

A METHODOLOGY FOR SELECTING PARETO OPTIMAL SOLUTIONS DEVELOPED BY A MULTI-OBJECTIVE AVI READER LOCATION MODEL

Surachet PRAVINVONGVUTH
Ph.D. Student Researcher
Civil & Environmental Engineering
Utah State University
Logan UT 84322-4110, USA
Tel: +1-435-797-2927
Fax: +1-435-797-1185
Email: surachet@cc.usu.edu

Piya CHOOTINAN
Ph.D. Student Researcher
Civil & Environmental Engineering
Utah State University
Logan, UT 84322-4110, USA
Tel: +1-435-797-2927
Fax: +1-435-797-1185
E-mail: piyachoo@cc.usu.edu

Anthony CHEN
Associate Professor
Civil & Environmental Engineering
Utah State University
Logan, UT 84322-4110, USA
Tel: +1-435-797-7109
Fax: +1-435-797-1185
E-mail: achen@cc.usu.edu

Sorawit NARUPITI
Associate Professor
Civil Engineering
Chulalongkorn University
Bangkok, 10330, Thailand
Tel: +66-2218-6460
Fax: +66-2251-7304
E-mail: kong@chula.ac.th

Abstract: In this paper, the methodology for selecting the most preferred solution from a set of non-dominated solutions developed by a multi-objective automatic vehicle identification (AVI) reader location model is proposed. Several criteria, which have not been considered in the multi-objective mathematical formulation, are incorporated in the selection method to differentiate the non-dominated solutions. Not only does the method assist the decision makers in dealing with this unmanageable set of solutions, it is also capable of taking into account the practicality issues of solution, which cannot easily be addressed in the mathematical model. The proposed method is applied to select the most preferred configuration of an AVI system for the simplified Irvine network in Orange County, California as a case study.

Key Words: automatic vehicle identification, location problem, multi-objective, multi-criterion, simple multi-attribute rating technique

1. INTRODUCTION

Automatic Vehicle Identification (AVI) has been considered as one of the emerging technologies used for collecting travel time information in a transportation network. A typical AVI system consists of AVI readers placed at various locations in the network, the transponders (or AVI tags) placed in the individual probe vehicles, and a central computer system. The basic idea of an AVI system is to match the unique identification (ID) numbers of individual probe vehicles when they pass through the AVI readers. Based on these AVI readings, various types of travel information can be processed and collected. In order to configure the AVI reader system (i.e., determine the number of AVI readers and their locations) that will benefit several applications (e.g., travel time estimation, origin-destination (O-D) estimation, incident detection, etc.), several criteria, usually conflicting, must

simultaneously be taken into account. Teodorovic, D. *et al.* (2002) is one of the few research groups studying the AVI reader location problem as a multi-objective optimization problem. Specifically, they determined the locations of a given set of AVI readers to maximize both the number of O-D pairs covered and the number of readings along the shortest path between each O-D pair. Chen, A. *et al.* (2004a) extended the model of Teodorovic, D. *et al.* (2002) to consider the cost of the AVI system as another objective and also relaxed the definition of O-D coverage to include not only the shortest path, but also all possible paths between an O-D pair. Recently, Pravinongvuth, S. *et al.* (2005) further redefined the definition of O-D coverage for the purpose of travel time collection and proposed a robust model to account for different demand patterns. In the later two studies, the multi-objective AVI location problems were solved by an evolutionary algorithm with an embedded shortest path algorithm (Chen, A. *et al.*, 2004b). The advantage of using the evolutionary computation approach is that it can generate a set of non-dominated solutions within a single run. Nevertheless, it oftentimes leaves the decision makers with the difficult task in selecting only one alternative to be implemented from this unmanageable set of non-dominated solutions.

In a broader perspective, multi-objective problems can be tackled by two different approaches, which are the multi-objective programming (MOP) and the multiple criteria decision making (MCDM) (Cohon and Rothley, 1997). The difference between these two approaches is due to the nature of problem in which, whether or not, alternatives are predefined. All of the prior works on the multi-objective AVI location problems discussed above are considered as the MOP approach, which mathematically determines the optimal solution (i.e., the number and locations of AVI readers). MCDM, on the other hand, is the process of selecting the most preferred alternative from the predefined set of alternatives. There are some studies outside transportation area (Rothley, 1999; Memtsas, 2003) that integrate these two approaches by using MCDM as an evaluation tool for selecting the most preferred solution from the solutions developed by MOP.

In this study, we apply the simple multi-attribute rating technique (SMART) (Edwards, 1971), one of the MCDM methods, to select the most preferred configuration of AVI reader system from a set of non-dominated solutions obtained by solving the multi-objective AVI reader location model of Pravinongvuth, S. *et al.* (2005). Evaluation criteria are developed and applied to the case study of the simplified Irvine network in Orange County, California. The selection methodology not only assists the decision makers in selecting the most preferred solution, it is also capable of taking into account the practicality issues of solution, which, most of the times, are difficult to model in the mathematical formulation.

2. MULTI-OBJECTIVE AVI READER LOCATION MODEL

The multi-objective AVI reader location model proposed by Pravinongvuth, S. *et al.* (2005) is used in this study. It can be considered as a MOP approach to determine the optimal configuration of AVI reader system that simultaneously optimizes three conflicting objectives. The notation and formulation used in this study are listed below.

- A** : Set of links in the network and $|A|$ is size of set **A**;
- W** : Set of O-D pairs and $|W|$ is size of set **W**;
- R_w** : Set of paths between O-D pair w ;
- ρ : Number of AVI readers allowed to install on each link;

- δ_{ar}^w : A path-link indicator denoting 1 if link a is on path r between O-D pair w , and 0 otherwise;
- γ_r^w : A path coverage indicator denoting 1 if both the first and the last link of path r between O-D pair w are equipped with AVI readers, and 0 otherwise;
- h_r^w : Flows on path r between O-D pair w ;
- x_a : An integer decision variable indicating the number of AVI readers to be installed on link $a \{0,1, \dots, \rho\}$;
- \mathbf{x} : A vector form of decision variable x_a .

$$\text{Maximize: } g(\mathbf{x}) = [-f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})]^T \tag{1}$$

Subject to:

$$x_a \in \{0,1, \dots, \rho\}, \quad \forall a \in \mathbf{A}, \tag{2}$$

where

$$f_1(\mathbf{x}) = \sum_{a=1}^{|\mathbf{A}|} x_a, \tag{3}$$

$$f_2(\mathbf{x}) = \sum_{w=1}^{|\mathbf{W}|} \min \left\{ \max \left\{ 0, \min_{\forall r \in R_w} \left\{ \sum_{a=1}^{|\mathbf{A}|} \delta_{ra}^w \gamma_r^w x_a - 1 \right\} \right\}, 1 \right\}, \tag{4}$$

$$f_3(\mathbf{x}) = \sum_{w=1}^{|\mathbf{W}|} \sum_{r=1}^{|\mathbf{R}_w|} h_r^w \max \left\{ 0, \sum_{a=1}^{|\mathbf{A}|} \delta_{ra}^w x_a - 1 \right\}. \tag{5}$$

Equation 1 presents a vector of objective functions to be optimized. It consists of three objectives given in equations 3 through 5. The first objective has a negative sign in order to convert the minimization problem to the maximization problem. Equation 2 represents the decision variable x_a indicating the reader location. The first objective as shown in equation 3 is to minimize the number of AVI readers. Equation 4 is to maximize the number of O-D pairs to be covered. An O-D pair w is covered only if all relevant paths between this O-D pair are covered. A relevant path r is covered only if at least two readers are installed along the paths and these two readers must be located on the first link and the last link of each path. The last objective in equation 5 is to maximize the number of readings. At the minimum, there must be at least two AVI readers on a path in order to have a reading. It is important to point out that a reading can be occurred even though a path is not covered. For example, if a path passing through two readers, it will definitely contribute one reading to objective 3 in equation 5 but the corresponding O-D pair (objective 2 in equation 4) may or may not be covered depending on the locations of these two readers. Solving this formulation gives a set of non-dominated solutions.

3. SELECTION METHODOLOGY

The result of solving the multi-objective AVI reader location model is a set of non-dominated solutions. Each solution is considered to be non-dominated because none of these solutions is better than others in all objectives. Even though it is beneficial to obtain a variety of solutions for the decision-making process, it is difficult for the decision maker to select the solution for implementation. Figure 1 presents the selection methodology proposed in this study.

According to Figure 1, the decision-maker has to develop evaluation criteria to differentiate among the non-dominated solutions. Afterward, the priorities of individual criteria must be established in order to compile the different units of the criteria into a single score for each solution. Last but not least, the most appropriate alternative is selected for detailed analysis. Each module is explained in detail as follows.

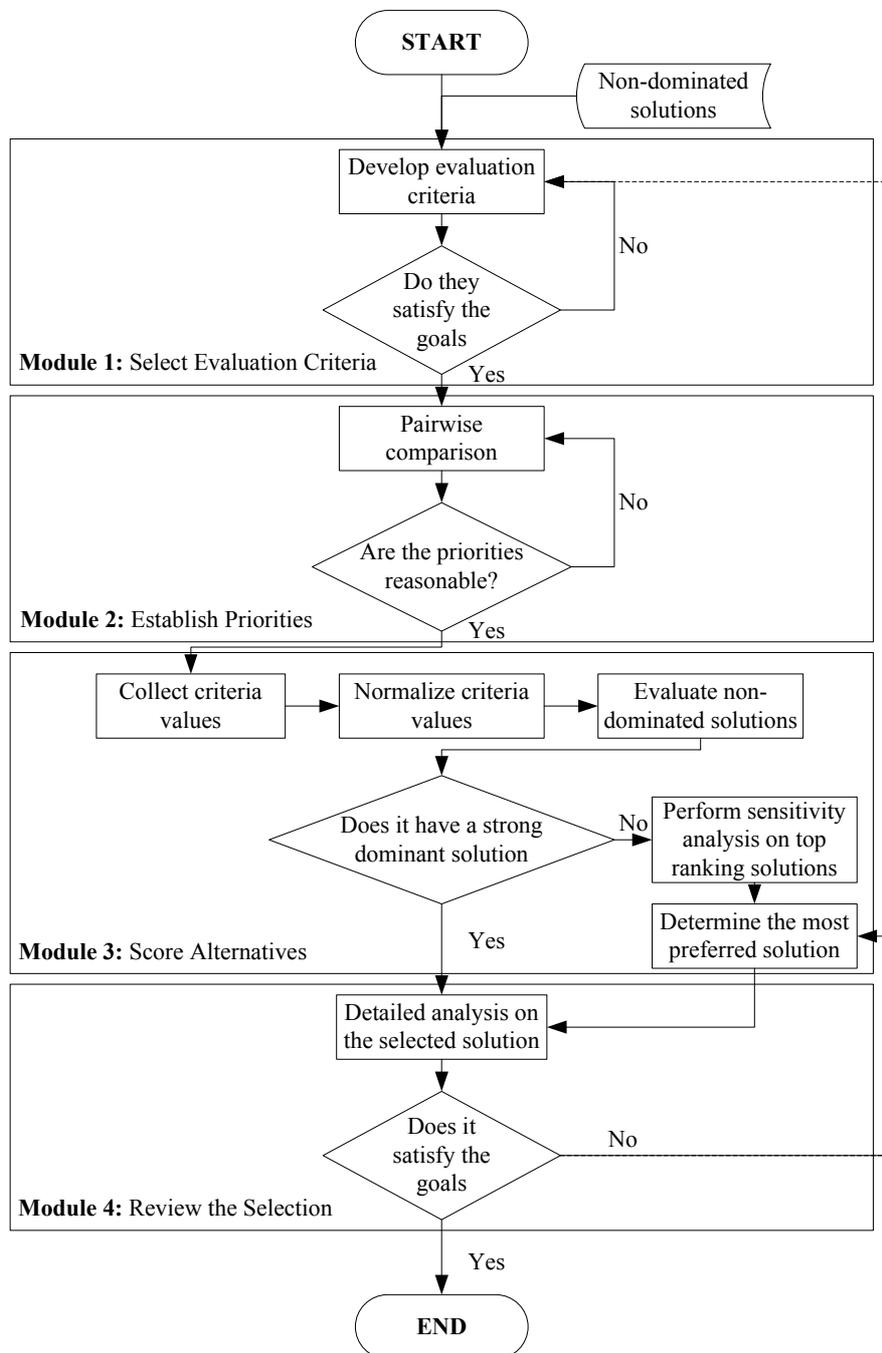


Figure 1. Selection Methodology

3.1 Module 1: Select Evaluation Criteria

In this module, the decision-maker has to construct a hierarchy tree consisting of all relevant criteria. Figure 2 shows the hierarchy tree used in this study, which includes criteria relating to the cost and benefit of the AVI system. There is only one criterion on the cost side, which is related to the number of AVI readers, while there are five criteria on the benefit side. Each criterion is discussed in detail as follows.

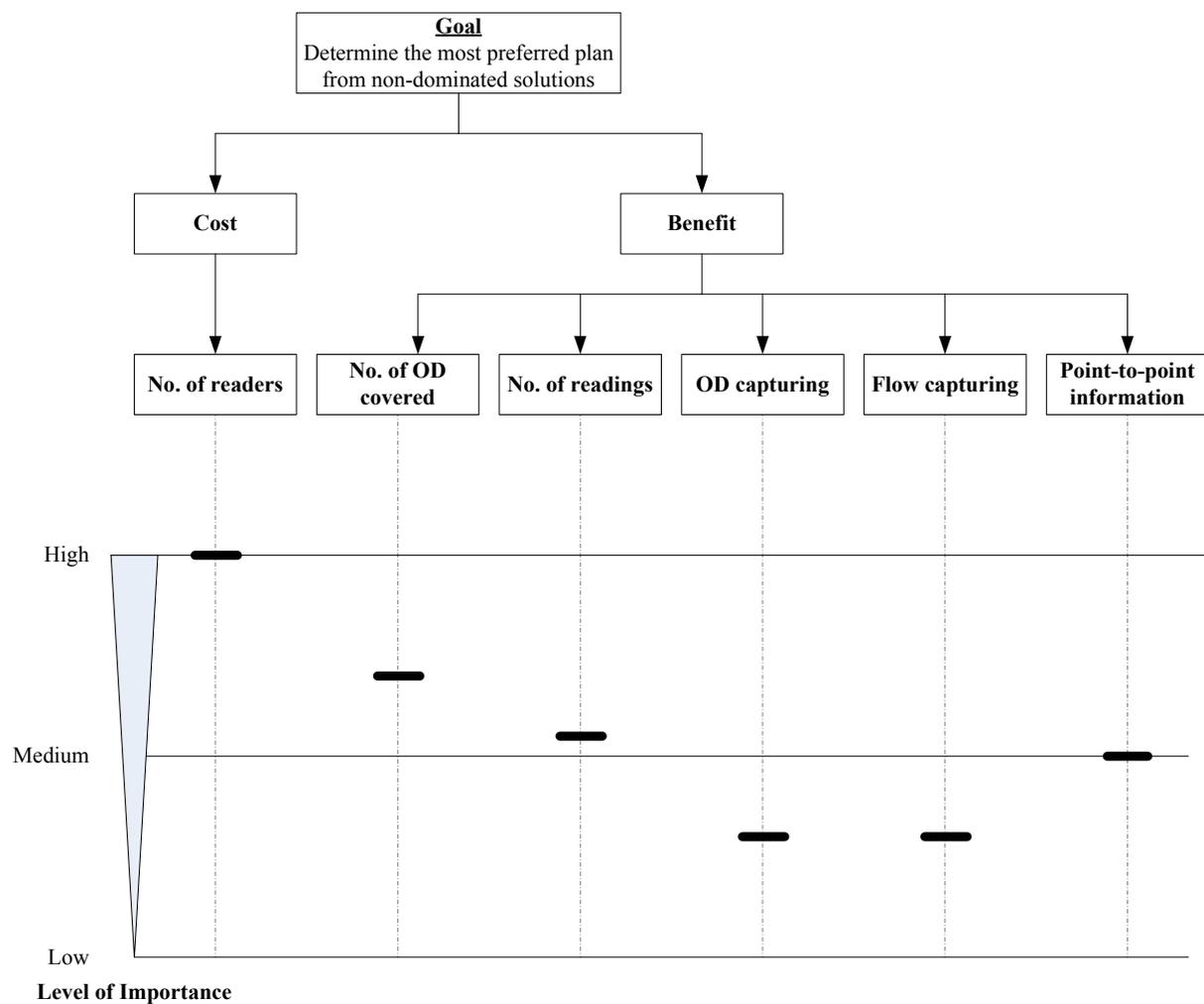


Figure 2. Hierarchy Tree

Criterion 1: Number of AVI readers – this criterion is basically a proxy of the installation cost. It is the total number of AVI readers to be installed in the network, which is objective 1 in equation 3. Clearly one may convert this objective to the monetary value if the estimated installation cost on each link is known. Even though this criterion is already used in the mathematical formulation to identify the non-dominated solutions, it is still necessary as one of the criteria for selecting the most preferred plan. It can be seen from Figure 2 that the number of AVI readers is the only criterion on the cost side. Hence, without this criterion, the solution with more AVI readers, which usually deliver higher benefit, will always be selected as the most preferred solution.

Criterion 2: Number of O-D pairs covered – this criterion represents the usefulness of the AVI system for O-D trip table estimation and electronic toll operation. For an O-D pair to be covered, all relevant paths of the O-D pair must have at least two readers, one on the first link and one on the last link of each path. This criterion is used in the maximum covering model (Church and ReVelle, 1974), and used as objective 2 in equation 4.

Criterion 3: Number of readings – this criterion indicates the amount of travel information being obtained from the AVI system. This criterion is used in the flow-capturing model (Hodgson, 1990), and used as objective 3 in equation 5.

Criterion 4: O-D capturing – the value of this criterion is obtained from the O-D capturing analysis (Chen, A. *et al.*, 2004a). It indicates the percentage of O-D pairs of which the vehicles are registered (i.e., pass through a pair of readers) at least a certain number of times (k) along all the paths. In the study, k is an integer between 1 and 10. In order to combine the benefit of different k values to one aggregate measure, weights of 0.50, 0.25, 0.13, 0.05, 0.02, 0.01, 0.01, 0.01, 0.01, 0.01 are assigned for $k = 1$ to 10, respectively. It is interesting to point out that if the weight assigned for $k = 1$ is one and zero for other k , this criterion is still different from criterion 2, because at $k = 1$ the O-D pair might not be covered.

Criterion 5: Flow capturing – it is similar to the O-D capturing criterion discussed earlier. Instead of regarding the number of O-D pairs, the percentage of path flows (relative to the total demand) in each category of k (the number of times vehicles registered along the path) is computed. This criterion is considered to distinguish the situation that there are several ways to obtain the same total number of readings (e.g., ten vehicles with one reading and one vehicle with ten readings). Similar to criterion 4, we assume the different weights for different k in order to obtain an aggregate measure of this criterion.

Criterion 6: Number of point-to-point information – this criterion is used to identify the number of pieces of information (e.g. travel time from reader at location A to reader at location B). It is developed to cope with the lack of ability to distinguish what kind of information is inherited in a reading in the other criteria. This criterion can be obtained by analyzing path flows. For example, if a vehicle passes through three readers (i.e., two readings), A, B, and C consecutively; there are two pieces of information occur from A to B and from B to C. Note that the information from A to C is redundant since it can be derived with pieces of information from A to B and from B to C. If there is another path that passes only readers B and C, this new path contributes to one more readings but not on the number of point-to-point information because the information from B to C exists already.

It is important to note that the first three criteria are derived from the MOP in equation 1 to 5. In this study, other three criteria (criteria 4 to 6), which have not been considered in MOP, are added to evaluate different non-dominated solutions for AVI reader location problem. Alternatively, criterion 4 to 6 might be put into the MOP objective function. However, in that approach, the model will be too complex with the presence of six objective functions and the inclusion of addition objectives could increase the number of possible Pareto solutions exponentially. After the evaluation criteria are determined, the decision-maker is asked to set the level of importance among criteria as shown in Figure 2. The reason for setting the level of importance qualitatively is because it might be difficult for decision-maker to quantitatively assign the priorities (e.g., weights). At the end of this module, we qualitatively know the relative important among all criteria.

3.2 Module 2: Establish Priorities

The main purpose of this module is to quantitatively establish the priorities of individual criteria (λ_i) in order to combine the criterion values into one overall score. The meaning of the term priority, which is normally used in MCDM, is the same as the term weight used in MOP. This module can be skipped if the decision-maker can quantitatively assign the priorities. However, in most of the cases, the decision maker may not be able to determine those priority values confidently. Thus, in this study, the pairwise comparison, which is the method to establish priorities in the Analytic Hierarchy Process (Saaty, 1977), is used to identify the values of these priorities. Here, the simple procedure for approximating the priorities proposed by Anderson, D. *et al.*, 2005 is presented.

Pairwise Comparison Matrix: The pairwise comparison matrix can be constructed by decision-maker as follows.

1. Construct a square matrix with the size equal to the number of criteria.
2. Fill the diagonal elements with the value of 1, which means equally important (e.g., the importance of criterion relative to itself).
3. For the elements that criterion A is more important than criterion B (see Figure 2 for the level of importance in the hierarchy tree), the decision-maker has to identify the relative importance of criterion A to criterion B . The importance scale (γ_{AB}) is rated from 1 (equally important) to 9 (extremely more important).
4. Fill out the rest elements (γ_{BA}) with the value of $1/\gamma_{AB}$.

Synthesization: After we obtain the pairwise comparison matrix, the priority of each criterion in terms of its contribution to the overall goal can be determined as follows.

1. Sum the values in each column of the pairwise comparison matrix.
2. Divide each element in the pairwise comparison matrix by its column sum. The new matrix is referred to as the normalized pairwise comparison matrix.
3. Compute the average of the elements in each row of the normalized pairwise comparison matrix. These averages are the priorities for criteria. The summation of these priorities is one.

Consistency: Since the pairwise comparison matrix is subjectively determined by decision-maker, the perfect consistency may not be achieved. The consistency ratio is design to measure the inconsistency of judgments. In practice, if it is greater than 0.10 (Anderson, D. *et al.*, 2005), the decision-maker is needed to revise his/her judgment; otherwise, the pairwise comparisons is considered reasonable. The consistency ratio can be determined as follows.

1. Multiply each value in the first column of the pairwise comparison matrix by the priority of the first criterion. Multiply each value in the second column of the pairwise comparison matrix by the priority of the second criterion. Continue this process for all columns.
2. Sum the values across the rows of the new matrix. The result vector is known as weighted sum.
3. Divide each elements of the weighted sum vector by the corresponding priority for each criterion.
4. Compute the average of those elements in the new vector. The result scalar is known as θ_{\max} .

5. Compute the consistency ratio (CR) by equation 6.

$$CR = \frac{\frac{\theta_{\max} - k}{k - 1}}{\varepsilon} \quad (6)$$

Where k is the number of criteria, and ε is the consistency index of a randomly generated pairwise comparison matrix. The value of ε is given as 0.58, 0.90, 1.12, 1.24, 1.32, 1.41, 1.45, and 1.49 for $k = 3$ to 10, respectively (Saaty, 1982).

At the end of this module, the decision-maker should re-check whether the priorities of individual criteria are reasonable or not. They should be compared with the qualitative thinking in the beginning, which is the level of importance in the hierarchy tree.

3.3 Module 3: Score Alternatives

After we get the evaluation criteria and their priorities, each alternative (in this study, it is referred to each non-dominated solution) can be scored and selected in this module. All non-dominated solutions are evaluated to get their criterion values. Because each non-dominated solution already has a numerical value for each criterion, the value can be normalized and named as criterion score by using equation 7. The implicit assumption in this equation is that the preference of each criterion increases linearly with the increment of criterion value. However, the assumption of linearity could be relaxed using nonlinear individual value functions (Cohon and Rothley, 1997).

$$\tilde{v}_i^n = \left(\frac{v_i^n - v_i^{\min}}{v_i^{\max} - v_i^{\min}} \right) \cdot 100 \quad (7)$$

Where,

- \tilde{v}_i^n = the score of a non-dominated solution n on criterion i (ranging from 0 to 100),
- v_i^n = the value of a non-dominated solution n on criterion i ,
- v_i^{\min} = the minimum value on criterion i among all non-dominated solutions,
- v_i^{\max} = the maximum value on criterion i among all non-dominated solutions.

Afterward, all non-dominated solutions can be evaluated to get the overall score (\tilde{v}^n) by weighting the priority (λ_i) on all k criteria as shown in equation 8, which is the key concept of SMART.

$$\tilde{v}^n = \sum_{i=1}^k \lambda_i \tilde{v}_i^n \quad (8)$$

If there is a strong dominant solution (i.e., the non-dominated solution has the highest overall score and the score is much higher than scores of other solutions), this solution can be selected. However, in most of the cases, the top solution might not be strongly dominant. The sensitivity analysis should be performed on the top ranking solutions. It will show how robust

this solution is by changing the criterion priority. The re-assigned overall scores when a criterion priority is changed can be computed as follows.

1. Compute the summation of priorities by excluding the criterion of interest.
2. Compute the priority proportion for each criterion (excluding the studied criterion) by dividing each criterion priority with the value from step 1.
3. Assign the priority value to the criterion of interest.
4. Compute the remaining value to the other priority values ($1 - \text{value from step 3}$).
5. Proportionally distribute the value from step 4 to each criterion ($\text{value from step 2} \times \text{value from step 4}$).
6. Repeat step 3 (varying the assigned value from zero to one).

After the sensitivity analysis is repeated on all criteria, the decision-maker can determine how robust the selected solution is relative to other solutions. For each criterion, the range of priorities that the selected solution is still the best can be revealed.

3.4 Module 4: Review the Selection

The most preferred solution from module 3 must be reviewed to ensure fulfillment of the criteria and goals of AVI reader location selection. If the selected solution does not satisfy the goals, the second best solution might be selected and re-reviewed. In such a special event that the decision maker cannot determine the most preferred plan; e.g., the candidate solutions have very similar performances, he/she might go back to module 1 and consider other criteria to distinguish the candidate solutions in other perspectives; then, select the most preferred solution.

4. NUMERICAL RESULTS

4.1 Network Description

A simplified Irvine network in Orange County, California, from Pravinvongvuth, S. *et al.* (2005) is used in this study. It consists of three major freeways (I-5, I-405, and SR-133) in the City of Irvine. The study area is approximately 2×2 mile². The Irvine network and associated traffic demand data were extracted from the Orange County Transportation Analysis Model (OCTAM), which contains data for the whole county. The OCTAM incorporates the best state-of-the-practice modeling components that are consistent with the new Southern California Regional Transportation Model recently released by the Southern California Association of Governments (SCAG). The OCTAM 3.0 has 2,940 traffic analysis zones (TAZ) within Orange County and 1,282 external stations (including cordon stations). The extracted network is composed of 63 nodes, 75 links, and 50 O-D pairs. The simplified network includes only freeway links, on-ramps, and off-ramps. There are 13 origins and 10 destinations. The most congested period, which is the PM peak (from 15:00 to 19:00), is used in this study. The total demand is 21,362 vph. The travel behavior is assumed to be a stochastic user equilibrium (SUE) traffic pattern with dispersion parameter of 1.00. Note that the path flows must be kept when performing the traffic assignment so that it can be used to evaluate the solutions.

4.2 Non-dominated Solutions

The non-dominated solutions (or Pareto solutions) are generated by solving the AVI reader location model in equations 1 to 5 and plotted in Figure 3. Because there are three objectives, the Pareto frontier forms a surface instead of a line. In this study, we assume that every link has a potential location and there is only one potential location on a link ($\rho = 1$). The distance-based genetic algorithm (GA) described in Chen, A. *et al.* (2004b) is modified to account the new definition of coverage by Pravinongvuth, S. *et al.* (2005). The GA parameters are set as follows: the population size is 128, the probability of crossover is 0.50, the mutation rate is 0.05, and the maximum number of generations is 100,000. There are 1,506 non-dominated solutions as depicted in Figure 3. Objective 1 ranges from 0 (do-nothing) to 75 readers (full installation). Objective 2 ranges from 0 (no O-D pair is covered) to 50 O-D pairs (all O-D pairs are covered). Objective 3 ranges from 0 (no reading) to 104,244 readings (when full installation). It is also interesting to point out that it needs at least 25 readers to cover all 50 O-D pairs, which is the solution of set-covering solution (Toregas and ReVelle, 1973) when using only objective 2 and using objective 1 as the constraint.

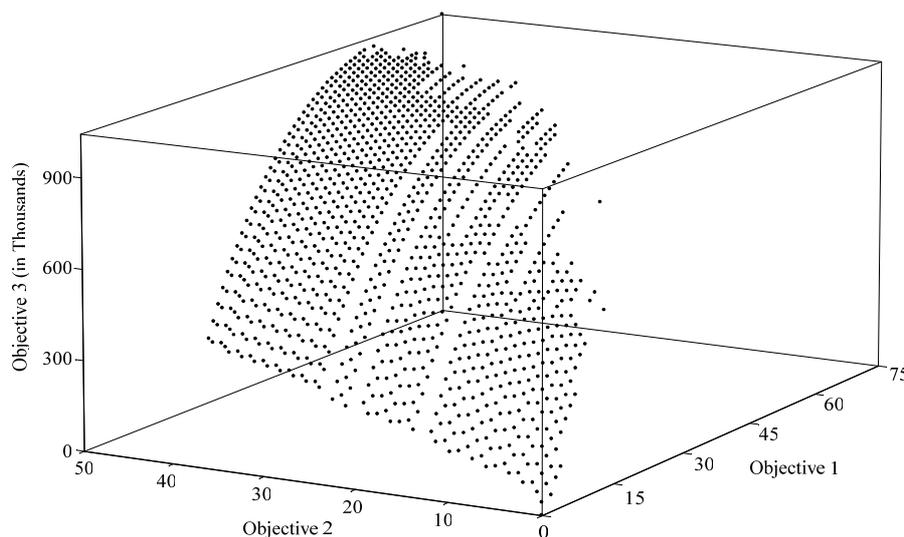


Figure 3. Non-Dominated Solutions

4.3 Solution Selections

As explain in module 1, six criteria are used to select the most preferred solution among the 1,506 non-dominated solutions obtained by solving the multi-objective AVI model (Pravinongvuth, S. *et al.*, 2005) for implementation. The pairwise comparison technique is used to establish the criterion priorities as explained in module 2. The pairwise comparison matrix is tabulated in Table 1(a). The importance scale is rated from 1 (equally important) to 9 (extremely more important). After the synthesization process, the priority table is obtained as shown in Table 1(b). The consistency ratio ($CR = 0.03$) is computed, which is lower than 0.10, so the decision maker's opinion is considered reasonably consistent. Then, all 1,506 non-dominated solutions are scored as explained in module 3. The top-five solutions are tabulated in Table 2. Solution 818 has the highest overall score but it is not a strong dominant solution.

Table 1. Priorities Establishment

(a) Pairwise Comparison Matrix

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Criterion 1	1.00	3.00	4.00	7.00	7.00	5.00
Criterion 2	0.33	1.00	3.00	6.00	6.00	4.00
Criterion 3	0.25	0.33	1.00	4.00	4.00	2.00
Criterion 4	0.14	0.17	0.25	1.00	1.00	0.50
Criterion 5	0.14	0.17	0.25	1.00	1.00	0.50
Criterion 6	0.20	0.25	0.50	2.00	2.00	1.00

(b) Priority Table

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Priority:	0.4315	0.2628	0.1391	0.0441	0.0441	0.0784

Table 2. Performance Scores of the Top-Five Solutions

Solution ID	Solution 818	Solution 852	Solution 835	Solution 785	Solution 887
Solution Rank	1	2	3	4	5
Score on Criterion 1	52	53	51	53	49
Score on Criterion 2	100	100	100	98	100
Score on Criterion 3	63	60	66	63	69
Score on Criterion 4	76	76	77	76	77
Score on Criterion 5	86	84	86	86	86
Score on Criterion 6	70	69	71	69	73
Overall Score	70.18	70.12	70.10	70.05	69.99

4.4 Sensitivity Analysis

Figure 4 shows the sensitivity analysis of the top-five ranking solutions. The vertical line on each plot indicates the current priority placed on that particular criterion. The slanted lines represent the overall scores for each alternative as a function of priorities placed on each criterion. The x-axis and y-axis range from zero to one and 0 to 100 which represent the range of possible priorities and overall scores, respectively. The six sensitivity plots for all criteria as shown in Figure 4 show the robustness within the approximately five percent ranges relative to the current priorities. The robust ranges of weight on each criterion are tabulated in Table 3. It can be seen that at the current set of priorities (where the vertical line is in Figure 4), solution 818 is the best. This solution is robust; i.e., its overall score is still the highest when the criterion priority changes within the ranges indicated in Table 3. However, this solution is quite sensitive to the priority on criterion 3. Solution 818 would still be the best if the priority on criterion 3 is in the range of 0.1196 to 0.1630. It is interesting to point out that if the decision maker gives a very high priority on criterion 3 such as 0.19 (see Figure 4), solution 887 turns to be the highest ranked solution. This makes sense as solution 887 has the highest score on criterion 3 (see Table 2). Thus, the more important the criterion 3 is, the higher the rank of solution 887 would be. In conclusion, the sensitivity analysis suggests that solution 818 is the most preferred solution and it is quite robust in changes of priority values.

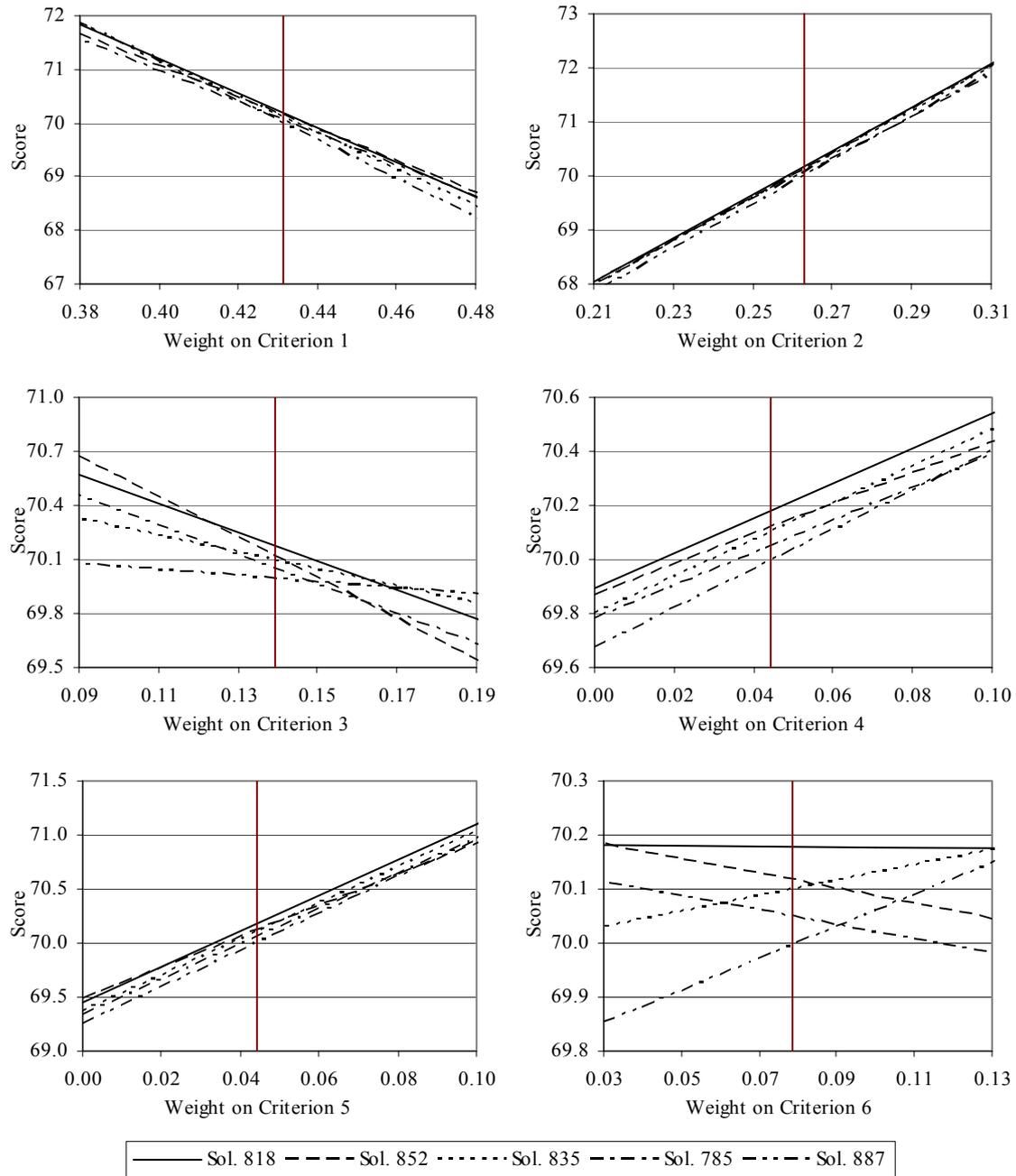


Figure 4. Sensitivity of the Top Five Solutions

Table 3. Robust Range of Weight on each Criterion

Criterion	Weight		
	Lower Bound	Current	Upper Bound
1	0.3956	0.4315	0.4572
2	0.2117	0.2628	1.0000
3	0.1196	0.1391	0.1630
4	0.0000	0.0441	0.2748
5	0.0172	0.0441	0.5143
6	0.0314	0.0784	0.1314

4.5 The Most Preferred Solution Performances

As explained in module 4, the most preferred solution (818) must be reviewed to ensure fulfillment of the criteria and goals of the AVI reader location problem. This solution requires 36 readers to be installed. It can cover all 50 O-D pairs and capture 65,955 readings. There are 54 pieces of point-to-point information. The reader location is depicted in Figure 5(a). The bold line indicates the link equipped with AVI readers. Compare to the flow pattern depicted in Figure 5(b), it can be seen that the location plan capture the high utilization part of the network. The O-D capturing and flow capturing analysis are conducted as shown in Figure 5(c) and 5(d), respectively. The O-D capturing analysis plots the percentages relative to the upper bound of 50 O-D pairs whereas the flow capturing analysis plots the percentages relative to the upper bound of 21,362 vph (i.e., total flows of O-D trip table). Because all O-D pairs are covered (i.e., all vehicles are registered at least one time), both plots cross the y-axis at 100%. k on the x-axis refers to the number of times that a vehicle is intercepted (or registered) by a pair of AVI readers. For illustration, in Figure 5(c), the percentage of O-D pairs intercepted by solution 818 at $k = 2$ is 52%. This means that there are 26 out of 50 O-D pairs in which vehicles are registered at least two times (or passing through at least three readers). Similarly, Figure 5(d) indicates that there are 67.28% of vehicles (14,372 out of 21,362) are registered at least two times.

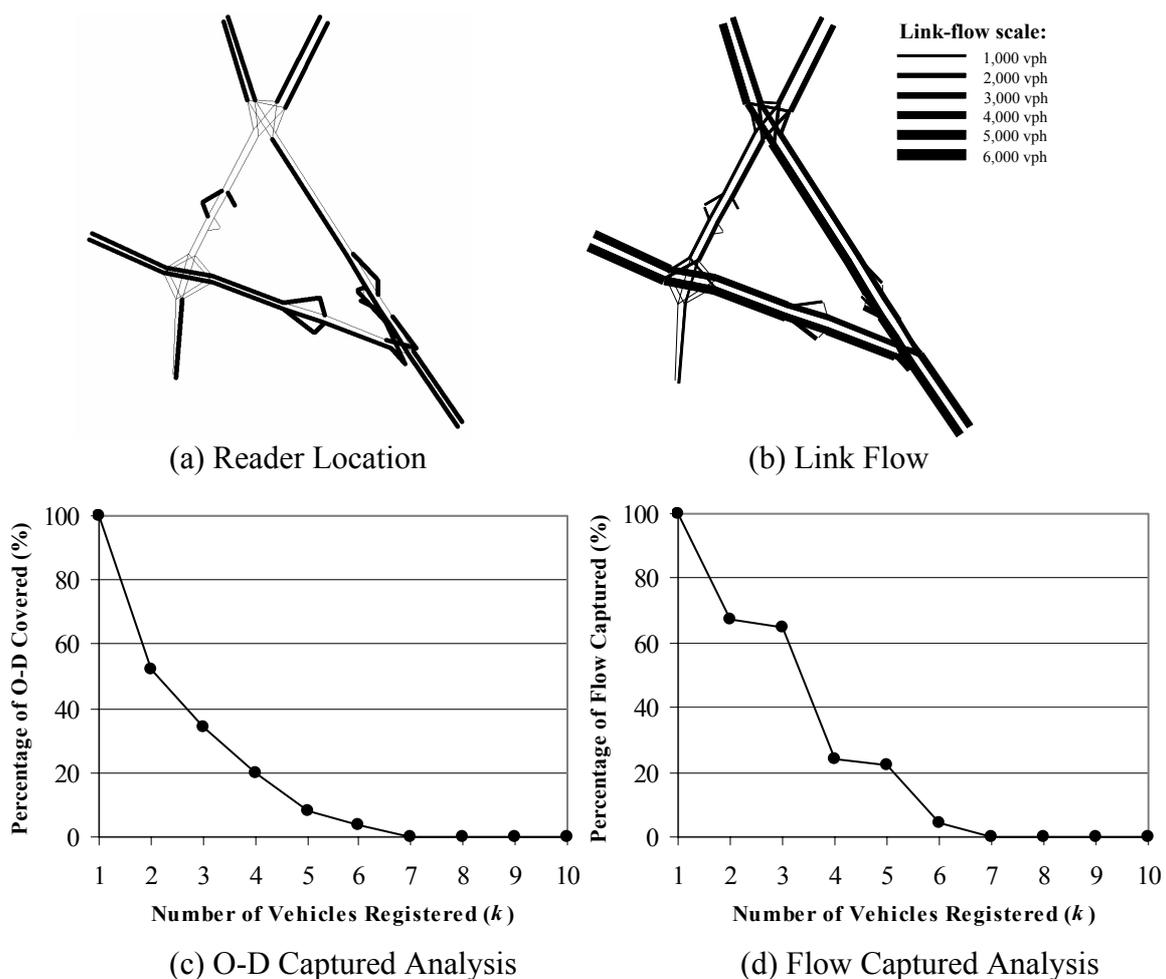


Figure 5. The Most Preferred Solution Performances

4.6 Result Discussions

The most preferred solution selected by the proposed methodology (based on SMART) is compared to other intuitive engineering methods as tabulated in Table 4. The lowest cost method (also well known as do-nothing alternative) provides an extreme solution that is the best in cost but the worst in benefits. On the other hand, the unlimited budget plan (i.e., installation at every links) provides the other extreme solution that is the worst in cost but the best in benefits. The normalized distance method is to find a compromise solution, because there is no solution in either MOP or MCDM that can satisfy all aspects simultaneously. Basically, this method is to find the solution that has the smallest normalized distance to the ideal point (i.e., the solution that has the highest score of 100 on all aspects). The normalized distance of solution n , $D(n)$, can be calculated by equation 9. The first distance ($k = 3$) is the normalized distance on the three objectives of the formulation whereas the others ($k = 6$) is the normalized distance on all six criteria of module 1.

$$D(n) = \left[\sum_{i=1}^k (\tilde{v}_i^n - 100)^2 \right]^{1/2} \quad (9)$$

As can be seen from Table 4, the selected solutions by other intuitive engineering methods are ranked as 1506th, 1145th, 239th and 140th out of 1,506 non-dominated solutions. This indicates that the other intuitive engineering methods do not allow the decision-maker to select the plan that most satisfies evaluation criteria.

Table 4. Comparison with Intuitive Selection Methods

Solution ID	Solution Selection Method	Solution Rank	Score on Criterion						Overall Score
			1	2	3	4	5	6	
818	SMART	1	52	100	63	76	86	70	70.18
2	Lowest Cost	1506	100	0	0	0	0	0	43.15
1	Unlimited Budget	1145	0	100	100	100	100	100	56.85
130	Distance on 3 Objectives	239	68	58	57	58	78	47	62.14
1219	Distance on 6 Criteria	140	44	94	81	84	90	73	68.30

5. CONCLUSION AND FUTURE RESEARCH

A methodology for selecting the most preferred plan from the set of Pareto optimal solutions obtained from solving the multi-objective automatic vehicle identification (AVI) reader location problem has been proposed in this study. It is recommended to keep the number of objectives in the MOP model as low as possible to minimize the computational efforts since the number of Pareto solutions grows exponentially with respect to the number of objectives. Only the primary quantitative measures should be treated as objectives in order to maintain a manageable size of the Pareto optimal solution set. Other secondary quantitative measures or qualitative measures can be used as additional criteria in the SMART method to select the most preferred solution among the many solutions in the Pareto set for implementation. The six criteria (including number of AVI readers, number of O-D pairs covered, number of readings, O-D capturing, flow capturing, and number of point-to-point information) are proposed to determine the most preferred AVI reader locations (Module 1: Select Evaluation Criteria). The proposed methodology allows the decision-maker to select the most preferred

plan from a large set of non-dominated solutions; i.e., selecting a single best solution from 1,506 non-dominated solutions. Even though the proposed methodology requires the decision-maker to understand the nature of problems and subjectively compare the importance of criteria in the pairwise comparison process (Module 2: Establish Priorities), the sensitivity analysis can remedy this subjective assumption by quantifying how robust the selected solution is subject to priorities changes (Module 3: Score Alternatives). Last but not least, the most preferred solution must be reviewed by performing a detailed analysis (Module 4: Review the Selection). A numerical example was conducted to illustrate the proposed selection methodology using a multi-objective AVI location model. As a result, the most preferred plan (solution 818) was selected from the 1,506 non-dominated solutions.

Since the AVI system is likely to be used in many transportation applications by many stakeholders, future research should explore other potential objectives, particularly on the qualitative measures that cannot be easily quantified, to be included in the selection methodology. On the computation perspective, other meta-heuristics should be explored to improve the efficiency of generating non-dominated solutions. Last but not least, the selection methodology can be applied to other transportation problems to select a most preferred solution from the Pareto set for implementation.

REFERENCES

a) Books and Books chapters

Anderson, D.R., Sweeney, D.J. and Williams, T.A. (2005) **An Introduction to Management Science, Quantitative Approaches to Decision Making**. 11th Edition, Thomson Learning, Ohio.

Cohon, J.L. and Rothley, K.D. (1997) Multiobjective methods. In C. ReVelle and A.E. McGarity (eds.), **Design and Operation of Civil and Environmental Engineering Systems**. John Wiley & Sons, New York.

Saaty, T.L. (1982) **Decision Making for Leaders, the Analytical Hierarchy Process for Decisions in a Complex World**. Wadsworth, California.

b) Journal papers

Chen, A., Chootinan, P., and Pravinongvuth, S. (2004a) Multiobjective model for locating automatic vehicle identification readers. **Transportation Research Record**, **1886**, 49-58.

Church, R. and ReVelle, C. (1974) The maximal covering location problem. **Papers of the Regional Science Association**, **Vol. 32**, 101-118.

Edwards, W. (1971) Social utilities. **Engineering Economist**, Summer Symposium Series 6.

Hodgson, M.J. (1990) A flow capturing location-allocation model. **Geographical Analysis**, **Vol. 22**, 270-279.

Memtsas, D.P. (2003) Multiobjective programming methods in the reserve selection problem. **European Journal of Operational Research**, **Vol. 150**, 640-652.

Rothley, K.D. (1999) Designing bioserve networks to satisfy multiple, conflicting demands. **Ecological Applications**, Vol. 9, No. 3, 741-750.

Saaty, T.L. (1977) A scaling method for priorities in hierarchical structures. **Journal of Mathematical Psychology**, Vol. 15, 234-281.

Teodorovic, D., Van Aerde, M., Zhu, F. and Dion, F. (2002) Genetic algorithms approach to the problem of the automated vehicle identification equipment location. **Journal of Advanced Transportation**, Vol. 36, No. 1, 1-21.

Toregas, C. and ReVelle, C. (1973) Binary logic solutions to a class of location problems. **Geographical Analysis**, Vol. 5, 145-155.

c) Papers presented to conferences

Chen, A., Chootinan, P. and Pravinongvuth, S. (2004b) An evolutionary approach for finding optimal automatic vehicle identification reader locations in transportation networks. **The 2004 Congress on Evolutionary Computation (CEC'2004)**, Portland, Oregon, June, 2004, IEEE Service Center, 1, 181-187.

Pravinongvuth, S., Chootinan, P. and Chen, A. (2005) Analysis of automatic vehicle identification reader locations under different travel demand patterns. **Presented at the 84th Annual Meeting Transportation Research Board**, Washington, D.C., January, 2005.