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PROPERTIES OF DYNAMIC TRAFFIC ASSIGNMENT WITH PHYSICAL QUEUES

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Abstract: Queue spillback is a common phenomenon in congested transportation networks. Nevertheless, traditionally, dynamic traffic assignment (DTA) problems are developed with the point-queue concept in which queue spillback is not captured. Indeed, one recent focus in DTA research is to capture this phenomenon and develop solution methods for the physical-queue DTA formulations. However, the properties of these problems, which have important implications on the theoretical advances and computational issues on transportation planning and operations, are not well recognized and understood. This paper summarizes the properties of physical-queue DTA, compares those with point-queue DTA, and discusses their implications. In particular, the interrelationship among properties including First-In-First-Out, causality, travel-time-link-flow consistency, and queue spillback are emphasized in this paper.

Key Words: Dynamic traffic assignment, point-queue models, physical-queue models, causality, First-In-First-Out, time-flow consistency

1. INTRODUCTION

The properties of dynamic traffic assignment (DTA) have important implications on the theoretical advances and computational issues on transportation planning and operations. These properties depend strongly on the two components of DTA: the travel choice principle and the traffic-flow component. The travel choice principle models travellers' propensity to travel, and if so, how they select their routes, departure times, modes, or destinations. In making such choices, travel time is one important element of their considerations. The travel choice principle can be formulated as a variational inequality problem, and it is established that the existence of solutions requires the mapping function of the problem to be continuous (Theorem 1.4 in Nagurney, 1993) whereas the uniqueness of solution further requires the mapping function to be strictly monotonic (Theorem 1.8 in Nagurney, 1993). Therefore, solution existence (uniqueness) requires route travel times to be continuous (strictly monotone) with respect to route flows.

The traffic-flow component depicts how traffic propagates on a transport network and hence governs the network performance in terms of travel time. Previous efforts focused on ensuring

1) flow conservation (the rate change of the number of vehicles is the difference between the link inflow and outflow at that time) and
2) time-flow consistency, the one-to-one mapping between traffic flow and traffic time.

These two considerations make the traffic flow model theoretically sound. Capturing actual traffic behaviour in the traffic-flow component is one important current research direction.

Indeed, past efforts have focused on capturing the following traffic behaviour:
1) First-in first-out (FIFO) (e.g., Tong and Wong, 2000; Huang and Lam, 2002): FIFO on the link level means that users who enter the link earlier will leave it sooner;
2) Causality (e.g., Friesz et al., 1993; Ran and Boyce, 1996.): Causality means that the speed and travel time of a vehicle on a link is only affected by the speed of vehicles ahead, and;
3) Queue spillback (e.g., Daganzo, 1994, 1995a; Kuwahara and Akamatsu, 2001; Rubio-Ardanaz et al., 2001; Szeto and Lo, 2004): Queue spillback refers to the end of queue spilling backwards in the network.

The above traffic behaviour governs the properties of DTA formulations such as the properties of route and origin-destination (OD) costs (e.g., continuity of route and OD costs), as well as solution properties (e.g., existence and uniqueness of solutions). However, these properties are not *fully* addressed in the literature. In addition, the interrelationship among FIFO, time-flow consistency, causality, and queue spillback are not *well* understood in the literature.

This paper addresses the properties of DTA with and without physical queue considerations, summarizes their similarities and differences, discusses their implications on theoretical advances and computational issues on planning and operations, and finally suggests future research directions to these two aspects. In particular, FIFO and causality are examined in details, and their interrelationship with time-flow consistency and queue spillback. The main finding is that the four considerations in the traffic flow model, namely causality, FIFO, time-flow consistency, and queue spillback are independent; Capturing one in the model does not imply capturing the other. This paper also highlights the key difference between the point-queue and physical-queue representations: the latter includes storage capacity in the resultant formulation to capture the effect of queue spillback.

The outline of this paper is as follows: Section 2 describes the five considerations in the traffic-flow component considered in the literature. Section 3 reviews the existing analytical formulation approaches for the traffic-flow component. Section 4 discusses the properties of DTA. Finally, section 5 gives concluding remarks.

2. FIVE CONSIDERATIONS IN THE TRAFFIC-FLOW COMPONENT

This section describes five independent considerations in the traffic-flow component: flow conservation, First-In-First-Out (FIFO), time-flow consistency, causality, and queue spillback. To facilitate discussion, we here introduce some notation and equations:

$u_a(\omega)$	inflow rate for link *a* at time ω (in vehicles per unit time)
$v_a(\omega)$	outflow rate for link *a* at time ω (in vehicles per unit time)
$U_a(\omega)$	cumulative inflow up to time ω
$V_a(\omega)$	cumulative outflow up to time ω
$\tau_a(\omega)$	travel time on link *a* at time ω
$x_a(\omega)$	number of vehicles on link *a* at time ω (or the link occupancy)

By definition, we have:

$$U_a(\omega) = \int_0^{\omega} u_a(s)ds \text{ , or } u_a(\omega) = \frac{\partial U_a(\omega)}{\partial \omega} \text{ , and} \tag{1}$$

$$V_a(\omega) = \int_0^{\omega} v_a(s)ds \text{ , or } v_a(\omega) = \frac{\partial V_a(\omega)}{\partial \omega}. \tag{2}$$

2.1 Flow Conservation

The flow conservation condition requires the number of vehicles on a link (link occupancy) at a particular time to be equal to the total inflow at the entry of that link at that time minus the corresponding total outflow at the exit. Mathematically, it can be expressed as:

$$x_a(\omega) = U_a(\omega) - V_a(\omega). \tag{3}$$

The above equation means that graphically, the vertical distance between the cumulative inflow and outflow at time ω gives the number of vehicles at that time, $x_a(\omega)$. Figure 1 illustrates the relationships among variables in (1)-(3).

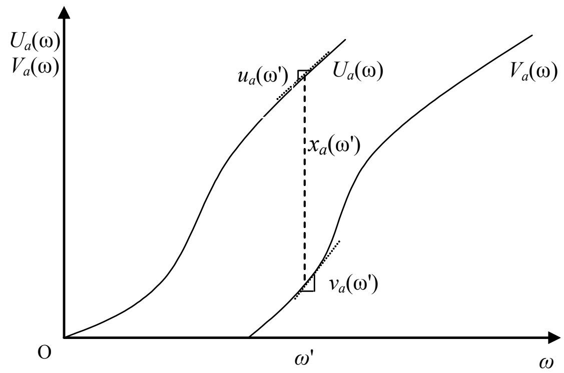

Figure 1 The Relationships among the Link Occupancy, Inflow Rate, Outflow Rate, Cumulative Inflow and Cumulative Outflow.

By taking the derivative to both sides of (3), we have an alternative expression for the flow conservation condition:

$$\frac{\partial x_a(\omega)}{\partial \omega} = u_a(\omega) - v_a(\omega). \tag{4}$$

Equation (4) relates the number of vehicles on link a, $x_a(\omega)$, to the inflow and outflow of link a whereas (3) relates the number of vehicles on link a to the cumulative inflow and outflow.

2.2 First-In-First-Out (FIFO)

Three FIFO properties are considered in the literature, namely link, route and OD. Link (Route) FIFO is satisfied if every user who enters the link (route) earlier will leave it sooner. Similarly, OD FIFO is satisfied if users on the same OD pair who depart the origin earlier will arrive the destination sooner. These three FIFO properties are defined through *travel time* and departure time, and play an important role in DTA problems, especially in describing *aggregate* flow propagation. Link FIFO can prevent unrealistic situation such as the fast traffic "jump over" the preceding slow traffic, and avoid the holding back problem (Carey and Subrahmanian, 2000). Although FIFO does not allow any realistic overtaking on a microscope level, in reality, road traffic tends to behave in a FIFO manner: Traffic which embarks on a road first will *on average* exit first (Carey, 1992). In particular, on a single-lane road and in a queue, no overtaking can be occurred and capturing FIFO for this situation in modelling is a must. Moreover, OD FIFO appears as a reasonable reflection of our experience, subject to the condition of no substantial overtaking happening and travellers having nearly perfect information about the network condition. In this paper, FIFO usually refers to Link FIFO, unless we specify which types of FIFO.

2.3 Time-flow Consistency, Inter-temporal Conservation or Flow Propagation

With Link FIFO and flow conservation, if we know cumulative inflows and outflows, we can derive the travel time of each vehicle. By definition, $U_a(\omega')$ is the total inflow up to time ω'. Let $V_a(\omega'')$ gives the same number of vehicles for exiting that link. Since flow is conserved, the total number of vehicles entering a link must be equal to the total number exiting that link:

$$U_a(\omega') = V_a(\omega'').$$

As no overtaking is allowed due to the FIFO condition, the vehicles leave as the same order as they enter, and the travel time of the last vehicle is equal to the exit time minus the corresponding entry time, i.e. $\omega'' - \omega'$. The travel time of other vehicles can be derived based on the similar analysis, and we find that the travel time of each vehicle J entering at time ω' is the horizontal distance between two cumulative curves for that vehicle. Mathematically, the travel time can be determined by:

$$U_a(\omega) = V_a(\omega + \tau_a(\omega)), \text{ or} \tag{5}$$

$$\tau_a(\omega) = V_a^{-1}[U_a(\omega)] - \omega. \tag{6}$$

The first term on the right hand side in (6) is the exit time and the second term is the entry time. Figure 2 shows the relationship between cumulative curves and travel times under flow conservation and link FIFO.

Taking the derivative of (5) and rearranging gives:

$$v_a(\omega + \tau_a(\omega)) = \frac{u_a(\omega)}{1 + \frac{d\tau_a(\omega)}{d\omega}}. \tag{7}$$

Condition (7) (or (5)) depicts the one-to-one relationship among inflow rate, outflow rate and travel time, and is known as time-flow consistency as it can ensure the consistency between travel time and link flow. This condition makes the traffic flow model theoretical sound. The condition is also called intertemporal conservation, in contrast to contemporaneous conservation (3) or (4), because it considers flows in different time. Condition (7) can be used for propagating traffic flow, and therefore is sometimes referred to as flow propagation.

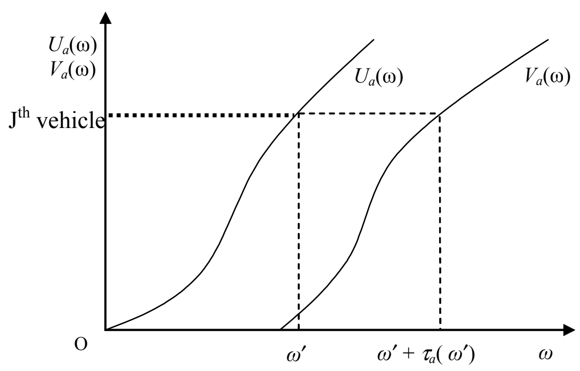

Figure 2. The Relationship between Cumulative Curves and Travel Times

In the literature, time-flow consistency (7) is assumed to ensure FIFO. Actually, the time-flow consistency condition (7) is not equivalent to FIFO. They are equivalent only if we assume that $u_a(\omega) \geq 0$ and $v_a(\omega) \geq 0$ for all ω. The following proposition and corollary emphasize this.

Proposition 1. (Time-flow consistency and FIFO relationship): With the assumption of the inflow $u_a(\omega) \geq 0$ and outflow $v_a(\omega) \geq 0$ for all ω, condition (7) or (5) is necessary and sufficient to ensure FIFO.
Proof: see propositions 3 and 4 in Carey (2004a).

Corollary 1: The condition (7) or (5) on its own (without ensuring $u_a(\omega) \geq 0$ and $v_a(\omega) \geq 0$ for all ω) is neither necessary nor sufficient for the travel time to satisfy FIFO.
Proof: see corollaries 1 and 2 in Carey (2004a).

2.4 Causality

Causality (or strict causality) refers to the property that the link travel times for traffic entering at time ω only depend on the traffic entering at time $\omega' \leq \omega$. This property means that the speed and *travel time* of a vehicle on a link is affected by the speed of vehicles ahead but not by vehicles behind. This property is consistent with vehicle following behaviour. We defined this property through travel time, not outflow. However, when time-flow consistency is ensured, travel time has a one-to-one relationship with link flow and hence causality can also defined by outflow: the outflow rates at time $\omega + \tau_a(\omega)$ only depend on the traffic entering at time $\omega' \leq \omega$, where $\tau_a(\omega)$ is the travel time for vehicles entering at time ω. If the speed and travel time of a vehicle is affected by vehicles ahead and *also* by vehicles behind we refer this as to partial causality, which cannot be observed in traffic. In this paper, causality stands for strict causality.

Partial and strict causality are not equivalent to (Link) FIFO. Moreover, causality is not the derived property from FIFO. The following proposition emphasizes their relationship.

Proposition 2. (Causality and FIFO relationship): Neither partial causality nor strict causality is necessary and sufficient for the travel time to satisfy FIFO.
Proof: See proposition 5 in Carey (2004a).

2.5 Spillback and Junction Blockage

Queue spillback refers to the end of queue spilling backwards in the network. When the queue spills backward passing the junction, the junction is blocked and this phenomenon is known as junction blockage. When the streets are short and the demand for them is high, queue spillback and junction blockage often occurs. This phenomenon must be captured in DTA modelling to get a realistic result.

Although these five considerations are important to be captured in the traffic-flow component, only flow conservation (discussed in 2.1.1) is considered in every traffic flow model but the other considerations are not captured by every traffic flow models, as discussed later.

3. FORMULATION APPROACHES OF THE TRAFFIC-FLOW COMPONENT

The formulation approaches can be classified based on the queue representation: point queue and physical queue, or whether it can capture queue spillback and junction blockage. The point-queue representation treats vehicles as points without physical lengths, whereas the physical-queue representation considers the vehicle lengths. Because of the fundamental difference in their assumption, only the physical-queue representation can capture junction blockage and queue spillback. In the following, we review the existing formulation approaches in each representation, which determine some DTA properties. Emphasis in this section will be on time-flow consistency and the key formulation difference between the two representations.

3.1 Point-queue Representation

In the point-queue representation, we have three existing approaches: the exit flow function approach (e.g., Carey and Srinivasan, 1993; Lam and Huang, 1995), the travel time function approach (e.g., Ran and Boyce, 1996; Yang and Meng, 1998; Huang and Lam, 2002), and the mixed approach (e.g., Yang and Huang, 1997).

3.1.1 The Exit Flow Function Approach

The exit flow function approach usually treats the outflow of a link (or a segment of link) as a non-decreasing function of the corresponding number of vehicles on the whole link (or the link segment) and/or as a function of inflow. Given the inflow rates and occupancies, the outflow rates can be determined by the flow conservation condition (4) and the predefined exit flow functions. The cumulative flows and hence travel times can then be obtained by (1), (2) and (6). This approach aims at finding a solution satisfying flow conservation (3) or (4), time-flow consistency (6) or (7), and the predefined exit flow functions.

3.1.2 The Travel Time Function Approach

The travel time function approach assumes that we have a (separable) travel time function on each link. The independent variables can include link inflow, outflow and occupancy. Given link inflows and link occupancies, we determine the travel times based on the travel time functions and flow conservation (4). We can then determine the outflows based on time-flow consistency (7). The key difference between the travel time function approach and the exit flow function approach is that the former starts with a travel time function and leaves the link outflows to be defined from the corresponding link inflows and travel times, while the latter starts with an exit flow function and leaves the link travel times to be defined later from the corresponding inflows and outflows. This implies that the former calculates link travel times before exit flows whereas the latter reverses the procedure and that the former employs predefined link travel time functions to calculate exit flows whereas the latter relies on predefined exit flow functions to determine exit flows. Alternatively, the travel time function approach can be viewed as finding a solution satisfying flow conservation (3) or (4), time-flow consistency (6) or (7), and the predefined travel time functions.

3.1.3 The Mixed Approach

In this approach, we require both predefined travel time functions and exit flow functions. The outflows are determined based on the exit flow functions but the travel times are determined *separately* by the travel time functions not derived from the corresponding exit flow functions or satisfied the time-flow consistency equation (7). Alternatively, the travel times are determined based on the travel time functions but the outflows are determined *separately* by the exit flow functions not derived from the corresponding travel time functions or satisfied time-flow consistency (7). In both cases, the exit flows and travel times calculated under this approach are not consistent, as opposed to the previous two approaches (Carey, 2004b). This approach formulates the traffic-flow component using flow conservation (3) or (4), and the predefined exit flow and travel time functions not satisfying time-flow consistency (6) or (7).

3.2 Physical-queue Representation

There are two approaches in the physical-queue representation: the advanced exit flow function approach (e.g., Kuwahara and Akamatsu, 2001 and Szeto and Lo, 2004), and the combined approach (e.g., Adamo et al., 1999 and Rubio-Ardanaz et al., 2001).

3.2.1 The Advanced Exit Flow Function Approach

The advanced exit flow functions are derived from the Lighthill and Whitham (1955) and Richards (1956) (LWR) model, the hydrodynamic or kinematic wave model of traffic flow, using the non-smooth flow-density relationship. The main difference between this approach and the original exit flow function approach for the point-queue representation is that the advanced approach considers the storage capacity in the exit flow function to capture the effects of physical queues. Similar to the exit flow function approach, the travel times are determined after the exit flows are known. This approach determines a solution to satisfy flow conservation (3) or (4), time-flow consistency (6) or (7), and the advanced exit flow functions.

3.2.2 The Combined Approach

This approach divides a link into a running segment and a queuing segment. Traffic propagation on the running segment is based on the travel time function whereas that on the queuing segment is based on the simplified advanced exit flow function, which is the exit flow function that only considers the downstream storage capacity but does not consider shockwaves. Since this approach adopts the simplified advanced exit flow functions, the link and route travel times are deduced based on cumulative curves, which are obtained after exit flow determination. This approach determines a solution to fulfil flow conservation (3) or (4), time-flow consistency (6) or (7), the advanced exit flow functions, and the travel time functions.

To sum up, *the physical-queue representation explicitly models the storage capacity of each link whereas the point-queue one does not*. In addition, the mixed approach under the point-queue representation does not ensure time-flow consistency whereas the others do. This implies that the queuing representation indeed has no relationship with time-flow consistency. The travel time and exit flow function approaches under the point-queue representation can ensure time-flow consideration but the mixed approach cannot. To emphasize this, we have:

Proposition 3: The queuing representation (whether queue spillback is captured) indeed has no relationship with time-flow consistency.

4. PROPERTIES OF POINT-QUEUE AND PHYSICAL-QUEUE DTA

This section discusses the properties related to actual traffic behaviour (including FIFO and causality), those related to developing efficient algorithms (Differentiability and monotone properties of route cost) and those related to existence and uniqueness of solutions (continuity of route and OD cost, and monotonicity). These properties are summarized in Table 1.

Table 1. A Comparison of the Properties between Point-queue and Physical-queue DTA

a) Properties Related to Actual Traffic Behaviour

Solution properties	**Point-queue problems**	**Physical-queue problems**
Causality	May or may not satisfy causality, depending on the choice of travel time or exit flow functions.	Obey causality.
Link FIFO	May or may not satisfy Link FIFO, depending on the choice of travel time or exit flow functions.	May or may not satisfy Link FIFO, depending on whether addition variables are introduced to capture Link FIFO.
Route FIFO	Satisfy Route FIFO if they satisfy Link FIFO.	
OD FIFO	Satisfy this property under the DUO condition and certain assumptions, but not satisfy under the SDUO conditions.	

b) Properties Related to Developing Efficient Algorithms, and Solution Existence and Uniqueness

Route cost properties	**Point-queue problems**	**Physical-queue problems**
Continuity w.r.t route flows	*Continuous* under mild assumptions.	Possibly *discontinuous.*
Monotonicity w.r.t route flows	Usually *non-monotonic.*	
Differentiability w.r.t route flows	*Differentiable* under differentiable link travel time functions and *non-differentiable* under continuous exit flow functions.	Possibly *non-differentiable.*
Continuity of OD travel time w.r.t demands	*Continuous* under mild assumptions.	Possibly *discontinuous.*
Existence	Must exist.	May not exist.
Uniqueness	*Non-unique* in terms of route flows and link flows.	

4.1 Properties Related to Actual Traffic Behaviour: FIFO and Causality

4.1.1 First-In-First-Out (FIFO) Properties

As stated before, three FIFO properties are considered in the literature, namely link, route and OD. The three FIFO properties in DTA depend on the choices of the travel flow component as well as the travel choice principle, as discussed below:

Link FIFO

a. Point-queue representation

We consider three approaches in the point-queue representation: travel time function, exit flow function, and mixed approaches. Under the travel time function approach, whether Link FIFO is satisfied depends on the choice of link travel time functions $h_a(.)$. For differentiable travel time functions, they satisfy FIFO for all the entry time ω if and only if $\frac{\partial \tau_a(\omega)}{\partial \omega} > -1$ holds for all the entry time (Carey, 2004a). Friesz et al. (1993) proved that the travel time function $\tau_a(\omega) = k_1 + k_2 x_a(\omega)$ obeys Link FIFO, where $\tau_a(\omega)$ is the link travel time at time ω; k_1 and k_2 are positive constants; $x_a(\omega)$ is the link flow at time ω. For the nonlinear travel time function $\tau_a(\omega) = h_a(x_a(\omega))$, Xu et al. (1999) showed that the travel time function satisfies FIFO under the following sufficient condition: the inflow rate is bounded above by B_a and the derivative of the travel time with respect to its link volume is bounded above by $1/B_a$, where B_a is a constant for link a. The above functions can satisfy the property $\frac{\partial \tau_a(\omega)}{\partial \omega} > -1$ and hence link FIFO. For non-differentiable travel time functions, Huang and Lam (2002) proved that the non-differentiable travel time function in the deterministic queuing model satisfies Link FIFO. However, Daganzo (1995b) proved that the travel time functions depending on the inflow $u_a(\omega)$ violate FIFO.

Under the exit flow function approach, whether solutions follow Link FIFO is determined by the choice of exit flow functions. For single traffic type, FIFO is ensured through the time-flow consistency condition and non-negative inflow and outflow assumption. Nevertheless, nearly all exit flow functions cannot capture Link FIFO when we consider multi-class traffic type, since in most cases, each traffic type has its own time-flow consistency condition. Under the non-negative inflow and outflow assumption, each vehicle satisfies FIFO with respect to its own traffic type but not the other traffic types, as there is no coordination among the travel time of each traffic type - The two traffic types enter the same link at the same time can leave in different time. There are two exceptions. One is the Simplified Cell Transmission Model proposed by Szeto (2003), which is obtained by dropping the storage capacity term in the Daganzo's (1994, 1995a) cell transmission model. Another was proposed by Smith (1993). Comparing with other exit flow functions, these two models/functions contain one more variable to store the information of the waiting time of each traffic packet. Leaving priority is given to those packets which have higher waiting time. Higher waiting time implies entering earlier. This means that both models allow the packets with higher waiting time to leave earlier, implying that Link FIFO is satisfied.

For the mixed approach, Link FIFO is not guaranteed. If the travel time functions, defined independently (e.g., Friesz linear travel time model) or defined by the exit flow function not using the time-flow consistency condition (e.g., $\tau_a(x_a(\omega)) = \frac{x_a(\omega)}{g_a(x_a(\omega))}$), satisfies the sufficient condition in Xu et al. (1999), then FIFO is guaranteed. Note that FIFO is not equivalent to time-flow consistency (see proposition 1). This means that a model can satisfy FIFO but not time-flow consistency. For instance, $\tau_a(x_a(\omega)) = \frac{x_a(\omega)}{g_a(x_a(\omega))}$ satisfies FIFO but not time flow consistency (Carey 2004b).

b. Physical-queue representation
Under the physical-queue representation, the choice of physical-queue traffic flow models controls whether Link FIFO is obeyed in the resultant DTA problems. For example, under the advanced exit flow function approach, the cell transmission model (Daganzo, 1994, 1995a) employs additional variables to ensure the traffic flow obeying FIFO.

In the combined approach, a careful selection of the travel time function and advanced exit flow function is required in order to capture Link FIFO. For example, in Adamo et al.'s model (1999), FIFO is ensured in the queuing segment and there is no restriction on the choice of travel time function on the running segment. If we use Friesz et al.'s. (1993) linear travel time function or a nonlinear travel time function under the sufficient condition in Xu et al. (1998), then FIFO is ensured.

<u>Route FIFO</u>
Like Link FIFO, whether route FIFO is ensured depends on the choice of the traffic-flow component, as Link FIFO implies Route FIFO (Wu et al., 1998).

<u>OD FIFO</u>
Unlike Link FIFO and Route FIFO, OD FIFO depends not only on the traffic-flow component but also the travel choice component. In the following, we discuss OD FIFO based on two common travel choice principles discussed in Ran and Boyce (1996): (i) Dynamic user optimal (DUO) and (ii) Stochastic dynamic user optimal (SDUO)

(i) DUO
We focus on two particular classes of DTA problems: route choice and simultaneously route and departure time choice. For the route choice DUO problems, Wu et al. (1998) proved that Route FIFO together with the DUO route choice conditions implies the OD FIFO condition. This also implies that whether OD FIFO is ensured or not depends on whether Route FIFO is ensured, which in turns depends on the choice of the traffic-flow component. In the DUO simultaneous route and departure time choice problems, travellers optimize for their total generalized costs, which compromise both the schedule delay cost as well as the travel time cost. Hence, the user-optimal simultaneous route and departure time choice may or may not constitute the shortest travel time route like the DUO route choice problems, since the early or late arrival penalties are part of the consideration. The question is whether the DUO simultaneous route and departure time choice problems would reproduce OD FIFO. Szeto and Lo (2004) gave an answer to the question and showed that OD FIFO is only maintained under link FIFO, and DUO and the following conditions on the costs:

1. The travel cost is the sum of the travel time and schedule delay costs;
2. The schedule delay cost is piecewise linear, and;
3. The unit travel time cost is higher than that of early arrival.

Note that the last condition is consistent with the empirical condition found by Small (1982). This means that the theoretical analysis together with the empirical results indicates that OD FIFO should uphold in reality. This finding is consistent with our experience, which reflects that OD FIFO generally holds in reality subject to overtaking not occurring frequently and all travellers having perfect information about the network status.

(ii) SDUO
Under the SDUO conditions, OD FIFO cannot be satisfied. In SDUO route choice problems and SDUO simultaneous route and departure time choice problems, travellers are assumed to

have imperfect information about the network condition. As a result, travellers departing at the same time at the same origin can select routes with different travel times and hence reach their common destination at different times.

4.1.2 Causality

a. Point-queue representation

Again, we consider three approaches under the point-queue representation. Under the travel time function approach, whether causality is satisfied depends on the choice of link travel time functions $h_a(.)$. Researchers proposed many travel time functions. *MOST* of them ensure causality. For example, Friesz et al. (1993) employed the travel time function which depends on the link occupancy; Travel time to be a function of its link inflow, occupancy, and outflow was proposed by Ran and Boyce (1996). None of the independent variables in these functions depend on the flows after the entry time. However, Carey (2004a) pointed out that the travel time function $\tau_a(\omega)=h_a(x_a(\omega+w))$ violates causality, where $h_a(.)$ is the travel time function, and w is a positive number, because the independent variable is a function of the link occupancy after the entry time.

For the exit function approach, the choice of exit flow functions determines whether the approach maintains causality. The exit flow function $v_a(\omega)=g_a(x_a(\omega))$ violates causality (Heydecker and Addison, 1998), or its derived travel time from time-flow consistency (5) violates causality. From flow conservation (3) , we have $x_a(\omega)=U_a(\omega)-V_a(\omega)$, where $U_a(\omega)$ and $V_a(\omega)$ are cumulative inflow and outflow at time ω. Because of the time-flow consistency condition (5) (or $V_a(\omega+\tau_a(\omega))=U_a(\omega)$), we have

$$x_a(\omega+\tau_a(\omega))=U_a(\omega+\tau_a(\omega))-U_a(\omega). \tag{8}$$

Hence, we have

$$g_a(x_a(\omega+\tau_a(\omega)))=g_a(U_a(\omega+\tau_a(\omega))-U_a(\omega)), \tag{9}$$

meaning that the outflow $g_a(x_a(\omega+\tau_a(\omega)))$ at time $\omega+\tau_a(\omega)$ depends on how much traffic has entered the time interval $(\omega,\omega+\tau_a(\omega)]$, and hence the derived travel time based on (5) or (7) violates causality. An exception of the exit flow function is

$$v_a(\omega+\tau_a(x_a(\omega)))=g_a(u_a(\omega),x_a(\omega)), \tag{10}$$

where the outflow (10) follows the time-flow consistency condition (7), and $\tau_a(x_a(\omega))$ in (7) is any existing travel time function that satisfies causality. Since the exit flow function (10) is derived from the travel time function satisfying causality using the one-to-one relationship, the exit flow function (10) can thus satisfy causality. An example of exit flow function satisfying causality is $v_a(\omega+\tau_a(\omega))=\dfrac{u_a(\omega)}{1+\dfrac{x_a(\omega)-x_a(\omega-1)}{\delta s_a}}$, where s_a and δ are the maximum exit rate and the discrete time interval. The corresponding travel time function has a form of $\tau_a(\omega)=\tau_a^0+\dfrac{x_a(\omega)}{\delta s_a}$ ensuring causality, where τ_a^0 is the free flow travel time. From the above discussion, we know that:

Proposition 4: Ensuring time-flow consistency is not sufficient to ensure causality.

For the mixed approach, the selection of predefined travel time functions determines whether the approach maintains causality. For example, Fernandez and de Cea (1994) employed a BPR type travel time function for time ω with the independent variable to be the link occupancy at that time. Thus, causality is ensured. Yang and Huang (1997) adopted the travel time function $\tau_a\left(x_a\left(\omega\right)\right)=\dfrac{x_a\left(\omega\right)}{g_a\left(x_a\left(\omega\right)\right)}$, where $g_a\left(x_a\left(\omega\right)\right)$ is an exit flow function. This travel time function satisfies causality but does not satisfy time-flow consistency in general (Carey 2004b). Thus we have:

Proposition 5: Ensuring time-flow consistency is not necessary to ensure causality.

Proposition 6. (Causality and time-flow consistency relationship): Ensuring time-flow consistency is neither necessary nor sufficient to ensure causality.
Proof: The result follows directly from propositions 5 and 6.

b. Physical-queue representation
As mentioned before, advanced exit flow functions are derived from the hydrodynamic theory. One fundamental property of this theory is that the wave speed is less than the vehicle speed. As a result, the trajectory of a vehicle can only be affected by conditions downstream of it (Heydecker and Addison, 1998) and hence advanced exit flow functions, including Cell transmission model (Daganzo 1994, 1995a), guarantee causality.

Whether the combined approach ensures causality relies on both travel time functions and advanced exit flow functions. The combined approach guarantees causality only if both travel time functions and advanced exit flow functions guarantee causality. In Adamo's model, causality is guaranteed, as the travel time function and the advanced flow function can ensure causality.

4.1.3 Implications from FIFO and Causality

Violations of Link FIFO and causality imply poor reflection of reality because traffic tends to behave in a FIFO manner (Carey, 1992) and that vehicle following is consistent with causality (Heydecker and Addision, 1998). Models that exhibit these violations are unreliable.

As shown in Table 1a, we observe that causality and FIFO have no relationship to spillback consideration. Causality and/or FIFO may or may not be ensured in both queue representations. More formally, we have:

Proposition 7. (Causality, FIFO and queue representation relationship): Causality and FIFO have no relationship to queue representation.

4.2 Properties Related to Solution Existence and Uniqueness and Development of Efficient Algorithms

The properties of the route cost mapping have implications not only on the development of efficient algorithms but also on solution existence and uniqueness. Some efficient algorithms rely on monotone, and differentiability properties of route travel costs. The continuity (strict monotonicity) of route or link cost is a pre-requisite for the existence (uniqueness) of solutions (Nagurney, 1993). These properties depend on the traffic-flow modelling assumption - whether queue spillback is considered - and the form of travel time or exit flow functions adopted. A detailed discussion on these topics can be found in Szeto (2003) and Szeto and Lo (2005). In the following, we highlight the similarities and differences of properties under the two queuing representations, and discuss their implications.

4.2.1 Similarities and Differences of Properties under the Two Queuing Representations

As seen in Table 1b, under both queuing representations, we observe that
(i) The route cost is non-monotonic with respect to route flows;
(ii) The route cost is non-differentiable with respect to route flows if certain traffic flow models are adopted, and;
(iii) Solutions may not be unique if they exist.
However, under the physical-queue representation, we find that
(i) Route costs may not be continuous with respect to route flows;
(ii) OD travel costs may not be continuous with respect to demands, and;
(iii) A Wardropian solution may not exist.

4.2.2 Implications

The non-monotone and non-differentiability properties in physical-queue DTA lead to difficulties in finding solutions if one exists at all, because the convergence of existing algorithms rely on either monotonicity or differentiability. One may rely on less restrictive algorithms such as genetic algorithm (e.g., Lo and Szeto, 2002) or simulated annealing (e.g., Friesz et al., 1992) for solutions.

The non-uniqueness of link flows implies that traffic can be predicted differently in various solutions. This raises the question of accuracy of the DTA models for various applications. Other than this, in actual applications, one must consider all possible solutions to cater for the worse-case scenarios in the planning and design.

All the unique properties are related to the existence of solutions to DTA problems. The implication of the discontinuity property of the supply function is that solutions may not exist for DUO problems with elastic demands. For any OD pair at any departure time, one can imagine that three situations can happen as shown in Figure 3. For cases (i) and (iii), a solution exists to the problem but for case (ii), no solution exists! For the fixed demand case, we clearly observe the trade-off between the existence of solutions and the levels of traffic dynamics captured; point-queue DTA solutions always exist whereas those for physical-queue problems may not.

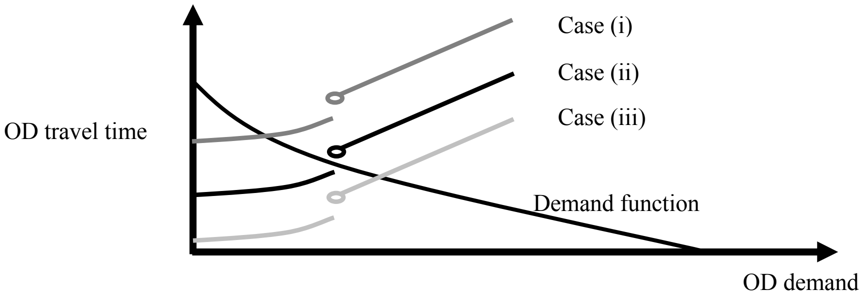

Figure 3. Three Possible Scenarios for the DUO Problem with Elastic Demands

5. CONCLUSIONS

This paper describes five important considerations in the traffic-flow component, namely flow conservation, FIFO, causality, time-flow consistency, and queue spillback, and summarizes the properties of DTA under both the point-queue and physical-queue representations, as shown in Table 1. The implications of properties to offline transport planning and online traffic control are discussed. Causality and FIFO are investigated in details, and their relationship with time-flow consistency, queue representation is addressed. The main findings are that causality is not a derived property from FIFO; Causality is not equivalent to FIFO; The time-flow consistency condition is not equivalent to FIFO; Time-flow consistency cannot ensure causality; Queue representation has no relationship with time-flow consistency nor FIFO and causality; the key difference between the point-queue and physical-queue representations is that the latter includes storage capacity in the resultant formulation to capture the effect of queue spillback.

The future research directions can be broadly classified by theoretical and computational issues. As mentioned before, a Wardropian solution may not exist in physical-queue DTA. To cope with this, in the future, one can relax the exactly equal travel time/cost assumption in DUO for developing new travel choice principles. One possible approach is to allow the travel times of all used routes to be unequal but their maximum difference must not exceed a tolerance or an aspiration level. That is, we develop a travel choice component based on the bounded-rationality behavioural notion. One can also develop other travel choice component that is behaviourally sound and consistent with actual traffic behaviour. Improving the traffic-flow component is another. One can also develop advanced traffic flow models based on the existing traffic flow models to capture realistic traffic behaviours such as shockwaves, queue formulation and dissipation, and queue spillback, lane changing behaviour, hysteresis phenomenon, etc., and use the unique mapping developed by Lo and Szeto (2002) to encapsulate the advanced traffic flow models in a DTA framework.

For the computational issue, developing efficient and/or convergent algorithms for DTA models and the bi-level models with DTA models as a lower level model is one. A possible approach for this is implementing parallelized genetic algorithm (e.g., Wong et al., 2001) for solving these models to increase the computation efficiency. Developing methods, such as using link performance functions at some stages of computation, to approximate the unique mapping of the traffic-flow component and to speed up the computation is another. Moreover, a large network involves many paths, even though most would not be used. A large path set makes path enumeration impossible. However, to deal with queue spillback properly, we must use path based DTA models. Research can be done on the path set generation in DTA models.

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