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A SIGNAL OPTIMIZATION MODEL INTEGRATING PEDESTRIAN CROSSINGS TO TRAFFIC MOVEMENTS

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Abstract: We proposed two types of signal optimization models formulated in BMILP to integrate pedestrian crossings into traffic movements under under-saturated traffic flow. The model simultaneously optimizes traffic and pedestrian movements to minimize weighted queues of primary queues during red interval and secondary queues during queue clearance time. A set of linear objective function and constraints set up to ensure the conditions with respect to pedestrian and traffic maneuvers. Optimization results from numerical examples illustrated that pedestrian green intervals using proposed models are greater than those using TRANSYT-7F, but opposite in the ratios of pedestrian green intervals to the cycle length for single pedestrian. In double pedestrian crossings, the differences among the ratios for the proposed models are smaller than those for TRANSYT-7F. The simulation results show that proposed models are superior to TRANSYT-7F in reducing delay, where the longer the pedestrian green interval the greater the effect.

Key Words: single pedestrian crossing, double pedestrian crossing, traffic movement, primary queues, secondary queues

1. INTRODUCTION

Most traffic signal optimization models assume that green intervals for pedestrian crossings are given as exogenous inputs such as minimum green intervals for straight-ahead movements. Based upon the conventional models, traffic engineers usually start with grouping the intervals into traffic streams, irrespective of the traffic and geometric conditions, and then the signal settings are determined. As the result, in reality, the green interval of the straight-ahead movement may not distribute adequately by the volume/saturation-flow of each movement. For example, in the case that green interval for pedestrian crossing is much larger than that of the straight-ahead traffic movement, the actual volume/capacity (v/c) of the straight-ahead movement will be much smaller than that either of the case without pedestrian crossing or of any other movement. The cycle length, also, will be much larger than that of the case without pedestrian crossing.

Therefore, the conventional approach of pre-grouping green intervals for pedestrian crossings into traffic streams may not be adequate for some situations. Moreover the conventional approach is to design the set of grouping green intervals for pedestrian crossings into traffic streams on a trial and error basis. However, for some complicated intersections, it is difficult to come up with an optimal set of the two green intervals. Especially, it is much more difficult at an intersection with double pedestrian crossings, which have two pedestrian crossings at each arm, because green intervals for pedestrian crossings can be distributed to the green intervals of some sequential traffic movements. In this paper we proposed two types of signal optimization models to integrate pedestrian crossings into traffic movements in binary mixed integer linear programming (BMILP), which are solvable by any standard branch-and-bound routine.

2. MODEL FORMULATION

2.1 General Notation and Terminology

This model simultaneously optimizes both traffic and pedestrian movements. A movement is the finest element into which traffic or pedestrian is divided in the traffic signal design in this paper, and is defined as follows. For traffic maneuver, a movement is defined as movement i for i = 1, 2,…,12, U2, U4, U6, U8; and for pedestrian maneuver, each pedestrian crossing defines as movement i for i = P1, P2,…, P8 as the intersection with double pedestrian crossings shown in Figure 1. For single pedestrian crossing (see Figure 4 in section 4), double pedestrian crossings above are integrated into a pedestrian crossing at an arm, i.e. P1 and P2 into P12, P3 and P4 into P34, P5 and P6 into P56, and P7 and P8 into P78; movement i for i = P12, P34, P56, P78 are defined as single pedestrian crossing, respectively.

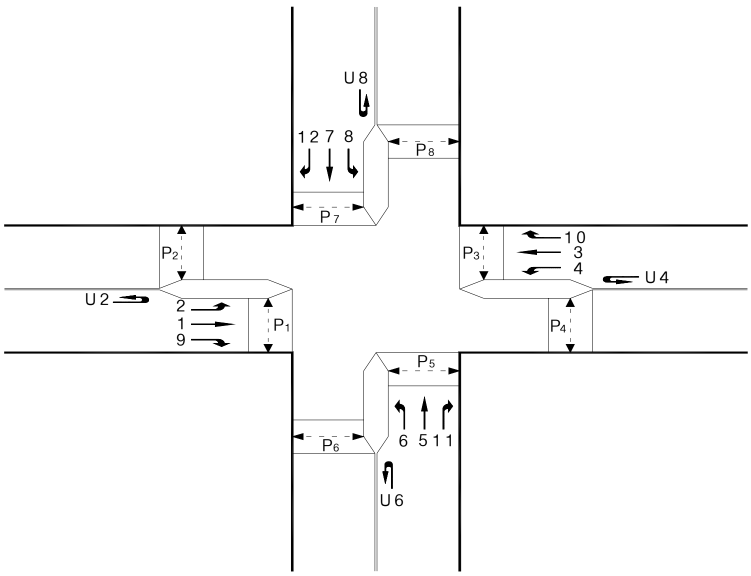


Figure 1. Movement Numbering Scheme at An Intersection with Double Pedestrian Crossings

One of compatible movements has right of way (green interval) irrespective of the green intervals for the others. Green intervals of incompatible movements, however, have to be separated each other. In other words, they cannot simultaneously have green intervals. Phase sequence depends upon the movements are whether compatible or not for pair of pedestrian crossings and traffic movements. Lane marking, whether exclusive or share lane marking for straight-ahead and left turn movement, is added to afore-mentioned requirements for pair of traffic movements. There is no any requirement to phase sequence for pair of single pedestrian crossings, but phase sequence for pair of double pedestrian crossings at an arm depends on letting pair of pedestrian movements whether simultaneously have identical green intervals or not. The potential compatible or incompatible relations for pair of all movements are illustrated in Table 1.

Table 1. Compatible or Incompatible Relations for Pair of All Movements

Move-ment		1	2	3	4	5	6	7	8	9	10	11	12	U2	U4	U6	U8	P12		P34		P56		P78	
																		P1	P2	P3	P4	P5	P6	P7	P8
1		X	X	X (o)	O	O	O	O	O	X	X	O	X	X	O	X	X	O	X	X	O	X	X	X	X
2		X	X	O	X	O	O	O	O	X	O	X	X	X	X	X	O	O	X	X	X	X	X	X	O
3		X (o)	O	X	X	O	O	O	O	X	X	X	O	O	X	X	X	X	O	O	X	X	X	X	X
4		O	X	X	X	O	O	O	O	O	X	X	X	X	X	O	X	X	X	O	X	X	O	X	X
5		O	O	O	O	X	X	X (o)	O	X	X	X	X	X	X	X	O	X	X	X	X	O	X	X	O
6		O	O	O	O	X	X	O	X	X	X	X	O	O	X	X	X	X	O	X	X	O	X	X	X
7		O	O	O	O	X (o)	O	X	X	O	X	X	X	X	X	O	X	X	X	X	X	X	O	O	X
8		O	O	O	O	O	X	X	X	X	X	O	X	X	O	X	X	X	X	X	O	X	X	O	X
9		X	X	X	O	X	X	O	X	X	X	X	X	X	X	O	X	X	X	X	X	X	O	X	X
10		X	O	X	X	O	X	X	X	X	X	X	X	X	X	X	O	X	X	X	X	X	X	X	O
11		O	X	X	X	X	X	X	O	X	X	X	X	X	O	X	X	X	X	X	O	X	X	X	X
12		X	X	O	X	X	O	X	X	X	X	X	X	O	X	X	X	X	O	X	X	X	X	X	X
U2		X	X	O	X	X	O	X	X	X	X	X	O	X	X	X	X	X	X	X	X	X	X	X	X
U4		O	X	X	X	X	X	X	O	X	X	O	X	X	X	X	X	X	X	X	X	X	X	X	X
U6		X	X	X	O	X	X	O	X	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
U8		X	O	X	X	O	X	X	X	X	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X
P 12	P1	O	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	P2	X	X	O	X	X	O	X	X	X	X	X	O	X	X	X	X	X	X	X	X	X	X	X	X
P 34	P3	X	X	O	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	P4	O	X	X	X	X	X	X	O	X	X	O	X	X	X	X	X	X	X	X	X	X	X	X	X
P 56	P5	X	X	X	X	O	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	P6	X	X	X	O	X	X	O	X	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
P 78	P7	X	X	X	X	X	X	O	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	P8	X	O	x	X	O	X	X	X	X	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Note: X and ○: compatibility and incompatibility for pair of movements, respectively;
( ): only for share lane for straight-ahead and left turn movement.

One characteristic of share lane depends upon how green interval is indicated to sharing movements. In order to avoid blocking each other, sharing movements should be a single stream and have identical signal indication introducing ( ) shown in Table 1. For pedestrian crossing, as afore-mentioned, Pij, ij = 12, 34, 56, 78, is defined as single pedestrian crossing and Pi, i = 1, 2,…,8, is defined as one of double pedestrian crossings at arm. Therefore Pij have to be subject to requirements for both Pi and Pj in Table 1. Double pedestrian crossings can or cannot have identical signal indications at each arm. If the double pedestrian crossings cannot have identical signal indications, they have to be incompatible each other, for example, ‘X’ for double pedestrian crossings of P1 and P2 have to be substituted by ‘O’ in Table 1.

2.2 Constraints

Traffic flows of all traffic movements are supposed to be under-saturated. Based on the conditions, the queues for all movements have to be cleared during green interval as shown in Figure 2. In the other words green interval for any movement has to be long enough to dissipate the queues of the movement. It is assumed that the departure traffic flow for movement i is described as two parts: s_i (veh/s) for saturation flow and q_i (veh/s) for arrival flow departure as shown in Figure 2.

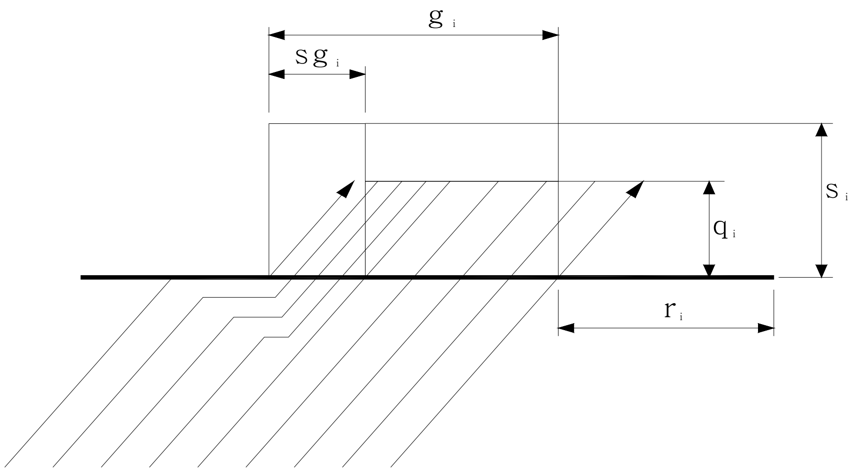


Figure 2. Departure Flows for Movement i at the Intersection

It is noticed that instead of using the cycle length as the decision variable, the reciprocal of cycle length $Z = 1/C$ is used in this model. The following variables are defined in the Figure 2, all of the time variables express as fraction of the cycle length, respectively:
g_i = ratio of green interval to the cycle length for traffic and pedestrian movement i, if the minimum duration of green for traffic or pedestrian crossing is denoted as $g_{\min,i}$ (s), then

$$g_i \geq Zg_{\min,i} \tag{1}$$

sg_i = ratio of green interval with saturation flow to the cycle length for traffic movement i (i.e. queue clearance time)

$$sg_i = \frac{q_i(1-g_i)+q_i sg_i}{n_i s_i} \tag{2}$$

where n_i = the number of lanes for traffic movement i

r_i = ratio of red interval to the cycle length for traffic and pedestrian movement i,

$$g_i + r_i = 1 . \tag{3}$$

Green interval for each traffic movement has to be long enough to dissipate the queues and arrivals of the movement. Based upon this constraint, green interval for movement i satisfy following constraint as

$$q_i \le n_i s_i sg_i + q_i (g_i - sg_i) \quad (4)$$

In addition to constraint above, we added another constraint to restrict the ratio of volume to capacity under 0.95 for movement i.

$$r_i \le 1 - \frac{q_i}{0.95 n_i s_i} \quad (5)$$

Effective green interval for pedestrian movement i required to clear an intersection crossing is computed according to following equation, which incorporates the effects of dispersion of platoons larger than 15 pedestrians.

$$g_i = 3.2 + \frac{L}{S_p} + (0.81 \frac{N_p}{W}) \text{ for } W > 3.0m, \quad (6)$$

$$g_i = 3.2 + \frac{L}{S_p} + (0.27 N_p) \text{ for } W \le 3.0m \quad (7)$$

where

L = the length of crossing distance (m)

S_p = average speed of pedestrians (m/s)

N_p = the number of pedestrians crossing during an interval (p)

W = the width of pedestrian crossing (m), and

3.2 = pedestrian start-up time (s)

For optimization of cycle length, let the minimum and maximum cycle lengths be $C_{\min}$ and $C_{\max}$, respectively. Instead of using the cycle length as the decision variable, the reciprocal of cycle length $Z = 1/C$ is used. The constraint on the cycle length can now be specified as

$$1/C_{\min} \le Z \le 1/C_{\max} \quad (8)$$

In order to guarentee feasible range of the cycle length, we let C be greater than or equal to the minimum cycle length (C_0) for undersaturated traffic flow and the range of the cycle length include the cycle length for minimizing delay (C_{web}) developed by WEBSTER (1966) as

$$C_0 = L/(1-Y) \quad (9)$$

$$C_{web} = (5 + 1.5L)/(1 - Y) \tag{10}$$

Green interval of each traffic movement or pedestrian crossing is specified by two variables: the start of green, ϕ_i, and green interval, g_i, for all traffic and pedestrian movements.
The signal phasing for movement i and j is modeled as equations for two pairs of clearance interval constraints of green interval suggested by Importa & Cantarella (1984) as follows:

$$\phi_i + g_i + ZY_{ij} \le \phi_j, \tag{11}$$

and

$$\phi_j + g_j + ZY_{ij} \le \phi_i + 1; \tag{12}$$

or

$$\phi_i + g_i + ZY_{ij} \le \phi_j + 1, \tag{13}$$

and

$$\phi_j + g_j + ZY_{ij} \le \phi_i, \tag{14}$$

where
Y_{ij} = the minimum clearance interval to be reserved between the end of green interval for movement i (j) and the start of green for movement j (i), in seconds; $Y_{ij} \ge 0$, if movement i and movement j are incompatible, otherwise, Y_{ij} is an arbitrary large negative value.

Since conditions of (11)-(14) are not linear forms, this problem should be transformed into following equivalent linear forms with a binary integer (α_{ij}) as follows:

$$\phi_i + g_i + ZY_{ij} - \alpha_{ij} \le \phi_j \tag{15}$$

and

$$\phi_j + g_j + ZY_{ij} + \alpha_{ij} \le \phi_i + 1 \tag{16}$$

where all traffic movements excepting both right turn and pedestrian crossings have to get one green interval located within the cycle length as

$$\phi_i + g_i \le 1$$

We set two types of models according to whether green intervals for straight-ahead movements meet those for pedestrian crossings or not. If green intervals for straight-ahead movements are subjected to those for pedestrian crossings, Model 1; otherwise, Model 2.

2.2 Objective Function

The traffic signal optimization problem contains many variables that in turn affect the system performance, individually or collectively. Since the delay is not linear form but strongly interrelated with queues, we introduced the minimization of the queues as the objective function. Under under-saturated traffic flow at the signalized intersection, the queues with respect to arrival traffic volume (vps), q_i, can be depicted as shown in Figure 3. Introducing the queues as the decision variables in the objective function, we classified the queues into two types of primary and secondary queues as shown in Figure 3, where the primary queues are queues occurring during red interval, $Q_{1,i}$, and the secondary queues during queue clearance time of green interval, $Q_{2,i}$, which can be set as

$$Q_{1,i} = q_i r_i \tag{17}$$

$$Q_{2,i} = \frac{q_i^2 r_i}{s_i - q_i} \tag{18}$$

Since the delay time for $Q_{1,i}$ is generally greater than that for $Q_{2,i}$, we introduced adjustment multiplier (K_i, $K_i \geq 1$) for favor of $Q_{1,i}$ to $Q_{2,i}$ in objective function as follows:

$$Minimize \quad \{K_i q_i + \frac{q_i^2 r_i}{(s_i - q_i)}\} r_i \tag{19}$$

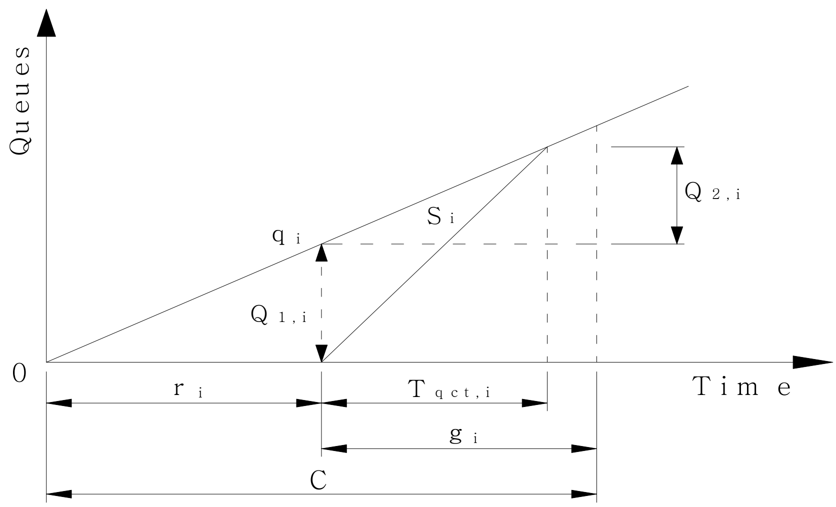


Figure 3. Queues for Under-saturated Traffic Flow at An Signalized Intersection

These problems mentioned are formulated as BMILP, which are solvable by any standard branch-and-bound routine. The binary integer variables are introduced to govern the order of signal displays, whereas the continuous variables (i.e., queues, cycle length, as well as signal timings including starts and green intervals) include the traffic movements and pedestrian crossings. Finally the signal timings are optimized to minimize sum of the weighted two types of queues in the BMILP, and the proposed model can therefore be formulated as the following BMILP:

Objective fuction: (19)

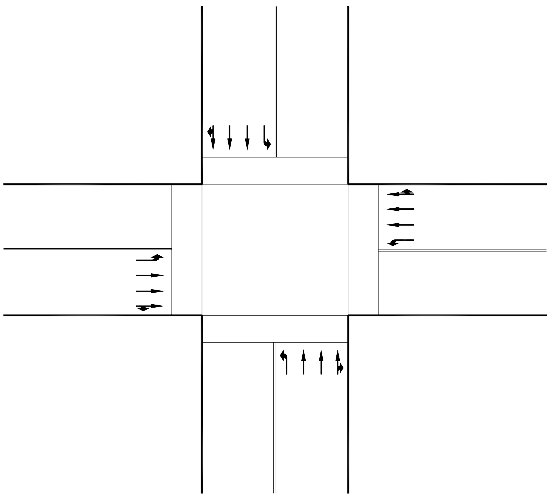
subject to constraints in (1) - (5), (8), (15), (16); one of (6) and (7).

4. APPLICATIONS AND EVALUATIONS OF MODEL

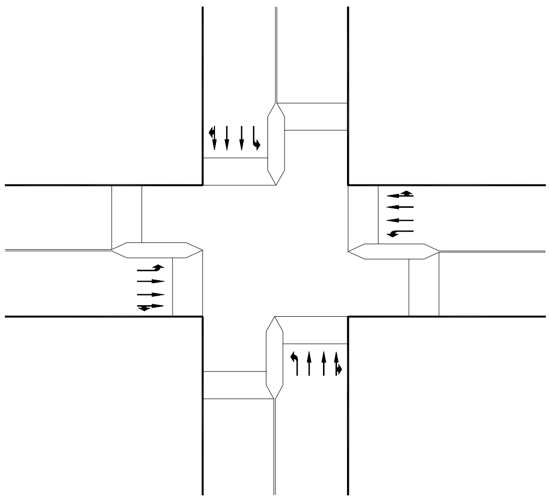
Consider a four-arm intersection with either single pedestrian crossing or double pedestrian crossings at each arm as shown in Figure 4. Each minimum green interval for double pedestrian crossings is regarded as 1/2 of that for single pedestrian crossing. The evaluations were performed for 4 cases:

- Case 1: 30 seconds (s) of green interval for single pedestrian crossing at each arm
- Case 2: 50 seconds (s) of green interval for single pedestrian crossing at each arm
- Case 3: 15 seconds (s) of green interval for double pedestrian crossings at each arm
- Case 4: 25 seconds (s) of green interval for double pedestrian crossings at each arm

We assumed some variables: traffic volume is 1800 pcu/h (80% for straight-ahead including right-turn, and 20% for left turn volume) at each arm, saturation flow rate 1800 pcu/h/lane for left turn traffic and 2000 pcu/h/lane for straight-ahead traffic, and minimum green intervals and yellow intervals 10 s and 3 s, respectively. Based on the conditions mentioned above, the LINDO package was employed to obtain solutions for the proposed models, and the TRANSYT-7F delay-based package for conventional model. TSIS microscopic simulation package was employed to evaluate these timing solutions.



For Case 1 and Case 2



For Case 3 and Case 4

Figure 4. Geometric Layout for Test Intersection

Table 2 – Table 5 illustrate the optimal signal timings for all cases of studies. For single pedestrian crossing, green intervals for pedestrian are equal to those for compatible straight-ahead traffic movements in TRANSYT-7F and Model 1, but the conditions are vanished in Model 2. For double pedestrian crossings, pedestrian green intervals are equal to or consecutively share those for compatible traffic movements, respectively.

Cycle lengths of Case 2 and Case 4 are longer than those of Case 1 and Case 3, respectively.

It can be seen that the longer the green interval for pedestrian crossing, the longer the cycle length under the identical pedestrian crossing type, which whether single or double pedestrian crossing. In single pedestrian crossings, pedestrian green intervals of the proposed models are longer than those of TRANSYT-7F but the ratios of pedestrian green intervals to cycle length for the proposed models are less than those for TRANSYT-7F. In double pedestrian crossings that some green intervals for pedestrian crossing have right of way every phase, the differences among the ratios of pedestrian green intervals to cycle length for the proposed models are smaller than those for TRANSYT-7F.

Table 2. Optimized Signal Timings for Case 1 (s)

Model		Phase1	Phase2	Phase3	Phase4	Cycle
TRANSYT	Phase					
	Time	24 (22)	31 (28)	24 (22)	31 (28)	110 (100)
Model 1	Phase					
	Time	27 (22.5)	33 (27.5)	27 (22.5)	33 (27.5)	120 (100)
Model 2	Phase					
	Time	27 (22.5)	33 (27.5)	27 (22.5)	33 (27.5)	120 (100)

Note: ( ) = % of green interval to cycle length

The proposed models produce same phase sequences as those of TRANSYT-7F for Case 1 and Case 2 (i.e. single pedestrian crossing), but do not for Case 3 and Case 4 (i.e. double pedestrian crossings). Phase sequences of the two proposed models are same for single pedestrian crossings but not for double pedestrian crossings.

Table 3. Optimized Signal Timings for Case 2 (s)

Model		Phase1	Phase2	Phase3	Phase4	Cycle
TRANSYT	Phase					
	Time	15 (12)	50 (38)	14 (11)	51 (39)	130 (100)
Model 1	Phase					
	Time	40 (22)	53 (28)	40 (22)	53 (28)	186 (100)
Model 2	Phase					
	Time	40 (22)	53 (28)	40 (22)	53 (28)	186 (100)

Note: ( ) = % of green interval to cycle length

Table 4. Optimized Signal Timings for Case 3 (s)

Model		Phase1	Phase2	Phase3	Phase4	Cycle
TRANSYT	Phase					
	Time	24 (22)	31 (28)	24 (22)	31 (28)	110 (100)
Model 1	Phase					
	Time	23 (23)	27 (27)	27 (27)	23 (23)	100 (100)
Model 2	Phase					
	Time	27 (27)	23 (23)	23 (27)	27 (23)	100 (100)

Note: ( ) = % of green interval to cycle length

Table 5. Optimized Signal Timings for Case 4 (s)

Model		Phase1	Phase2	Phase3	Phase4	Cycle
TRANSYT	Phase					
	Time	32 (23)	39 (28)	30 (21)	39 (28)	140 (100)
Model 1	Phase					
	Time	32 (27)	32 (27)	28 (23)	28 (23)	120 (100)
Model 2	Phase					
	Time	23 (23)	27 (27)	23 (23)	27 (27)	100 (100)

Note: ( ) = % of green interval to cycle length

Table 6 illustrates simulation results for Case 1 and Case 3 using TSIS package. It shows that two proposed models reduced delay times than those for conventional model using TRANSYT-7F, where the effect of single pedestrian crossing (Case 1) was much greater than that of double pedestrian crossing (Case 2). For the two models, Model 1 and Model 2 produce same results.

Table 6. Delay Times of Case 1 and Case 3 for the shorter pedestrian green interval (s/veh)

Case	TRANSYT (A)	Model 1 (B)	Model 2 (C)	A-B	B-C
Case 1	186.7	51.2	51.2	135.6	0.0
Case 3	47.4	46.5	46.5	0.9	0.0

Table 7 illustrates simulation results for Case 2 and Case 4. It can be seen that two proposed models reduce delay times than those for conventional model using TRANSYT-7F similarly to Table 1, where the effect of single pedestrian crossing (Case 2) was much greater than that of double pedestrian crossings (Case 4). For the two models, Model 1 and Model 2 produced same result for single pedestrian crossing (Case 2) but Model 2 is superior to Model 1 for double pedestrian crossings (Case 4).

Based on Table 6 and Table 7, the proposed models decrease the delay for Table 7 more than that for Table 6. It can be seen that the longer the pedestrian green interval the greater the effect of reducing the delay in any pedestrian crossing. The delays of Table 6 for the shorter pedestrian crossing intervals are less than those of Table 7 for the longer pedestrian crossing intervals.

Table 7. Delay Times of Case 2 and Case 4 for the longer pedestrian green interval (s/veh)

Case	TRANSYT (A)	Model 1 (B)	Model 2 (C)	A-B	B-C
Case 2	216.6	72.1	72.1	144.5	0.0
Case 4	56.1	54.1	48.0	2.0	8.2

5. CONCLUSIONS AND RECOMMANDATIONS

In this paper we proposed two types of signal optimization models formulated in BMILP to integrate pedestrian crossings into traffic movements under under-saturated traffic flow. In this model, both traffic and pedestrian movements are simultaneously optimized to minimize the weighted two types of queues: which are primary queues during red interval and secondary queues during queue clearance time. We assess higher priority to primary queues than to secondary queues for reflecting actual delay time. A set of linear constraints has been set up to ensure the conditions mentioned above.

Numerical examples are given to demonstrate the effectiveness of the proposed models. The examples are classified into 4 cases by minimum green intervals for pedestrian crossings and the number (1 or 2) of pedestrian crossings located at an arm. Optimization results illustrated that pedestrian green intervals using proposed models are greater than those using TRANSYT-7F but opposite for the ratios of pedestrian green intervals to the cycle lengths for single pedestrian crossing. In the case of double pedestrian crossings that some green intervals for pedestrian crossing have right of way every phase, the differences among the ratios of pedestrian green intervals to cycle length for the proposed models are smaller than those for TRANSYT-7F.

The simulation results show that proposed models are superior to conventional model using TRANSYT-7F in reducing delay time, where the longer the pedestrian green interval the greater the effect. For the two models, Model 1 is superior to Model 2 for double pedestrian crossings that need longer pedestrian green intervals.

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