : lead time at location i .

3.2 Problem Formulation

Let α be a converting factor from Euclidean distance to monetary unit and let the inventory cost at retailer i, denoted by $q_i(w_i)$, the inventory and routing problem formulation can be addressed as follows.

IRP

$$\min_{x,y,w} \quad \alpha \sum_{i,j,k} c_{ij} x_{ijk} + \sum_i q_i(w_i) \tag{1}$$

$$\text{subject to} \quad w_i \geq u_i \tau_i(x, y), \quad i = 1,...n \tag{2}$$

$$\sum_{i=0}^{n} w_i y_{ik} \leq b_k, \qquad k = 1,..., K \tag{3}$$

$$\sum_{i=1}^{n} w_i \leq Q \tag{4}$$

$$\sum_{k=1}^{K} y_{ik} = \begin{cases} K, i = 0 \\ 1, i = 1,..., n \end{cases} \tag{5}$$

$$y_{ik} = 0, \quad or \quad 1, \quad i = 1,...n, \quad k = 1,..., K \tag{6}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk}, j = 0,..., n, \quad k = 1,..., K \tag{7}$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ik}, i = 0,..., n, \quad k = 1,..., K \tag{8}$$

$$\sum_{(i,j)\subset S\times S} x_{ijk} \leq |S| - 1, \quad S \subseteq \{1,..., n\}, \quad 2 \leq |S| \leq n - 1, \quad k = 1,..., K \tag{9}$$

$$x_{ijk} = 0, \quad or \quad 1, \quad i = 0,...n, \quad j = 0,..., n, \quad k = 1,..., K \tag{10}$$

$$\text{where} \quad q_i(w_i) = A\frac{u_i}{\tau_i} + \frac{h}{2} w_i \tag{11}$$

Consider a fixed route k, retailer i with $y_{ik} = 1$, let $Y_k = \{i : y_{ik} = 1\}$ the problem in (1)-(11) can be decomposed as the following two closely related problems: the inventory allocation problem and a number of traveling salesman problems (TSP). For the inventory allocation problem, it can be expressed as follows.

IA-1

$$\min_{w_i} \quad \sum_{i=1}^{n} q_i(w_i) = \sum_{i=1}^{n} A\frac{u_i}{w_i} + \frac{h}{2} w_i \tag{12}$$

$$\text{subject to} \quad w_i \geq u_i \tau_i(x, y), \quad i = 1,...n \tag{13}$$

$$\sum_{i=1}^{n} w_i \leq Q \tag{14}$$

$$\sum_{i \in Y_k} w_i \leq b_k, \qquad k = 1,...,K \tag{15}$$

Let $W_k = \sum_{i \in Y_k} w_i, \quad k = 1,...,K$ and let $Q_k(W_k)$ solve the following EOQ-like problem:

<u>*IA-2*</u>

$$\min_{w_i} \quad \sum_{i \in Y_k} q_i(w_i) = \sum_{i \in Y_k} A \frac{u_i}{w_i} + \frac{h}{2} w_i \tag{16}$$

$$\text{subject to} \quad w_i \geq u_i \tau_i(x, y), \ \ i \in Y_k \tag{17}$$

Therefore the inventory allocation problem in (12-15) can be re-written as follows.

<u>*IA-3*</u>

$$\min_{W_k} \quad \sum_{i=1}^{k} Q_k(W_k) \tag{18}$$

$$\text{subject to} \quad W_k \geq \sum_{i \in Y_k} u_i \tau_i(x, y) \tag{19}$$

$$\sum_{k=1}^{K} W_k \leq Q \tag{20}$$

$$W_k \leq b_k, \qquad k = 1,...,K \tag{21}$$

where $Y_k = \{i : y_{ik} = 1\}$. For a fixed Y_k, $k = 1,...,K$, consider x_k solves TSP(Y_k), which can be expressed as follows.

<u>*TSP*</u>

$$\min_{x^k} \quad \sum_{i,j} c_{ij} x_{ijk} \tag{22}$$

$$\text{subject to} \qquad \sum_{i=0}^{n} x_{ijk} = 1, \qquad j = 0,...,n \tag{23}$$

$$\sum_{j=0}^{n} x_{ijk} = 1, \quad i = 0,\ldots,n \tag{24}$$

$$\sum_{(i,j)\subset S\times S} x_{ijk} \le |S| - 1, \quad S \subseteq \{1,\ldots,n\}, \quad 2 \le |S| \le n-1 \tag{25}$$

$$x_{ijk} = 0, \quad or \quad 1, \quad i = 0,\ldots n, \quad j = 0,\ldots,n \tag{26}$$

In the inventory allocation problem (18-21), the lead time at retailer i, $\tau_i(x, y)$ is determined by solving the vehicle routing problem (VRP) given by w_i, which can be expressed as follows.

VRP

$$\min_{x,y} \quad \sum_{i,j,k} c_{ij} x_{ijk} \tag{27}$$

subject to

$$\sum_{i=0}^{n} w_i y_{ik} \le b_k, \quad k = 1,\ldots,K \tag{28}$$

$$\sum_{k=1}^{K} y_{ik} = \begin{cases} K, i = 0 \\ 1, i = 1,\ldots,n \end{cases} \tag{29}$$

$$y_{ik} = 0, \quad or \quad 1, \quad i = 1,\ldots n, \quad k = 1,\ldots,K \tag{30}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk}, j = 0,\ldots,n, \quad k = 1,\ldots,K \tag{31}$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ik}, i = 0,\ldots,n, \quad k = 1,\ldots,K \tag{32}$$

$$\sum_{(i,j)\subset S\times S} x_{ijk} \le |S| - 1, \quad S \subseteq \{1,\ldots,n\}, \quad 2 \le |S| \le n-1, \quad k = 1,\ldots,K \tag{33}$$

$$x_{ijk} = 0, \quad or \quad 1, \quad i = 0,\ldots n, \quad j = 0,\ldots,n, \quad k = 1,\ldots,K \tag{34}$$

For the inventory allocation problem IA-1, IA-2, IA-3, since the lead time τ is determined by the vehicle routing problem (27-34), which is implicitly expressed as a VRP solution and is not a closed form, which can be directly solved. For the traveling salesman problem and the vehicle routing problem, the inventory replenishment amounts, w, are not determined without solving the inventory allocation problems. It has been noted by Federgruen and Zipkin (1984), and Chien, T. et al. (1989), that exact solution for the inventory routing problem can be difficult to find due to the interrelationships between inventory replenishments and routing patterns. In the following section, we propose a new solution procedure to deal with such closely related problems and develop a new class of strategies with tractable computation efforts as demonstrated in the later sections.

3.3 Solution Procedure

Let $Y = \{Y_k, k = 1,..., K\}$, $X = \{X_k, k = 1,..., K\}, W = \{W_k, k = 1,..., K\}$ where $W_k = \sum_{i \in Y_k} w_i$, and

$\tau = \{\tau_k, k = 1,..., K\}$ where $\tau_k = \sum_{i \in Y_k} \tau_i$. The solution set of the *IA-3* problem is denoted by $I3(Y, \tau)$ and thus $W \in I3(Y, \tau)$, and solution set of *IA-2* problem is denoted by $I2(Y_k, \tau_k)$ and thus $W_k \in I2(Y_k, \tau_k)$. The solution set of *TSP* is denoted by $TSP(Y_k, W_k)$ and thus $X_k \in TSP(Y_k, W_k)$. The solution set of *VRP* is denoted by $VRP(W)$, and thus $Y \in VRP(W)$, and $\tau \in VRP(W)$. The proposed solution procedure for the variable lead-time inventory routing problem in (1-11) can be conducted as follows. Let superscript t denote the replenishment cycle index.

STEP0. Given a set of routing patterns, $Y^{(t)}$, and initial lead-times $\tau^{(t)}$. Set index $t = 0$.

STEP1. Solve the inventory allocation problem IA-3 in (18-21) and find the optimal inventory replenishment $W^{(t)}$, such that $W^{(t)} \in I3(Y^{(t)}, \tau^{(t)})$. Also solve the traveling salesman problem TSP in (22-26) and find the sequence of visiting orders to each retailer, $X_k^{(t)}$, on a given route k such that $X_k^{(t)} \in TSP(Y_k^{(t)}, W_k^{(t)})$, $k = 1,..., K$.

STEP2. Improve $X_k^{(t)}$, for $k = 1,..., K$ by TSP-MOD procedure.

STEP3. Solve the vehicle routing problem VRP in (27-34) and find a new set $Y^{(t+1)}$ such that $Y^{(t+1)} \in VRP(W^{(t)})$ via the VRP-COS procedure. Update the new lead-time set $\tau^{(t+1)}$ by multiplying the converting factor α such that $\tau^{(t+1)} \in VRP(W^{(t)})$. Set $t \leftarrow t + 1$.

STEP 4. Termination test. For a given value, T_{MAX}, if $t = T_{MAX}$ then stop; otherwise return STEP1.

3.4 Implementation heuristic

In this section, a new class of implementation heuristics for conducting STEPs 1-3 in the solution procedure is developed, for which a better mutually consistent solutions for problem IA-1 and VRP can be found in comparison with two individually separate solutions. Consider the inventory allocation problem IA-2 with zero lead-time, it becomes a classical type of Economic Ordering Quantity (EOQ) problem. Suppose in each period of inventory replenishment cycle, the inventory replenishment amount is determined by the lead-time demand where the variable lead-time is the result of vehicle routings from the previous replenishment cycles. For this implementation heuristic, we consider a smoothing forecast for current lead-time by conducting the moving average method of the previous lead times in the following way. For each retailer i, $i=1,...,n$, we have

$$w_i^{(t)} = u_i \tau_i^{(t)} \tag{35}$$

A smooth forecast of the lead-time, $\tau_i^{(t)}$, by conducting the moving average method over N replenishment cycles can be expressed as

$$\tau_i^{(t)} = \frac{1}{N}(\tau_i^{(t)} + \tau_i^{(t-1)} + ... + \tau_i^{(t-N+1)}), \qquad t \geq 1 \tag{36}$$

On the other hand, for the traveling salesman problem, a tour construction is considered by using the sweep method and the improvement heuristic, TSP-MOD, which is conducted as follows.

TSP-MOD

T-STEP1. For a given route k, construct an initial tour, X_k, by sweep method such that each retailer with replenishment $w_i^{(t)}$ within this tour is serviced and the corresponding route distance is minimized.

T-STEP2. Improve current tour X_k by interchange visiting retailers such that a lower route distance can be achieved by 2-opt or 3-opt procedure.

T-STEP3. Iterate T-STEPs 1-2 until no improvement is achieved.

For the vehicle routing problem, when taking into vehicle capacity into account, it can be regarded as multiple TSPs, which can be heuristically solved as follows.

VRP-COS

V-STEP1. Conduct TSP-MOD for each fixed route k.

V-STEP2. Check the feasibility of each routing k, $k=1,...,K$. If route k' violate the feasibility of VRP in (28-34), remove the visiting retailer by inventory replenishment in increasing orders until the feasibility is satisfied.

V-STEP3. Make new routes k'' to include the removed retailers and satisfy the feasibility test.

V-STEP4. Improve current routes by using the branch interchange technique conducted in the following composites.

V-COS1. Use 2-opt first to interchange visiting retailers within the same tour until no improvement in minimizing routing distance.

V-COS2. Use 3-opt secondly to interchange visiting retailers within the same tour until no improvement in minimizing routing distance.

V-COS3. Use 2-opt first to interchange visiting retailers across different tours until no improvement in minimizing routing distance.

V-COS4. Use 3-opt secondly to interchange visiting retailers across different tours until no improvement in minimizing routing distance.

V-STEP3. Iterate procedures for V-COS1-4 until no improvement is achieved.

4. NUMERICAL COMPUTATIONS

In this section, the proposed new class of implementation heuristics given in section 3 is conducted at two randomly generated instances of the inventory and routing problems. Consider the instances of interest, the retailers are scattered around the X-Y coordinates, where the integer points $X \in [-100,100]$ and $Y \in [-100,100]$. The daily demand rate, $u_i, i = 1,...,n$, is set as 5 items. The ordering cost, *A*, is set \$1200 and the inventory cost, *h*, set as \$1.0 per day per item. The converting factor from the Euclidean distance to monetary unit, α, is set \$150 per unit distance. For the first instance, the number of retailers is 12 and for the second instance, the number of retailers 30. The number of vehicles is 10 and capacity 3900 units. The depot capacity is set as 50000 units.

Regarding the time periods of planning horizon, 10 replenishment cycles are taken into account, i.e. $T_{MAX} = 10$. The performance indices are expressed as three kinds of costs, that is, the transportation cost evaluated from the vehicle routing problem, the inventory cost evaluated from the inventory allocation problem and the total cost, which is the sum of the transportation and inventory costs. Four kinds of stock policies are analyzed in this numerical experiment, which are generated from equations (35-36). Consider the conventional approach in solving the inventory allocation and vehicle problems, an EOQ-based stock policy is used where the lead-time is regarded as fixed and accordingly the inventory allocation and vehicle routing problems are separately solved iteratively until the termination condition holds.

Computational results are summarized in Tables 1-2. For the first instance, as it seen from Table 1, the proposed class of implementation heuristics, N=1,2,3,4, all outperformed the EOQ-based stock policy by yielding approximately 10% improvement in the transportation cost and in the inventory cost with 11.6% improvement. For the total cost of the sum of the transportation and inventory costs, the proposed class of implementation heuristics achieved nearly 12% improvement over that did the EOQ-based stock policy. For the second instance, n=30, as it seen from Table 2, again, the proposed class of implementation heuristics outperformed the conventional EOQ-based stock policy approximately 1.36% in the total cost, and achieved approximately 2% improvement in the inventory cost. However, as far as the transportation cost is concerned, in the second instance, the EOQ-based stock policy achieved slightly better performance by 0.17% than the proposed class of implementation heuristics, when N=3,4. Numerical experiments are conducted on Sun SPARC machine and coded by C++ computer language. Total computation times are within 1 minute of CPU time.

Table 1. Cost Comparison For Various Stock Policies When N=12

stock policy / Cost	EOQ-based	N=1	N=2	N=3	N=4
Transportation cost	226410	199991	202930	200905	200905
Inventory cost	386400	348188	336287	336691	335705
Total cost	612810	548179	539217	537596	536610

Table 2. Cost Comparison For Various Stock Policies When N=30

stock policy / Cost	EOQ-based	N=1	N=2	N=3	N=4
Transportation cost	299966	299689	299800	300477	300600
Inventory cost	820000	814384	809319	806753	804083
Total cost	1119966	1114073	1109119	1107230	1104683

5. CONCULSIONS AND DISCUSSIONS

In this paper, we considered a combined problem of the inventory allocation and vehicle routing problems for one depot and many geographically dispersed retailers when variable lead-times have been taken into account in congested urban logistics networks. Mathematical programs were given for this combined problem and the solution procedure was developed to heuristically solve this complicated problem due to the nature of vehicle routing problem and variable lead-time demands. Numerical experiments have been conducted at two randomly generated inventory routing problems. The proposed class of implementation heuristics was carried out where the proposed stock policies gave good results by yielding relatively lower inventory costs in comparison with another stock policy. Consider the variations of instances for the inventory routing problem, more computations are being undertaken in order to investigate the efficiency and robustness of the proposed solution procedure. On the other hand, consider the retail industry now facing demand chain management, investigations may continue to be undertaken for simultaneously taking both side’s benefits, i.e. the supply side and demand side, and discuss the appropriateness of cost minimization in our future work.

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REFERENCES

Anily, S and Federgruen, A. (1990) One warehouse multiple retailer system with vehicle routing costs, **Management Science**, **Vol. 36, No. 1**, 92-114.

Bard, J., Huang, L., Jaillet, P. and Dror, M. (1998) A decomposition approach to the inventory routing problem with satellite facilities, **Transportation Science**, **Vol. 32**, 189-203.

Bertazzi, L., Paletta, G. and Speranza, M.G. (2002) Deterministic order-up-to level policies in an inventory routing problem, **Transportation Science, Vol. 36, No. 1**, 119-132.

Campbell, A., Clarke, L., Kleywegt, A.J. and Savelsbergh, M.W.P. (1998) The inventory problem. In Crainic, T.G.. and Laporte, G.. (eds.), **Fleet Management and Logistics**, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Chan, L.M.A, Federgruen, A., and Simchi-Levi, D. (1998) Probabilistic analysis and practical algorithms for inventory-routing models, **Operations Research**, **Vol. 46**, 96-106.

Chien, T., Balakrishnan, A. and Wong, R.T. (1989) An integrated inventory allocation and vehicle routing problem, **Transportation Science, Vol. 23,** 67-76.

Dror, M., Ball, M. and Golden, B. (1985) A computational comparison of algorithms for the inventory routing problem, **Annals of Operations Research, Vol. 4**, 3-23.

Dror, M. and Ball, M. (1987) Inventory/routing: reduction from an annual to a short period problem. **Naval Research Logistics Quarterly, Vol. 34**, 891-905.

Federgruen, A. and Zipkin, P. (1984) A combined vehicle routing and inventory allocation problem, **Operations Research, Vol. 32,** 1019-1037.

Golden, B., Assad, A. and Dahl, R. (1984) Analysis of a large scale vehicle routing problem with an inventory component. **Large Scale Systems, Vol. 7**, 181-190.

Jaillet, P., Bard, J., Huang, L. and Dror, M. (2002) Delivery cost approximations for inventory routing problems in a rolling horizon framework. **Transportation Science, Vol. 36, No. 3**, 292-300.