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DEVELOPMENT OF A SIMULATION ALGORITHM FOR DSO SOLUTION ON A SIMPLE NETWORK

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Abstract This study develops a DSO (Dynamic System Optimum) simulation algorithm first, which can reproduce the traffic condition so as to minimize total travel time on condition that time-dependent OD demand is given. Next, the algorithm is verified by applying to a simple network with one OD pair. Recently most of dynamic traffic simulation models reproduce traffic flows dynamically under the principle of DUO (Dynamic User Optimal). On the other hand, DSO solution must provide practical information for the effective traffic control such as ramp metering control and road pricing measure. However, there is no simulation algorithm seeking for the DSO solution Therefore, in this study, DSO simulation algorithm has been developed in accordance with Dynamic Marginal Cost equilibrium principle, which equilibrates DMC of all routes. As a result of the verification, it has been confirmed that the proposed model and algorithm efficiently solved DSO solution.

Key Words: Dynamic system optimal, Traffic assignment, Traffic control

1. INTRODUCTION

Being important working tools for governments and consultants who have to provide effective transportation planning, traffic assignment models have received a great deal of attention from academic and other analysts.

Traffic assignment models are classified into 2 types: static model and dynamic model. In past researches, most studies deal with the former model which the final result represents a steady state after road users have finished making their adjustments or the marginal shifts have settled down. However, dynamic model explicitly deal with change over time, either continuously or in increments.

A broad class of transportation networks can be described as user optimal. Here travel patterns are set up by individual users each choosing the cheapest way to arrive at his/her respective destination, rather than having his/her travel pattern dictated by a choice consistent with some aggregate system optimal. On the other hand, in some cases, the traffic pattern can be regulated by some central authority, as for example, a network used for the transportation of military supplies or for a railroad network. It is obvious that in this case, the problem which the central authority faces is to determine the traffic pattern which minimizes the total cost over the whole network.

That the two above criteria lead generally to flow traffic patterns was observed first by Wardrop (1952), who calculated the flow patterns according to the above two criteria for the case of a network consisting of two nodes connected by n independent paths and for a special cost function. He discussed briefly the case of general network and sketches the equilibrium equations; however he did not discuss their solution.

Two basic principles of traffic assignment which were first enunciated by Wardrop (1952) are:

A) *The principle of equal average travel time* -- the average travel time on all routes actually used are equal, and less than that on any unused route.

B) *The principle of minimum total travel time* -- the total travel time in the network as a whole is minimized.

Clearly, these two assignments are different in principle and results, except in the case of uncapacitated roads and flow-independent link travel times, which are difficult to find in real-life traffic condition. Moreover, the selfish driver is anti-social in that the total system time in the user-optimal flow pattern is always greater than that in the system-optimal flow pattern.

In Dynamic User Optimal (DUO) assignment, vehicles are assumed to choose their routes based on present instantaneous travel times, while in Dynamic Use Equilibrium (DUE) assignment, they are assumed to choose their routes according to the travel time actually experienced. On the other hand, from global or social perspective, Dynamic System Optimal (DSO) assignment is useful to minimize the total travel time in the whole network.

Although several analysis on dynamic traffic assignment which explicitly deal with congestion have been reported, most of them deal with DUE or DUO principles [Bernstein *et al.* (1993), Heydecker *et al.* (1996), Kuwahara and Akamatsu (1997, 2001), Ran *et al.* (1996), Smith (1993), Yang *et al* (1994), Han (2003)]. However, a few analytical studies do exist on DSO assignment. The several models have been proposed in the literature for the DSO assignment since Merchant and Nemhauser (1978a, b) formulated a mathematical program for system-optimal dynamic traffic assignment in a network with multi origins and a single destination. They made an unrealistic assumption that all links are uncapacitated and that in each time interval the number of vehicles departing a link is a function only of the volume on that link. Therefore, congestion caused by blocking of one link by another congested link is not modeled. In last decade, several studies purposed mathematical formulation; however these models also included unreasonable assumption, in which exit volumes from a link is determined by the number of vehicles on the link [Wei *et al*. (1990), Lafortune *et al*. (1993)]. Yang *et al.* (1997) analyzed dynamic road pricing for DSO solution but did not include route choice. Kuwahara *et al* (2001) reported that Dynamic Marginal Cost (hereafter DMC), which plays an important role in the DSO assignment, equal sum of free flow travel time, *FFTT*, and "congestion time" defined as the time duration from current time t to the queue vanishing time. On the other hand, there are many studies for the DSO assignment which adopted bi-level programming and heuristic algorithm [Yang and Bell (1997), Maher *et al*. (2001), Chiou (2003)].

This study develops a DSO simulation algorithm using the DMC concept formulated by Kuwahara *et al*. on condition that time-dependent OD demand is given, and the algorithm is verified by applying to a simple network with one OD demand and two alternative routes.

2. DSO ASSIGNMENT

2.1 Dynamic marginal cost

In order to solve the DSO solution, the DMC of all routes should be equilibrated. By making traffic demands be assigned so as to equilibrate DMC of all routes, DSO solution can be established by Kuwahara *et al*. (2001) as below. The definition of DMC is, "The dynamic marginal cost of an additional unit of traffic demand at time t is the additional cost (e.g. travel time) with respect to all of demand." A simulation algorithm is proposed based on the DMC concept.

In a simple network with one alternative route and single OD, it has a single bottleneck with the capacity of μ, arrival rate into and departure rate from the bottleneck is given as $\lambda(t)$ and $D(t)$ respectively. The cumulative numbers of vehicles arriving at and departing from the bottleneck by time t, $A(t)$ and $D(t)$ respectively, waiting time of each individual, $w(t)$,and also queue at the bottleneck, appeared from time t_0 and vanished at time t_e, are shown in the Fig.1. Queues are developed under the First In First Out queue discipline. Under this condition with one route, every demand must take at least constant free flow travel time, FFTT.

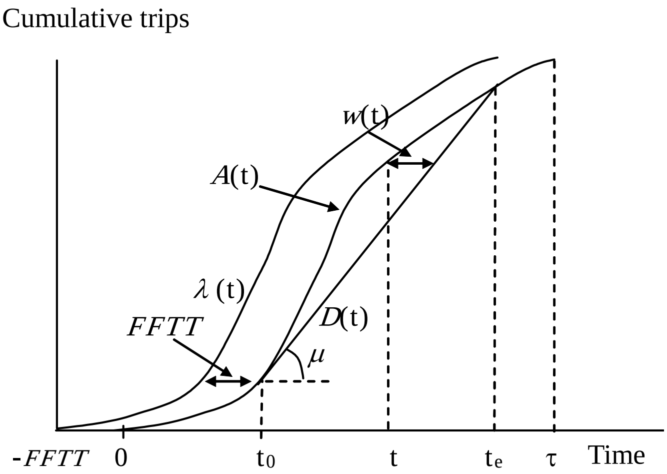


Figure 1 Cumulative Curve

$$
\begin{cases}
MC(t) = FFTT \qquad \text{if} \quad t < t_0\,, \quad t > t_e & (1)\\
\begin{aligned}
MC(t) &= FFTT + w(t) + \int_t^{t_e} \frac{1}{\mu}\lambda(u)du \\
&= FFTT + w(t) + \frac{1}{\mu}(A(t_e) - A(t)) \\
&= FFTT + w(t) + \{t_e - (t + w(t))\} \\
&= FFTT + t_e - t \qquad \text{if} \quad t_0 < t \le t_e
\end{aligned} & (2)\\
\begin{cases} MC^+(t) = FFTT + t_e - t \\ MC^-(t) = FFTT \end{cases} \qquad \text{if} \quad t = t_0 & (3)
\end{cases}
$$

Equation (2) implies that, in contrast with the static marginal cost, the shift in arrival rate at time t affects the travel time of all vehicles arriving at the bottleneck after time t until the queue vanishes.

Equation(3) implies that, when the arrival rate is equal to the capacity and queue does not appear, DMC(MC(t)) has two values, $MC^+(t)$ and $MC^-(t)$, which describe how much travel time change due to additional and subtracted single unit arrival rate at time t respectively. If the demand rate $\lambda(t)$ increases by $d\{\lambda(t)dt\}$, $w(t+dt)$ increased by $d\{\lambda(t)dt\}/\mu$; because a queue appears at time t. On the other hand, if the demand rate $\lambda(t)$ decreases by $d\{\lambda(t)dt\}$, $w(t+dt)$ has never been changed and its value has stayed zero because queue has never appeared at time t. As a whole, the change will influence to all vehicles arriving at the bottleneck within the time queue appears. Furthermore, if the arrival rate is keep on the capacity during some time interval, DMC has taken two values within the interval. This is one of the most important properties of DMC.

In order to avoid the confusion, the meaning of usual travel time and DMC are emphasized. Usual travel time is sum of FFTT and waiting time (arrow line in Fig.2), $w(t)$, of an individual. On the other hand, DMC equals FFTT plus congestion time (gray line), $t_e - t$.

To achieve DSO solution, the method is to determine the flow that cause marginal cost of all routes in the network is equilibrated.

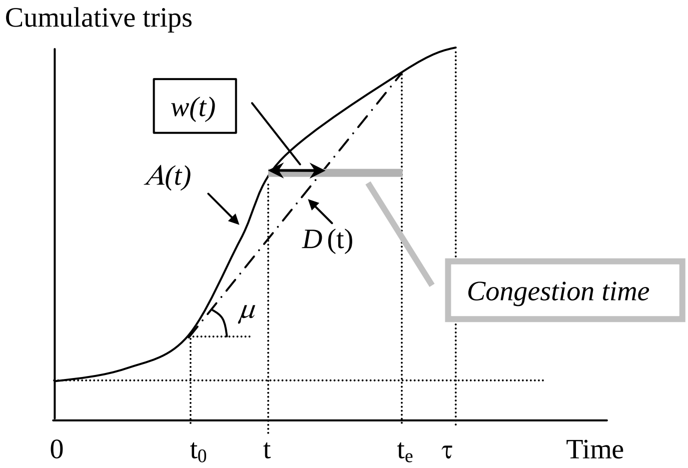


Figure 2 Travel Time vs. DMC

2.2 DSO Solution

In case of simple network, 2 alternative routes (freeway and arterial) with one-to-one OD demand shown in Fig.3, according to DUO principle, all demand choose freeway until its travel time is greater than that of arterial. As a result, the DMC curve can be illustrated as shown in Fig.4(c).

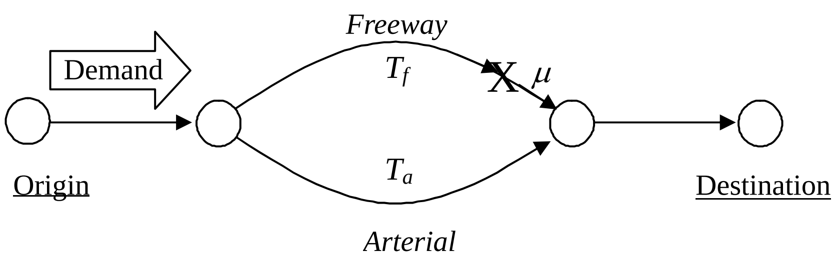


Figure 3 Simple Network

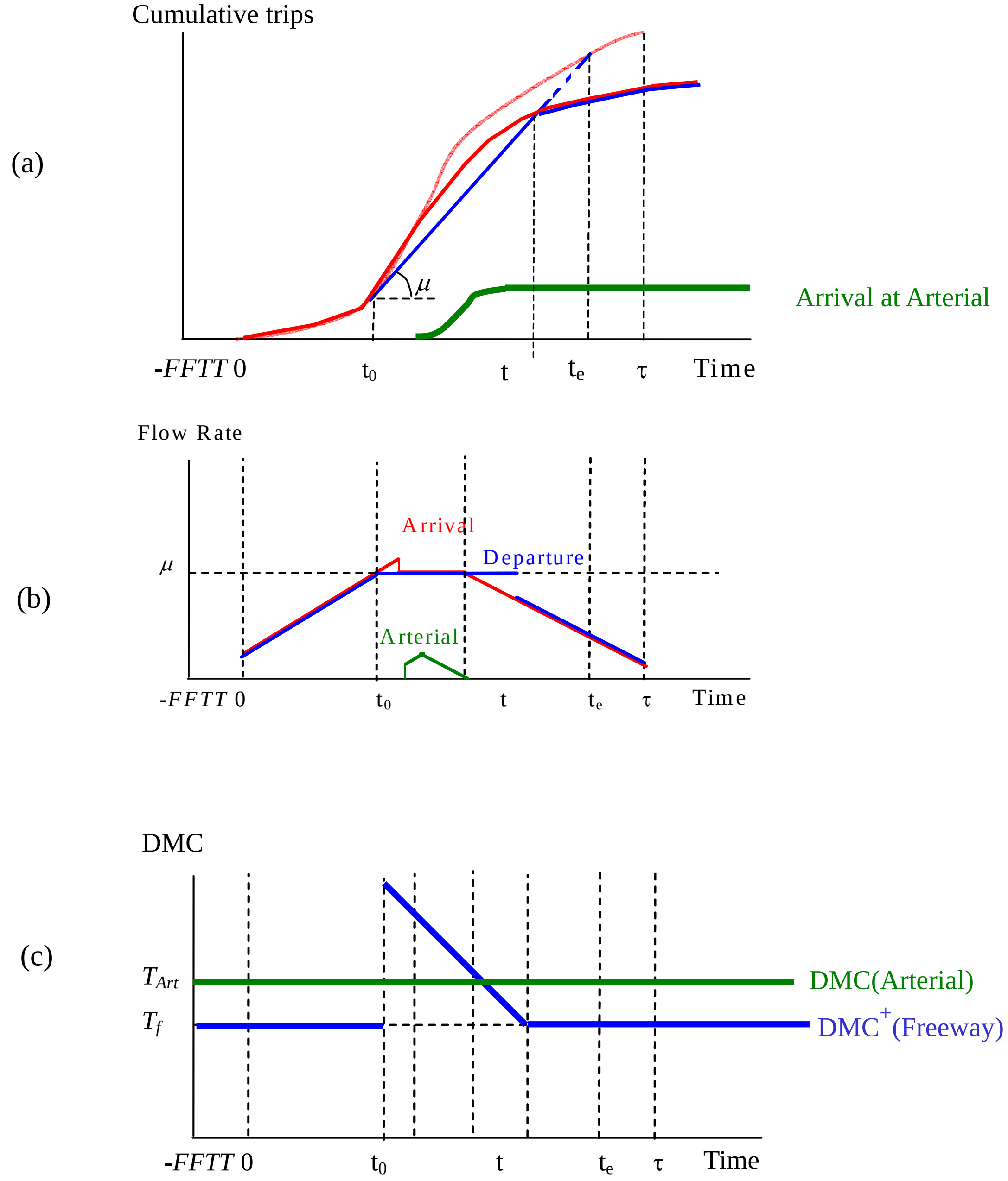


Figure 4 Cumulative Curve, Flow Curve, and DMC Curve (DUO)

On the other hand, for DSO solution, the exceeding demand from capacity is assigned to choose their routes based on DMC instead. Therefore, at time immediately after queue starting time, t_0 in Fig.5(b), exceeding demand change their route from freeway to arterial because DMC^+ (Freeway) is greater than DMC (Arterial). This implies that even if the freeway travel time is always less that arterial travel time (under DUO, everyone uses the freeway), we could reduce the total travel cost by shifting the flow just equals to bottleneck capacity to the arterial as long as DMC^+ (Freeway) is greater than DMC (Arterial), t_0-t in Fig.5(c).

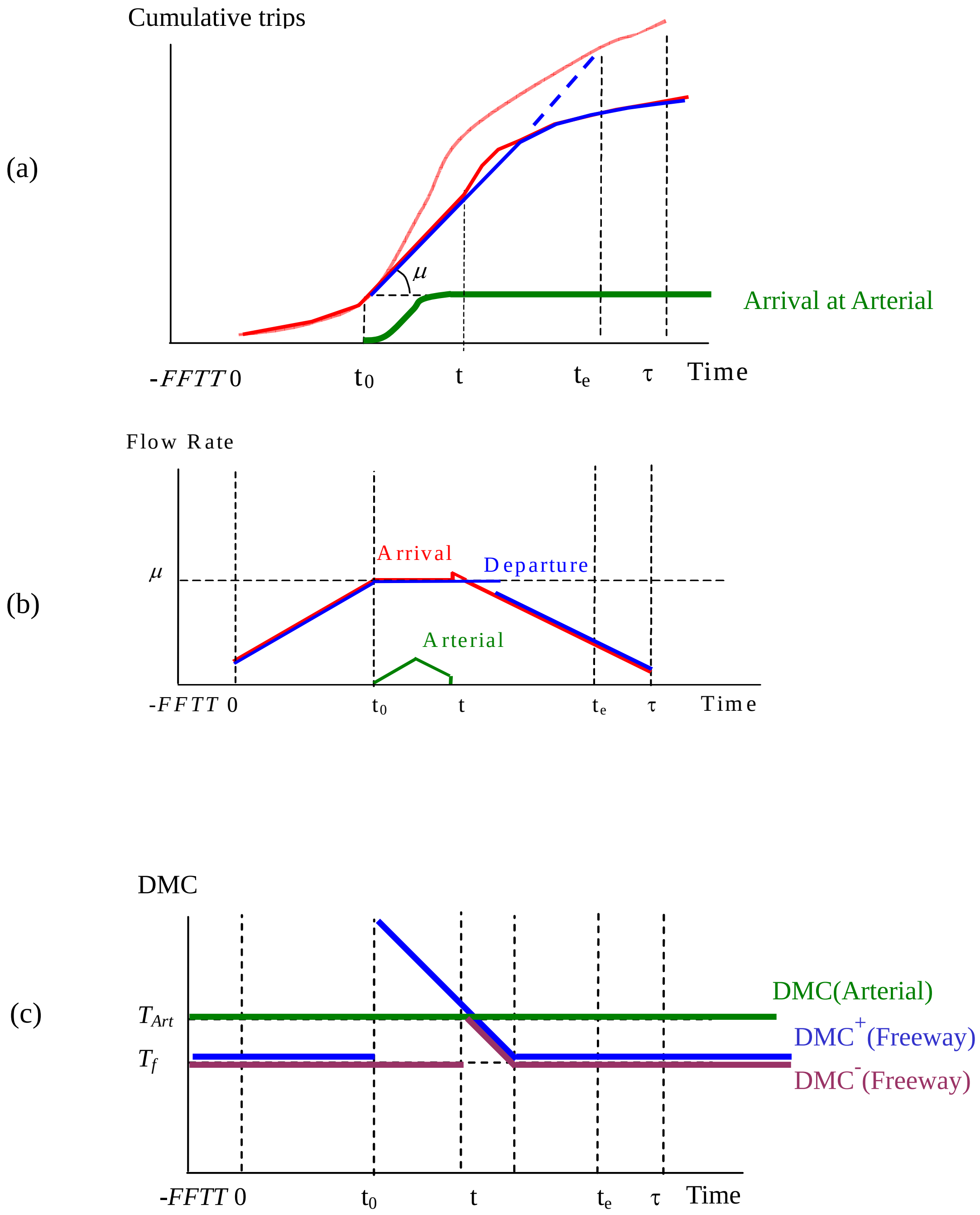


Figure 5 Cumulative Curve, Flow Curve, and DMC Curve (DSO)

3. DSO ALGORITHM

In order to solve the DSO solution, we use the concept of DMC. Under the condition that time-varying OD demand is fixed and given, we propose a new DSO algorithm using dynamic traffic simulation which iteratively obtains the approximate DSO solution. Two types of network are considered, one has travel time cost and the other has DMC on each links.

They are called "usual network" and "DMC network" respectively. And two groups of vehicles are considered: group of vehicles choosing their routes on usual network and on DMC network. They are called uncontrolled group and controlled group respectively.

3.1 Controlled group and uncontrolled group

Demand is separated into 2 groups: uncontrolled group and controlled group. Vehicles belonging to uncontrolled group are considered to choose their route based on usual network. On the other hand, vehicles in controlled group are considered to choose their route based on DMC network. The basic concept of this algorithm is how to determine which vehicles should be shifted from uncontrolled group to controlled group. DSO is achieved that demand being equal to bottleneck capacity choose bottleneck route and exceeding demand choose detour route. If all vehicles are dealt with controlled group, they choose the detour route and no vehicle chooses this bottleneck (fastest) route anymore. As a result, the bottleneck has become vacant and its DMC has equaled to free flow travel time, which is smaller than another one. Therefore, all vehicles return to choose the bottleneck route again in the next iteration, and they will choose the bottleneck route and the detour route alternately after this iteration, called *hunting phenomenon*. Thus only some of vehicles will be shifted to controlled group.

3.2 Criteria for shifting vehicles

As expressed above, just some of vehicles in short time interval should be shifted to controlled group. The criteria for determining those suitable vehicles are as following;

The exceeding volume

If all vehicles' OD are identical, the vehicles that arrive bottleneck after the capacity is already full, exceeding volume, should be shifted to controlled group. In other word, the vehicle in short time interval whose arrival time is later than queue occurring time at bottleneck should be shifted.

Actually, in ordinary arrival curve, arrival vehicle is limited by capacity; however, ideal arrival curve can be illustrated in Fig.6, and the exceeding demand from capacity is focused on. These vehicles will be shifted to controlled group. In order to figure volume curve like Fig.7, we would like to introduce new concept "cumulative delay". Each driver naturally encounters delay in some links along his/her route. These delays will be cumulated and will be cut from actual arrival time, *AAT,* of BN-link. The ideal arrival time, *IAT,* will be obtained. Then the number of vehicle at t=IAT will be added 1 unit.

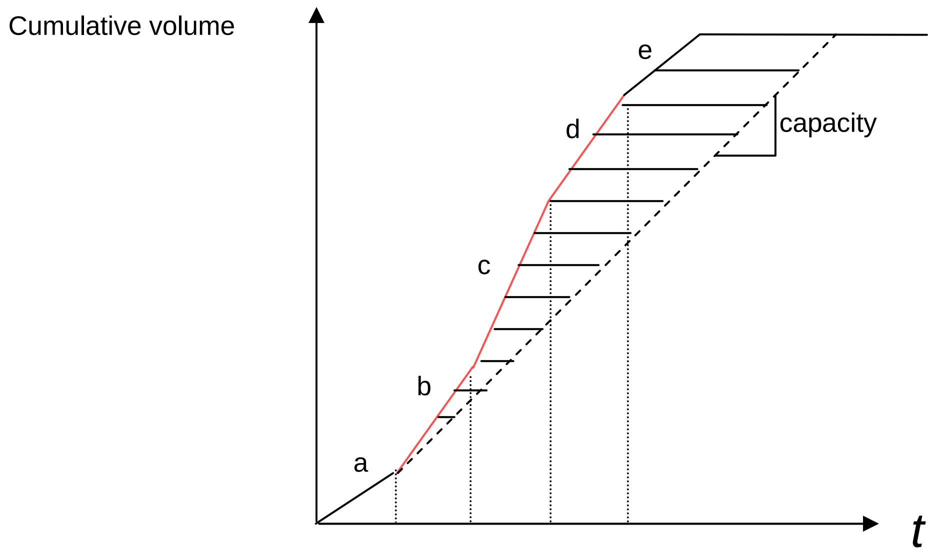


Figure 6 Ideal Cumulative Curve

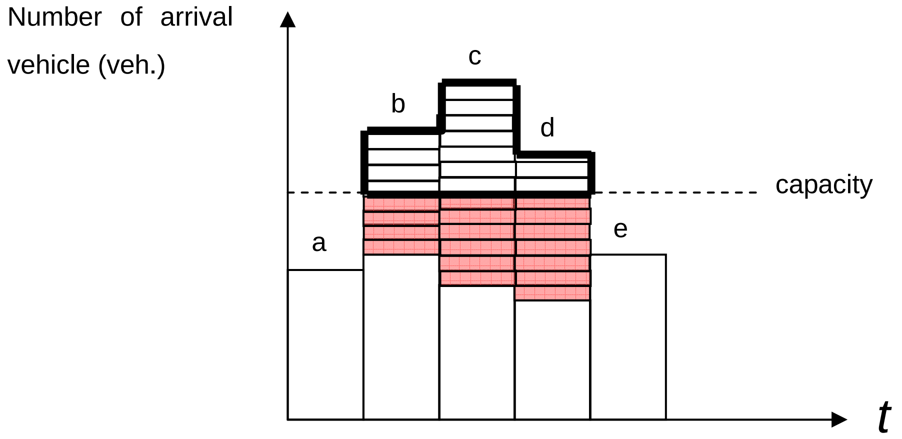


Figure 7 Ideal Arrival Curve

The second best route concept

In general network, it is more complicated because waiting vehicles at bottleneck are belonging to various OD pairs. The rule for loading demand, called 2nd best route concept has been introduced. The vehicles that have the minimum difference between travel time of the 1st best route and that of the 2nd best route should be shifted. In other word, their travel time is smaller increase. From the network depicted in Fig.8, there are two OD demands, OD1 and OD2, whose 1st best route is dotted-line and gray line respectively. And the travel time is a_1 minutes for OD1 and a_2 minutes for OD2.

If the congestion or non-reoccurring incident, e.g. accident, occurs at sharing link, illustrated in Fig.9, some vehicles in both OD pairs change their route. Accordingly we understand their 2nd best routes and their travel time, say b_1 and b_2 minutes for OD1 and OD2 respectively. Therefore, the difference between the 1st best route and the 2nd one of OD1 and OD2 is b_1-a_1 and b_2-a_2 respectively. In case of $b_1-a_1 < b_2-a_2$, waiting vehicles belonging to OD1 at the earliest bottleneck link have priority to be shifted. In other word, the vehicles belonging to OD1 can be assigned through sharing link until the link is full or no demand in OD1. In latter case, some of vehicles in OD2 should be assigned until the link is full.

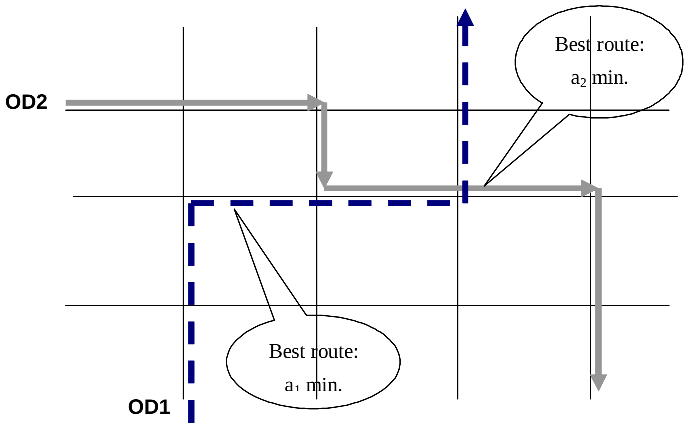


Figure 8 The Network for Explaining 2nd Best Route Concept (1)

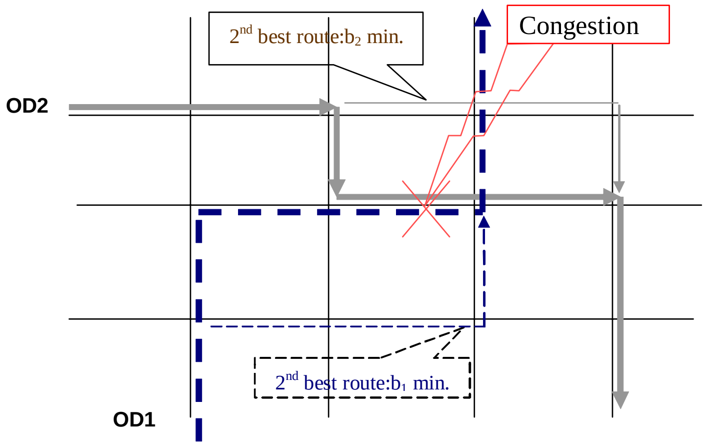


Figure 9 The Network for Explaining 2nd Best Route Concept (2)

3.3 Usual network & DMC network

In usual network, all link costs are evaluated by instantaneous link travel time. The link costs change dynamically due to inflow and capacity. Conversely, in DMC network, all link costs are calculated as FFTT + congestion time, evaluated by using queue vanishing time calculated by previous iteration; therefore, the time-dependent link costs are predetermined in each iteration. Two types of congestion are considered; waiting queue at bottleneck, and full capacity: flow rate is nearly capacity (no existing queue). However, if there is no congestion in network, DMC network is same as usual network.

3.4 Framework

In 1st iteration, all vehicles are considered to belong to uncontrolled group and they are assigned to each of the route in accordance with DUO principle. From the result of this assignment, DMC of each link can be evaluated using the duration of the queue. Additionally cumulative curves, which describe ideal arrival time (actual arrival time minus delay.) and departure time of each vehicle, can be illustrated. Accordingly, the number of exceeding vehicles in each time interval from its capacity can be evaluated by using this curve. If congestion queues occur, the earliest queue is focused on and the waiting vehicles, shaded area in Fig.10(b), which are the exceeding demand from the bottleneck capacity in the short time interval (δt, say 1 minute), from queue starting time, are shifted from uncontrolled group to controlled group because DMC^{+} (Freeway) is greater than DMC (Arterial). As a result, in the next iteration, only the exceeding demand from the bottleneck capacity in the short time interval are shifted to controlled group and they choose their routes depend on DMC network instead of usual network and the other vehicles choose their routes depend on usual network. Consequently, exceeding demand in next time interval is evaluated. This process is repeatedly implemented until no vehicle shifts his route even it belongs to controlled group.

However, in case of many OD pattern, the waiting vehicles, in shaded area in Fig.10(b), are comprised of vehicles belonging to various OD pairs. Thus, the 2nd best route concept is applied for shifting the most suitable vehicles in each time interval according to Chapter 3.2. And they will choose the minimum DMC route instead of minimum travel time route, quickest route.

Based on the proposed algorithm, the process is roughly stated as follows.

Step 0. Implement program normally for DUO solution. The earliest BN link and congestion time of all links are obtained.

Step 1. Evaluate individual's second best route choice and its travel time.

Step 2. Based on the second best route concept, shift waiting vehicles in short time interval from queue appearing time at the earliest BN link from uncontrolled group to controlled group. Thus they perhaps change their route simultaneously. The result shows the earliest BN link, congestion time of all links, and definitely total travel cost.

Step 3. If there is no shifting vehicle, then stop. Otherwise, go to step 2.

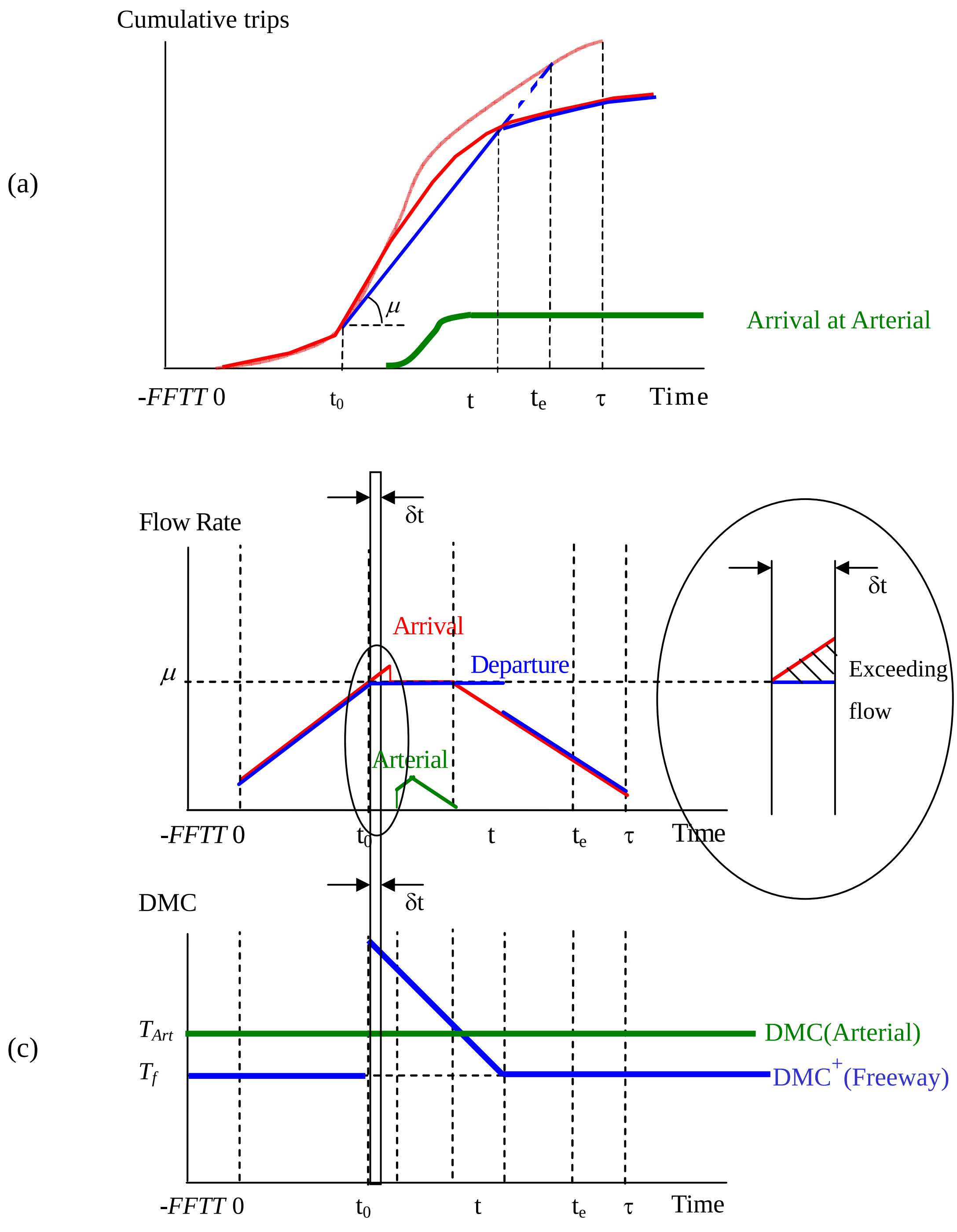


Figure 10　Cumulative Curve, Flow Curve, and DMC Curve (DUO)

4. VERIFICATION RESULTS

4.1 Simple network with one OD, two alternative routes

In order to verify the DSO algorithm, it is applied on the simple network depicted in Fig.11, of a parallel freeway and an arterial street on each of which one bottlenecks exists with one-to-one OD demand shown in Fig.12. Flow – Density(Q-K) relationship also shown in Fig. 13. And we assume that free flow travel time via freeway (T_f) is less than that via an arterial street (T_a), and the toll on freeway is not taken into account in this study.

By using this example, the iterative process is terminated at 59th iteration and the last condition also illustrated in Fig. 14(c) and Fig. 15(c). In final iteration, exceeding flow from the capacity belongs to controlled group and they choose arterial instead of freeway. Therefore, there is no waiting vehicle at the entrance of bottleneck, freeway for this network.

Under this condition, total cost in the network is minimized, as shown in Fig. 16. And the total travel cost was around 0.65 million seconds which is about 60 percent of that in DUO solution, 1.1 million seconds.

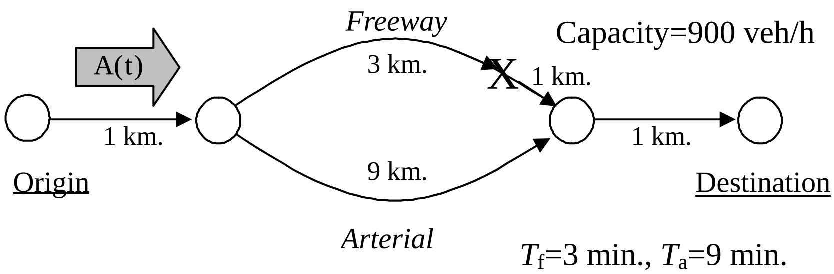


Figure 11 Study Network

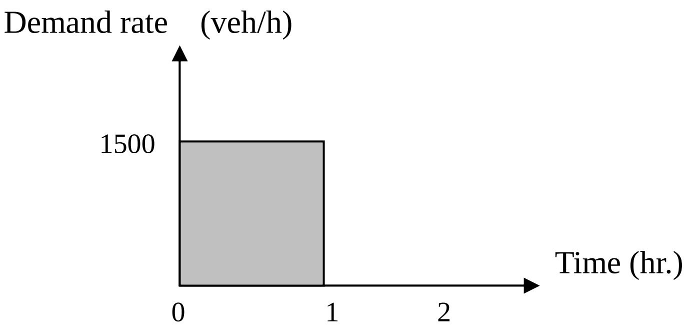


Figure 12 Demand Curve

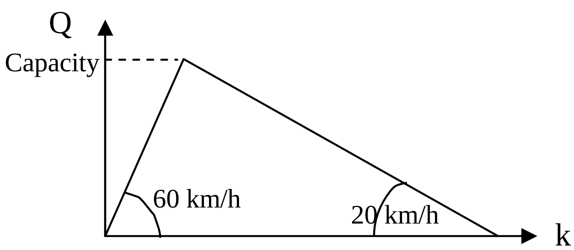
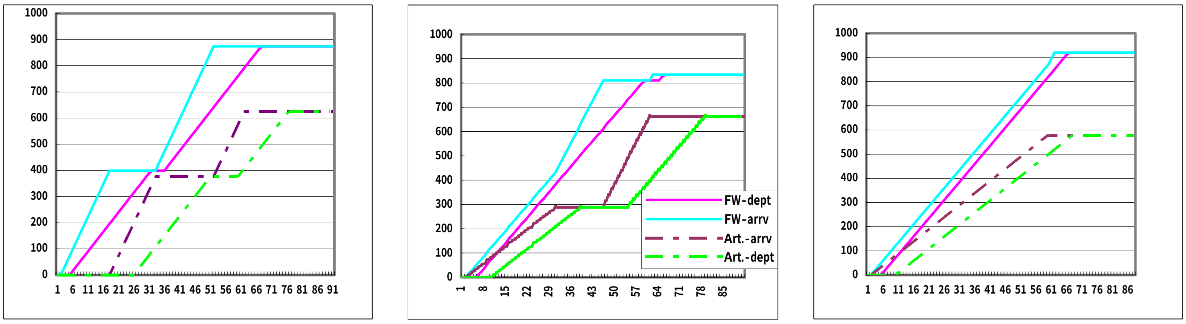
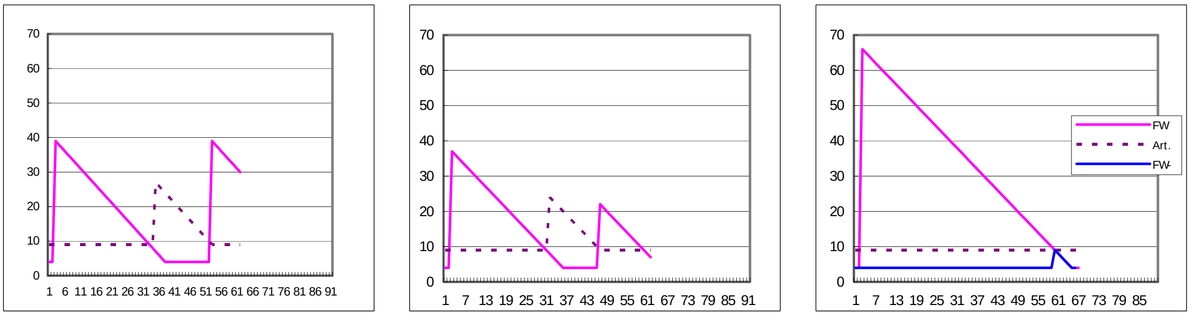


Figure 13 Q-K Relationship of all links



***(a)**: The first iteration.* ***(b)**: The 30th iteration.* ***(c)**: The final iteration (59th).*

Figure 14 The cumulative numbers of both links



***(a)**: The first iteration.* ***(b)**: The 30th iteration.* ***(c)**: The final iteration (59th).*

Figure 15 Dynamic Marginal Cost

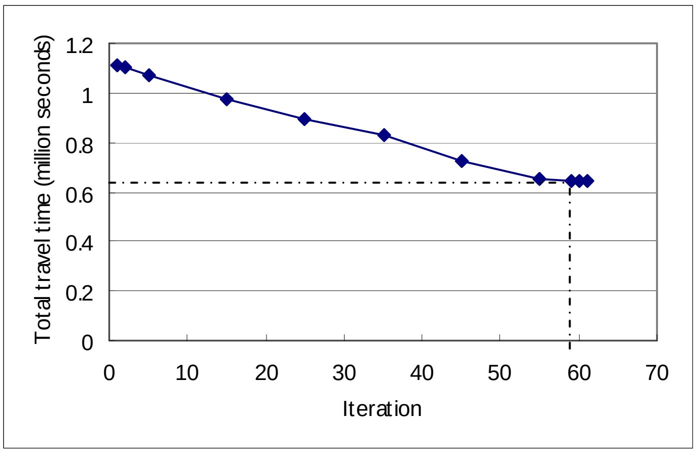


Figure 16 TTC Curve

5. CONCLUSION AND FUTURE SCOPE

In this paper, a dynamic traffic simulation is developed for the DSO solution, and the proposed simulation algorithm is confirmed to achieve the DSO solution by applying to just a simple network.

DMC plays an important role in seeking for DSO solution and DSO can be achieved by equilibrating the DMC of each route at all times. The dynamic traffic simulation algorithm based on this equilibrium concept has been developed, which can calculate DSO solution. Then, the simulation algorithm is verified using a simple network, and its good performance is confirmed. However, the verification result does not indicate the validity of the algorithm for general networks. Therefore, research effort will be addressed to develop as well as extend to the analysis to a general network.

Nevertheless at the moment the major limitations of this simulation algorithm are, in the author's opinion, as following;

- Individuals have identical cost function
- Mode choice and mixed flow vehicles (cars, buses, trucks) are ignored
- Parking and other source of congestion are ignored

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