Journal of the Eastern Asia Society for Transportation Studies, Vol. 6, pp. 2092 - 2107, 2005

MULTIMODE STOCHASTIC DYNAMIC SIMULTANEOUS ROUTE/DEPARTURE TIME EQUILIBRIUM PROBLEM WITH QUEUES

ShuGuang Li
Doctor student
School of electrical engineering, Xi'an JiaoTong University, Xi'an 710049, China
Email: lxlsg@vip.sina.com

YanMing Su
Professor
School of electrical engineering, Xi'an JiaoTong University, Xi'an 710049, China
Email: lxlsg@sohu.com

Abstract

We propose a multimode stochastic dynamic simultaneous route/departure time equilibrium model. The mode-classes typically refer to different vehicle types such as passenger cars, trucks, and buses sharing the same road space. Each vehicle type has its own characteristics, such as free flow speed, vehicle size. We extend single mode deterministic point model to multimode deterministic point model for modeling the asymmetric interactions among various modes. We prove the multimode deterministic point model within each mode-user meets the FIFO discipline, and the speeds of different modes approach consistent during congestion. In order to avoid route enumeration, we present a stochastic dynamic network loading algorithm for simultaneous route and departure time choice maintaining the structure of the DYNASTOCH algorithm of Ran and Boyce (1996), and prove the algorithm does generate the result of the logit flow assignment, and then the diagonalisation algorithm is used to solve multimode stochastic dynamic simultaneous route/departure time equilibrium problem. Finally, the model and algorithm are tested by a numerical example in a simple network.

Keywords: dynamic traffic assignment, multimode deterministic point model.

1. INTRODUCTION

Dynamic traffic assignment problem has attracted the attention of many researchers with the development of intelligent transportation systems, particularly route guidance system and advanced traffic management system. The extensions of dynamic traffic assignment problem are proposed to include more factors such as departure time choices, multiple transportation modes etc in order to achieve more realistic traffic forecasts. The difference between various modes is typically based on vehicle characteristics, e.g. passenger cars and trucks. Interactions between the mode-users sharing the same road infrastructure are taken into account.

Traditionally almost all traffic simulation models include multiple vehicle types. For example, in the mesoscopic simulation model DYNASMART (see Mahmassani *et al* (1994)), passenger cars, bus and trucks can be simulated with onboard vehicle guidance equipment based on vehicle type and information availability. However, all but few macroscopic traffic assignment models assume single vehicle type and driver type, usually an average driver in a passenger car. Since the differences in vehicle types are so big, single vehicle type used in the traffic model cannot achieve more realistic traffic flow conditions. Dafermos(1972) applied the extended traffic assignment model to multimode traffic that is capable of taking the diversity of different vehicle types, sharing the same transportation network. Wynter (1995) discussed the behavioral incoherence in static multi-class assignment models by analyzing the multi-class travel cost functions. They derived a mathematical structure for multi-class cost functions to avoid this incoherence. Ran and Boyce (1996) were among the first to use an analytical approach to modeling multimode DTA. Bliemer and Bovy (2003) extended single mode dynamic traffic assignment to multimode dynamic assignment model, considering different driving characteristic, network usage and route choice behavior. However, the multimode dynamic traffic assignment model proposed by Bliemer and Bovy does not consider the link exit capacity. In order to model asymmetric interactions between different vehicle types such as passenger cars, trucks, buses sharing the same road space, we extend single mode deterministic queue model proposed by Li.Jun (2000), Huang (2002), Han (2003) to multimode deterministic queue model. In this true multimode DTA model, not only the route and departure time choice process will be different for each mode, but also the traffic flow operations are mode specific.

In this paper, firstly a continuous-time multimode dynamic traffic network model is presented. Then the FIFO discipline of the link travel time function within each mode is proved and the speeds of different modes approach consistent during congestion. In section 3,we extend continuous-time model to discrete-time model. In section 4, multimode stochastic dynamic simultaneous route/departure time equilibrium problem is formulated as a VI problem. In section 5, we present a new stochastic dynamic network-loading algorithm for stochastic dynamic simultaneous route/departure time choice in order to avoid route enumeration, and then the diagonalisation algorithm is used to solve the VI problem. Finally, a numerical example is given to illustrate the advantages of the proposed method.

2. CONTINUOUS TIME NETWORK MODEL

We consider a network $G(N, A)$ composed of a finite set of nodes, N, and a finite set of directed links, A. Let R be the set of origin nodes, and S be the set of destination nodes. Let a be a link, and let p be a route which is simply an acyclic ordered set links, $\{a_1,a_2,\ldots,a_n\}$, that connects an origin and a destination. K_{rs} represents the set of all feasible route set between an OD pair rs. Here, Let M denote the set of all modes, examples of modes include passenger cars, trucks and public transportations, etc. Denote pcu_m: as the passenger car equivalents parameter of mode m (here, denote pcu as the unit of passenger car)

Notations:

$v_{am}(t),u_{am}(t)$: link a inflow and outflow rate of mode m travelers at time t;

$t_{am}(t)$: travel time experienced by mode m travelers on link a at time t;

$q_{am}(t)$: queue length of mode m travelers on link a at time t;

$v_{am}(k),u_{am}(k)$: link a inflow and outflow rate of mode m travelers during time interval k;

$t_{am}(k)$: travel time experienced by mode m travelers on link a during time interval k

$q_{am}(k)$: queue length of mode m travelers on link a during time interval k;

$t^{rs}_{pm}(k),c^{rs}_{pm}(k)$: travel time and cost of mode m travelers on path p of OD pair rs during interval k;

$f^{rs}_{pm}(k)$: inflow rate of mode m travelers entering path p of OD pair rs during time interval k;

Q^{rs}_{m}: travel demand between OD pair rs.

2.1 Link Dynamic Function

Here, following Kuwahara and Akamatsu (1997), Jun.Li (2000), Huang (2002), S. Han (2003), a single-mode deterministic point queue model is extended to multimode deterministic point queue model. We assume each link consists of two distinct parts. The first part is the running part of the link that each mode of travelers can run according to the each mode of free-flow velocity. In other words, each mode of travelers experiences the constant running time t_{am} to the exit queue part of link (there assume $t_{a1}<t_{a2}<\ldots<t_{aw}$, due to the difference of the velocity of various modes under un-congested traffic conditions. For example, the velocity of car is higher than that of truck under un-congested traffic conditions). The second part is the exit queue part (the vehicle of mode m is assumed to be a point without length). The queue delay experienced by mode m travelers is caused by the limited link exit capacity (in *pcu*), or the maximum link exit flow rate (in *pcu/h*).

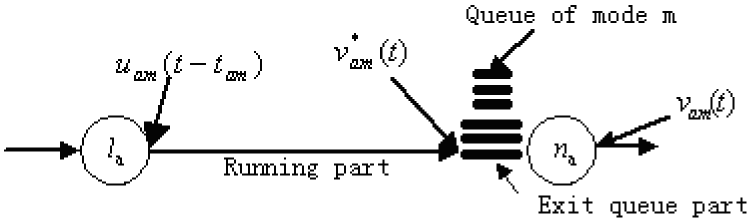

Figure 1. Link Flow propagation Conditions

The link flow propagation conditions for mode m travelers are depicted in Fig.1. The flow of mode m travelers entering link a at time $t-t_{am}$, $u_{am}(t-t_{am})$, experiences the constant running time t_{am} to arrive at the exit queue of the link, and the arrival flow rate of mode m travelers to the exit queue at time t on link a is $v^*_{am}(t)$, the flow rate of mode m travelers exiting from the exit queue is $v_{am}(t)$. Thus, the link dynamic functions can be formulated as follows.

Running part:

$$v^*_{am}(t)=u_{am}(t-t_{am}) \quad \forall a,m,t \tag{1}$$

Queue part:

$$\frac{dq_{am}(t)}{dt}=v^*_{am}(t)-v_{am}(t) \quad \forall a,m,t \tag{2}$$

Let $q_{am}(t)$ be the number of mode m travelers waiting in the queue at time t on link a. It is to say that the queue length marginal change of mode m travelers is equal to the difference between the arrival flow rate of mode m travelers to exit queue part at time t and the departure flow rate of mode m travelers from exit queue part at time t on link a.

2.2 Link Exit Flow Function

We will use the following assumptions for deriving multimode link exit flow function

(1). The class-specific pcu_m parameter that transforms the effect of mode m into passenger car equivalents is fixed under all traffic conditions.

(2). The mixture of the travelers of various modes is homogenous on the link.

The temporal and spatial interactions of travelers of various modes mainly appear in the exit queue part of the link. If the sum of queue length of travelers of various modes, $\sum_m pcu_m \cdot q_{am}(t)>0$(in *pcu*), or the sum of arrival flow rate of travelers of various modes to the exit queue $\sum_m pcu_m \cdot v^*_{am}(t) \geq s_a$ (in *pcu/h*), then the arrival flow to exit queue mustn't wait and the departure flow rate exiting the link exit queue is s_a. According to assumption (2), the link outflow rate of mode m travelers from the exit queue can be calculated as follows.

$$v_{am}(t) = \frac{v^*_{am}(t)}{\sum_m pcu_m \cdot v^*_{am}(t)} s_a \quad \forall a,m,t \tag{3}$$

If $\sum_m pcu_m \cdot q_{am}(t)$=0, or $\sum_m pcu_m \cdot v^*_{am}(t) < s_a$. Mode m travelers will pass the exit queue without delay. Thus, the link exit function can be formulated as follows.

$$v_{am}(t) = \begin{cases} \dfrac{v^*_{am}(t)}{\sum_m pcu_m \cdot v^*_{am}(t)} s_a & if \ \sum_m pcu_m \cdot q_{am}(t) > 0 \ \ or \ \sum_m pcu_m \cdot v^*_{am}(t) \geq s_a \\ v^*_{am}(t) & otherwise \end{cases} \quad \forall a,m,t \tag{4}$$

Furthermore, substituting equation (4) into equation (2), The link exit queue function can be expressed as follows.

$$\frac{dq_{am}(t)}{dt} = \begin{cases} v^*_{am}(t) - \dfrac{v^*_{am}(t)}{\sum_m pcu_m \cdot v^*_{am}(t)} s_a & if \ \sum_m pcu_m \cdot q_{am}(t) > 0 \ \ or \ \sum_m pcu_m \cdot v^*_{am}(t) \geq s_a \\ 0 & otherwise \end{cases} \quad \forall a,m,t \tag{5}$$

2.3 Link Travel Time Function

The queue delay of mode m travelers on the link can be determined only by the total queue length of various modes in the exit queue on that link. Mode m travelers entering link a at time t will spend the travel time t_{am} to arrive at the exit queue. On the other hand, they will enter into the exit queue of the link at time $t+t_{am}$ and the number of total queue length (in *pcu*) in the exit queue at time $t+t_{am}$ is $\sum_m pcu_m \cdot q_{am}(t + t_{am})$. Thus, the queue delay on link a for mode m travelers entering link a at time t can be given as

$$d_{am}(t) = \frac{\sum_m pcu_m \cdot q_{am}(t + t_{am})}{s_a} \quad \forall a,m,t \tag{6}$$

The total link travel time for mode m travelers entering link a at time t is the sum of the travel time of the running segment and the queue delay. As below.

$$t_{am}(t) = t_{am} + d_{am}(t) = t_{am} + \frac{\sum_m pcu_m \cdot q_{am}(t + t_{am})}{s_a} \quad \forall a,m,t \tag{7}$$

The following lemma proves the preservation of the FIFO discipline within each mode-user when dynamic link travel time of mode m travelers is computed using above equations (6), (7).

Till now, we can consider single-mode point queue model as a special case of the above model formulated in this paper. If only one vehicle type (truck or car) appear on a link, let the link inflow rate of the other vehicle types be 0, and then the above model (1)-(7) is equivalent to single-mode point queue model.

Blemier(2003) and Toint and Wynter (1996) given the multiple transportation mode link travel time functions for dynamic (static) traffic assignment and indicate the speeds of different modes approach consistent during congestion. On the other hand, there is no reason to believe that a certain mode-user is able to drive (much) faster than other mode-users during congestions.

***Lemma* 1**: the ratios of link travel times of travelers of various modes approach 1 as congestion is approached.

Proof: Where $q_a(t^*)$ represents the total queue length (in *pcu*) in the exit queue at time t^* on link *a*, $q_a(t^*) = \sum_m pcu_m \cdot q_{am}(t^*)$. While approaching congestion, we can assume $q_a(t^*)$ becomes high enough during period $[t_1, t_2]$. On the other hand, we can consider the link queue delays in the exit of the link *a* experienced by mode m travelers entering link at time *t*, $d_{ai}(t), d_{aj}(t), i, j \in M$, are high enough too during period $[t_1, t_2]$. Thus, we can neglect the effects of the free flow travel time of travelers of various modes on link *a*. Finally, we obtain

$$\frac{t_{ai}(t)}{t_{aj}(t)} = \frac{t_{ai} + d_{ai}(t)}{t_{aj} + d_{aj}(t)} \to 1$$

The proof completes.

***Lemma* 2**: the dynamic link travel time function of mode *m* travelers meets the FIFO condition. On the other hand, a user of a certain modes entering a link later should not leave that link earlier than another user of that same mode that already drives on the same link. The FIFO condition only needs to be satisfied within each mode, but need not be satisfied across modes under non-congested conditions. According to Lemma 1, the FIFO may be satisfied across modes during congestions..

Proof: According to equation (5)

We can obtain

$$\sum_m pcu_m \cdot dq_{am}(t + t_{am}) / dt = \begin{cases} \sum_m pcu_m \cdot v_{a1}^*(t + t_{am}) - s_a \\ 0 \end{cases} \quad \forall a, m, t \tag{8}$$

Thus

$$\sum_m pcu_m \cdot dq_{am}(t + t_{am}) / dt \geq -s_a \quad \forall a, m, t \tag{9}$$

Here

$$\tau_{am}(t) = t + t_{am}(t) = t + t_{am} + \frac{\sum_m pcu_m \cdot q_{am}(t + t_{am})}{s_a} \quad \forall a, m, t \tag{10}$$

$$\frac{d\tau_{am}(t)}{dt} = 1 + \frac{\sum_m pcu_m \cdot dq_{am}(t + t_{am}) / dt}{s_a} \geq 0 \quad \forall a, m, t \tag{11}$$

The proof completes.

3. DISCRETE TIME NETWORK MODEL

In the discrete time formulation, Time horizon *T* is divided by a finite number of time interval $k \in \{1, 2, \Lambda, n\}$. Let δ be the length of time interval. In other words, we have *n* time intervals; index *k* represents the time interval $[(k-1) \cdot \delta, k \cdot \delta)$. Here we assume the time horizon is large enough so that no vehicles will remain on the network in the terminal time interval *n* and the length of the time interval is small enough that the discrete network model is close to its continuous time counterpart.

We assume the link inflow and departure flow rates of mode *m* travelers, $v_{am}(k), u_{am}(k)$ are constant during the time interval *k*. on the other hand, the total link travel time $t_{am}(k)$, the length of exit queue

$q_{am}(k)$ and the queue delay $d_{am}(k)$ of mode m travelers are state variable and are defined exactly at time $k \cdot \delta$ and are not constants during the time interval $[(k-1)\delta, k\delta)$.The calculations of the state variable for non-integer time make use of the round-off or linear interpolation way.

3.1 Link Dynamic Function

The discrete-time link dynamic function on link a can be expressed as
running part:

$$v^*_{am}(k) = u_{am}(k - t_{am}) \quad \forall a, m, k \tag{13}$$

Queue part:

$$q_{am}(k) - q_{am}(k-1) = (v^*_{am}(k) - v_{am}(k)) \cdot \delta \quad \forall a, m, k \tag{14}$$

Where $q_{am}(k)$ represents the link exit queue length of mode m travelers during time interval k on link a, $v^*_{am}(k)$ and $v_{am}(k)$ represent the arrival flow rate and departure flow rates to exit queue part of mode m travelers during time interval k on link a, respectively.

3.2 Link Exit Flow Function

If $\sum_m pcu_m \cdot q_{am}(k) = 0$, then $q_{am}(k) = 0, \forall m$. Substituting it into equation (14), then we obtain

$$v_{am}(k) = v^*_{am}(k) + q_{am}(k-1)/\delta \quad \forall a, k, m \tag{15}$$

If $\sum_m pcu_m \cdot q_{am}(k) > 0$, Substituting it into equation (14), we obtain

$$\sum_m pcu_m \cdot (v^*_{am}(k) + q_{am}(k-1)/\delta) \geq s_a \tag{16}$$

Based on equation (4) and mixture homogenous assumption, the link departure flow rate of mode m travelers can be calculated as follows.

$$v_{am}(k) = \frac{v^*_{am}(k) + q_{am}(k-1)/\delta}{v^*_a(k)} \cdot s_a \quad \forall a, k, m \tag{17}$$

Where

$$v^*_a(k) = \sum_m pcu_m \cdot (v^*_{am}(k) + q_{am}(k-1)/\delta) \tag{18}$$

Thus, the discrete time exit flow function can be expressed as

$$v_{am}(k) = \begin{cases} \dfrac{v^*_{am}(k) + q_{am}(k-1)/\delta}{v^*_a(k)} \cdot s_a & if \quad v^*_a(k) \geq s_a \\ v^*_{am}(k) & otherwise \end{cases} \quad \forall a, k, m \tag{19}$$

The discrete time link queue function can be expressed as

$$q_{am}(k) = \begin{cases} v^*_{am}(k) \cdot \delta + q_{am}(k-1) - \dfrac{v^*_{am}(k) \cdot \delta + q_{am}(k-1)}{v_a(k)} \cdot s_a & if \quad v^*_a(k) \geq s_a \\ 0 & otherwise \end{cases} \quad \forall a, k, m \tag{20}$$

3.3 Link Travel Time

Total link travel time function for mode m travelers entering link a during time interval k is the same with that in continuous time case as equation (7).

$$t_{am}(k) = t_{am} + \frac{\sum_m pcu_m \cdot q_{am}(k + t_{am})}{s_a} \quad \forall a, k, m \tag{21}$$

3.4 The Actual Route Travel Time And Travel Cost Functions

Actual travel time to traverse the route $p=\{a_1,a_2,...,a_n\}$ for mode m travelers entering into the network during time interval k can be calculated using the following nested function.

$$t_{pm}^{rs}(k) = t_{a1m}(k) + t_{a2m}(k + t_{a1m}(k)) + \Lambda + t_{anm}(k + t_{a1m} + \Lambda + t_{an-1m}) \quad \forall rs, p, k, m \tag{22}$$

Where $t_{a1m} = t_{a1m}(k), t_{a2m} = t_{a2m}(k + t_{a1m}(k))$ $(\forall m)$ for short.

Considering the schedule delay cost function of mode m travelers as follows.

$$Sch_{sm}(k) = \begin{cases} \beta\left[t_s - \Delta_s - k - t_{pm}^{rs}(k)\right] & if\ t_s - \Delta_s > k + t_{pm}^{rs}(k) \\ \gamma\left[k + t_{pm}^{rs}(k) - t_s - \Delta_s\right] & if\ t_s + \Delta_s < k + t_{pm}^{rs}(k) \\ 0 & otherwise \end{cases} \quad \forall k, m \tag{23}$$

Denote $\left[t_s - \Delta_s, t_s + \Delta_s\right]$ as the desired time interval of the travelers for arrival at the destination s in the network. Where $t_s - \Delta_s$ is the commuter's desired earliest arrival time of the travelers at the destination s, $t_s + \Delta_s$ is the desired latest arrival time of the travelers at the destination s. β and γ are the unit cost of schedule delay early and late of the travelers at the destination s, respectively.

Therefore, the travel cost of a trip from origin r to destination s on route p for mode m travelers leaving origin r during time interval k can be expressed as follows

$$c_{pm}^{rs}(k) = \alpha_m t_{pm}^{rs}(k) + Sch_{sm}(k) \quad \forall rs, p, k, m \tag{24}$$

Where α_m is a convention factor of mode m travelers to transform the route travel time into the route travel cost. According to the empirical results, we assume that $\gamma_m > \alpha_m > \beta_m$ holds

4. MULTIMODE STOCHASTIC DYNAMIC SIMULTANEOUS ROUTE/DEPARTURE TIME EQUILIBRIUM

The definition of multimode stochastic dynamic simultaneous route/departure time equilibrium is given as follows.

Definition 1: *For each user-mode and for each origin-destination (OD) pair, the perceived route travel costs experienced for all users of mode m,regarding of the departure time, is equal and minimum, and less than (or equal to) the perceived route travel costs for users of mode m on any unused route.*

This definition is similar with the definition of multimode deterministic dynamic simultaneous route/departure time equilibrium presented by Blemier (2002). Mathematically, by adopting the logit model, Multimode stochastic dynamic simultaneous route/departure time equilibrium conditions can be characterized by the following expressions.

$$f_{pm}^{rs}(k) = \frac{exp(-\theta_m \cdot c_{pm}^{rs}(k))}{\sum_p \sum_k exp(-\theta_m \cdot c_{pm}^{rs}(k))} \cdot Q_m^{rs} \quad \forall rs,p,k,m \tag{25}$$

These equilibrium conditions can be expressed as below.

$$\hat{C}_{pm}^{rs}(k, f^*) \begin{cases} = \hat{C}_{m,\min}^{rs} & if\ f_{pm}^{rs*}(k) > 0 \\ > \hat{C}_{m,\min}^{rs} & if\ f_{pm}^{rs*}(k) = 0 \end{cases} \quad \forall rs, k, p, m \tag{26}$$

$$\sum_{p}\sum_{k} f_{pm}^{rs*}(k) = \frac{Q_m^{rs}}{\delta} \qquad \forall rs, m \tag{27}$$

$$\hat{C}_{pm}^{rs}(k, f^*) = c_{pm}^{rs}(k) + \frac{1}{\theta_m}\ln\frac{f_{pm}^{rs*}(k)}{Q_m^{rs}}\delta \quad \forall rs, k, p, m \tag{28}$$

$$f_{pm}^{rs*}(k) \geq 0 \qquad \forall rs, k, p, m \tag{29}$$

Where Equation (28) represents the calculation of the perceived travel cost of route p between OD pair rs for mode m travelers, the first term on the left side of Equation (28) is the actual path travel cost, the second term represents the stochastic value of the perceived travel cost. $f_{pm}^{rs*}(k)$ represents the inflow rate of route p between OD pair rs for mode m travelers entering network during time interval k, $\hat{c}_{min\ m}^{rs}$ is the minimum unit perceived travel cost for mode m travelers between OD pair rs, Q_m^{rs} is the given demand for mode m travelers between OD pair rs. f is the vector of all route inflow rates for mode m travelers. Equation (27) represents the flow conservation constraint and Equation (29) states the non-negativity constraint.

Equation (26) expresses that any used route and chosen departure time (by mode m travelers) has minimal route cost for that OD pair. All other unused routes and un-chosen departure times are equally or more expensive. Obviously, the set of cheapest departure times and routes may be different for different mode-travelers.

The equivalent variational inequality (VI) formulation of multimode stochastic dynamic simultaneous route/departure time equilibrium (26) can be stated as follows:

Lemma 3. Above equilibrium problem (26) is equivalent to the following **VI** problem: find a vector $f^* \in \Omega$ such that

$$\sum_{rs}\sum_{p}\sum_{k}\sum_{m} \hat{c}_{pm}^{rs*}(k, f^*)(f_{pm}^{rs}(k) - f_{pm}^{rs*}(k)) \geq 0 \qquad \forall f \in \Omega \tag{30}$$

Where Ω is defined as the set of all $\boldsymbol{f}$ satisfying the following constraints

$$\Omega = \left\{ \mathrm{f} \middle| \sum_{p}\sum_{k} f_{pm}^{rs}(k) = \frac{Q_m^{rs}}{\delta}, f_{pm}^{rs}(k) \geq 0, \forall rs, m \right\}$$

The above multinomial logit function (equation 25) for modeling travelers' simultaneous path and departure time choice behaviors is a very simplistic model that might give unrealistic result of prediction since they neglect the impacts of path overlap. In further studies, a general C-logit, PS-logit and Probit model are used.

5. ALGORITHM

The optimal solution to variational inequality (30) can be found in the framework of the diagonalisation method (Ran and Boyce, 1996). Before describing the diagonalisation method in detail, we will see how we can perform stochastic dynamic network loading, which is essential to find feasible link flow patterns. Note that this paper develops a stochastic dynamic network loading method considering the logit-based route and departure time choices. The logit-based route and departure time choice function can be expressed as follows:

$$P_p^{rs}(k) = \frac{\exp(-\theta \cdot c_p^{rs}(k))}{\sum_p \sum_k \exp(-\theta \cdot c_p^{rs}(k))} \quad \forall rs, p, k \tag{31}$$

5.1 Dynamic Stochastic Network Loading Method

In this section, stochastic dynamic network loading method for the logit-based route and departure time choice is proposed. This network loading method is similar to the method proposed by Dial's STOCH for stochastic static network assignment and the method proposed by Ran and Boyce's DYNASTOCH for stochastic dynamic network assignment. In this study, we consider only the logit model for stochastic dynamic simultaneous route/departure time choice. The method maintains the structure of the DYNASTOCH method, so only deals with reasonable routes, and assigns the demand between OD pair *rs* to the link of the network according to the actual link travel cost. (The stochastic dynamic network loading method is denoted as SRD-DYNASTOCH)

In order to reflect the effects of schedule delay cost in the algorithm, we extend the original network to include the dummy link with schedule delay cost, $c_m^{s's}(k) = sch_{sm}(k), \forall m$ as shown in Fig.2.

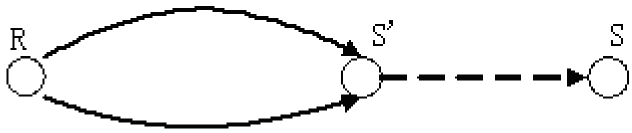

Figure 2. Extended Network

Step 1: Calculation of link likelihood

Compute the minimum actual travel cost $\pi_{is}(k)$ for travelers departing node *i* during time interval *k*. calculate the link likelihood, $L_{(i,j)}(k)$,for each link (*i,j*) during each time interval *k*:

$$L_{(i,j)}(k) = \begin{cases} \exp(\theta[\pi_{is} - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & if \quad C_o^{rs} > C_o^{js} \\ 0 & otherwise \end{cases} \quad i \in r \tag{32}$$

$$L_{(i,j)}(k) = \begin{cases} \exp(\theta[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & if \quad C_o^{is} > C_o^{js} \\ 0 & otherwise \end{cases} \quad i \notin r \tag{33}$$

Where equations (31) and (32) express the calculation way of the link likelihood when the head node *i* of link (*i,j*) is and isn't the origin *r*, respectively.

$\pi_{is}(k)$ is the minimum travel cost from *i* to *s* by departing the node *i* during time interval *k*

π_{rs} is the minimum route travel cost from origin *r* to destination *s* for all departure time. $\pi_{rs} = \min\{c_p^{rs}(k), \forall p, k\} \quad \forall rs$

C_o^{is} is the ideal travel cost from *i* to *s* when there is no flow in the network

$t_{(i,j)}(k)$ is link travel time experienced by travelers entering into link (*i,j*) during interval *k*

$c_{(i,j)}(k)$ is link travel cost experienced by travelers entering into link (*i,j*) during interval *k*.

Step 2: backward pass

By examining all nodes *j* in ascending sequence with respect to $\pi_{is}(k)$ from the destination *s*, calculate $w_{(i,j)}(k)$, the link weight for each link (*i,j*) during each time interval *k*:

$$w_{(i,j)}(k)=\begin{cases}L_{(i,j)}(k) & if \quad j=s \\ L_{(i,j)}(k)\cdot \sum_{(j,k)\in A(j)} w_{(j,k)}(k+t_{(j,k)}(k)) & otherwise\end{cases} \tag{34}$$

Where *A(j)* is the set of links starting from node *j,* When the origin *r* is reached, stop

Step 3: forward pass

Consider all nodes *i* in descending sequence with respect to $\pi_{is}(k)$, starting with the origin *r*. when each node *i* is considered during each time interval *k*, compute the inflow to each link (*i,j*) during each time interval *k* using the following formula:

$$v_{(i,j)}(k)=\begin{cases} q^{rs}\cdot \dfrac{w_{(i,j)}(k)}{\sum_k \sum_{(i,k)\in A(i)} w_{(i,k)}(k)} & if \quad i=r \\ \left\{\sum_{(k,i)\in B(i)} u_{(k,i)}(k)\right\}\cdot \dfrac{w_{(i,j)}(k)}{\sum_{(i,k)\in A(i)} w_{(i,k)}(k)} & otherwise\end{cases} \tag{35}$$

Where, *B(i)* is the set of links ending at node *i.* When the destination *s* is reached, stop

The flow generated by the algorithm is equivalent to a logit-based flow independent route/departure time assignment between each OD pair, given the reasonable route set is fixed in order to produce a convergence solution. Proofs of the algorithm see appendix.

4.2 Diagonalisation Method

Here, we propose the diagonalisation method to solve multimode stochastic dynamic simultaneous route/departure time equilibrium problem. The method is similar with that of Ran and Boyce (1996), and Han (2003). The method consists of the outer and inner iterations. Outer iteration includes the updating estimation of actual link travel time or link inflow rate. Inner iteration calculates the link inflow updating direction and the auxiliary link inflow rate by the method of successive averages. The method can be expressed as follows.

Step 0: initialization: Set outer iteration counter *i*=1, perform SRD-DYNASTOCH for the given demand $Q_m^{rs}, \forall rs, m$ according to free flow travel time and travel cost, find initial link inflow rate

$u_{am}^{i}(k), \forall a, \forall k, m$

Step 1: inner iteration (MSA)

Step1.0 initialization: set inner iteration counter *j*=1, $u_{am}^{j}(k)=u_{am}^{i}(k), \forall a,k,m$

Step1.1: calculate link travel time and travel cost $t_{am}^{j}(k), c_{am}^{j}(k), \forall a,k,m$ using $u_{am}^{j}(k), \forall a,k,m$.

Step 1.1 direction finding: perform SRD-DYNASTOCH for the given demand $Q_m^{rs}, \forall rs, m$, according to current actual link travel time and travel cost $t_{am}^{j}(k), c_{am}^{j}(k), \forall a,k,m$. This generates auxiliary link flow $\hat{u}_{am}^{j}(k), \forall a,k,m$.

Step 1.2 move: update flow pattern as

$$u_{am}^{j+1}(k)=u_{am}^{j}(k)+\lambda^{j}(\hat{u}_{am}^{j}(k)-u_{am}^{j}(k)), \forall a,k,m$$

Step 1.3 convergence test of inner iteration: if

$\sqrt{\sum_m \sum_a (u_{am}^{j+1}(k) - u_{am}^{j}(k))^2} \Big/ \sum_m \sum_a (u_{am}^{j}(k)) \leq \gamma$ (γ is a predetermined tolerance) or j is equal to a given number, then stop; otherwise, go to step 1.1 and set j=j+1.

Step 2 convergence test of outer iteration: if the convergence criteria are satisfied or i is equal to a given number, stop; otherwise, go to step 1 and set i=i+1.

The step size λ^j is a predetermined value, we set $\lambda^j = 1/j$,or $\lambda^j = 1$ .in order to maintain correct flow propagation constraint, we calculate new link flow pattern by directly updating link choice probability (Han(2003)) or using pure network loading.

6 .NUMERICAL EXAMPLE

In this section, the above algorithm is used to find the solution for multimode stochastic dynamic simultaneous route/departure time equilibrium model and we perform only one iteration in the inner iteration of the diagonalization method according to Sheffi (1985)'s advice.

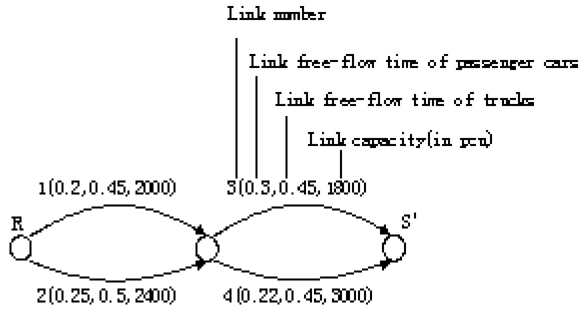

Figure 3. Example Network

Two different modes on the network are passenger cars (mode 1) and trucks (mode 2), respectively. The example network includes 4 links, 1 origin, and 1 destination as shown in Fig.3. All free flow link travel times of passenger cars and trucks and link exit capacity (in *pcu*) are given in this figure. Other input data are: $\alpha_m = 6.4$ (RMB/h), β_m=4 (RMB/h), $\gamma_m = 22$ (RMB/h), Δ_m=0.25h,k_m^*=9.0h, $\theta_m = 0.1$ (m=1,2),set T be from 5 to 11.A.m.and K=600, δ=0.6min. The demands of passenger car and truck are 4000 and 4000, respectively

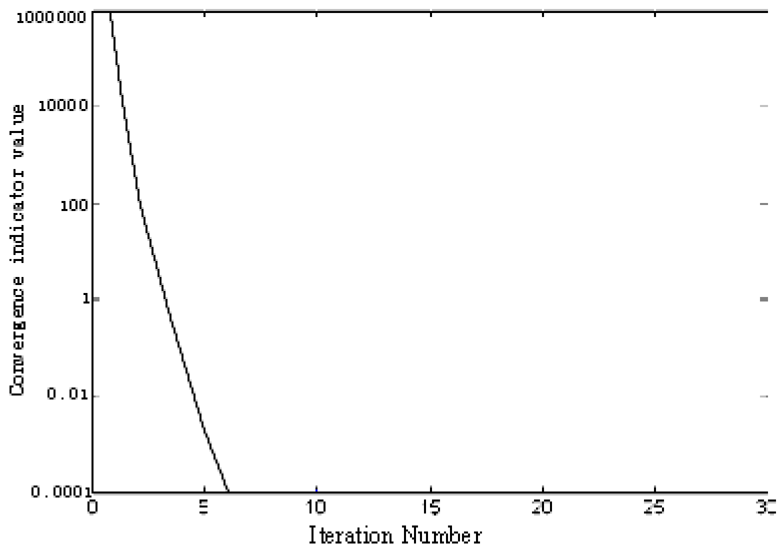

Figure 4. Convergence Of The Algorithm

The value of the convergence indicator of the algorithm is decreasing as the iteration number increase as shown in Fig.4. When the iteration number amounts to 10, the values of the convergence indicator approximate very much small value. After 10 iterations, the value of the convergence indicator changes very small.

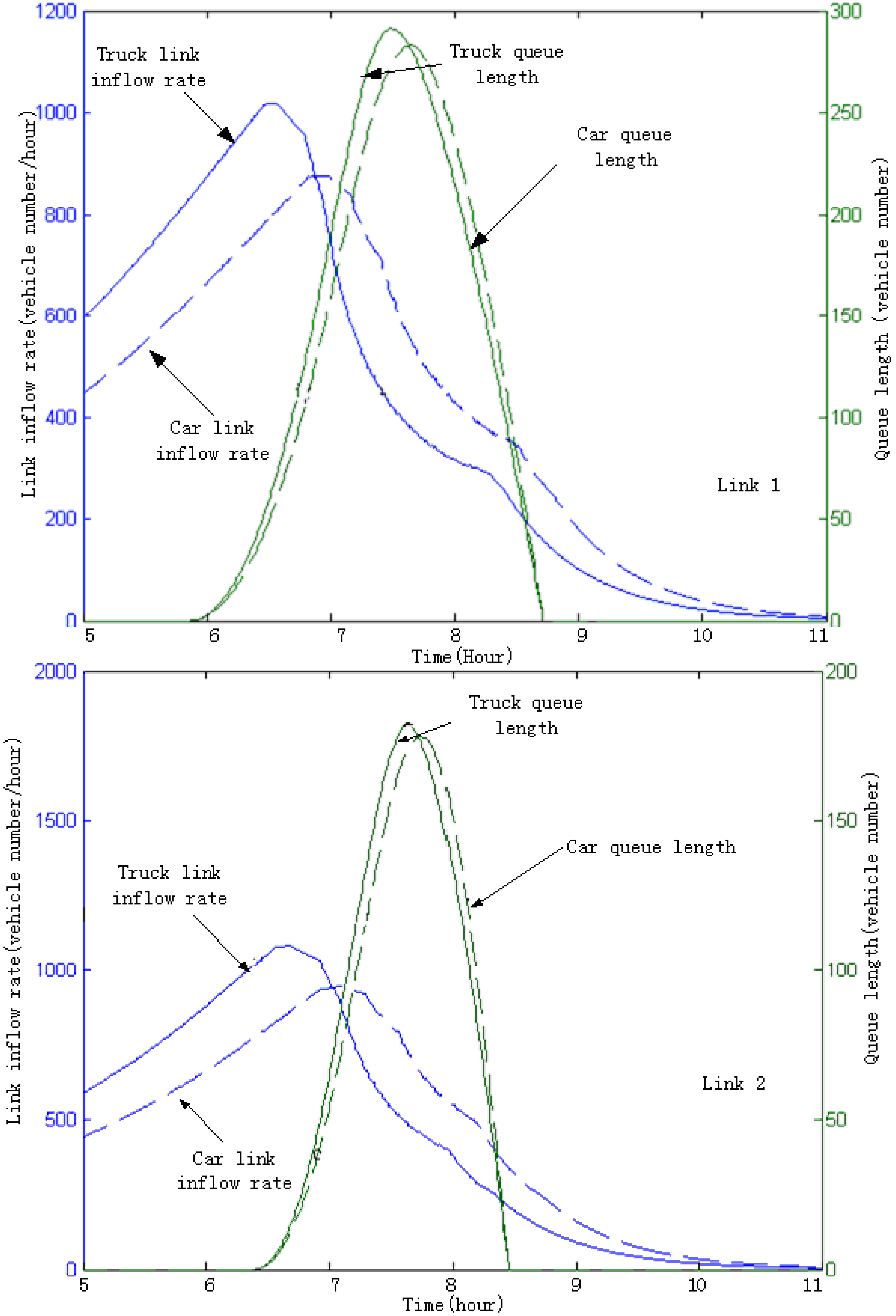

Figure 5. Link Inflow Rates And Queue Lengths Of Trucks And Passenger Cars (Links 1,2)

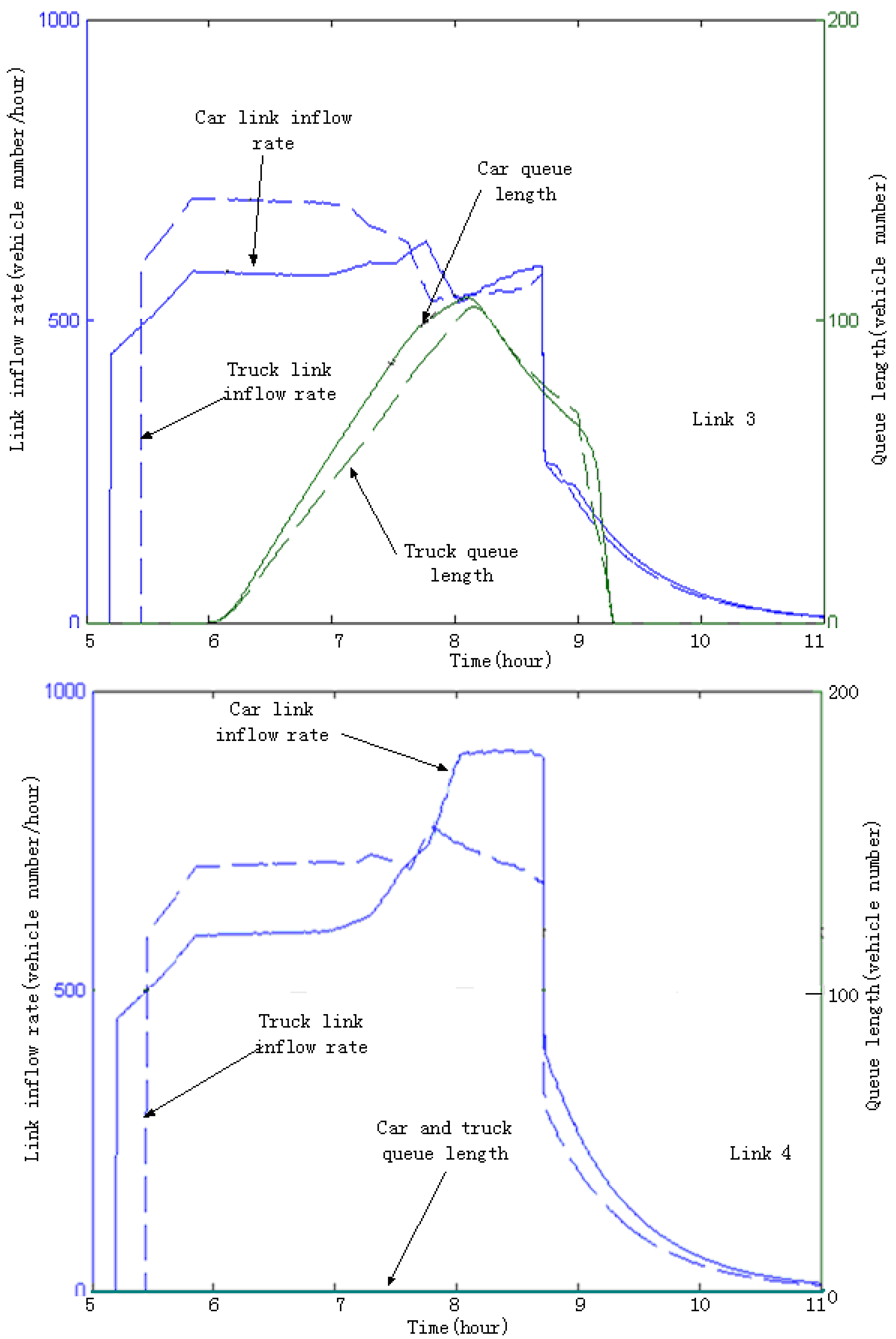

Figure 6. Link Inflow Rates And Queue Lengths Of Trucks And Passenger Cars (Links 3,4)

Fig.5 gives the link inflow rates and the queue lengths of trucks and passenger cars on links 1 and 2. We can find the departure time of truck travelers is generally earlier than passenger car travelers on links 1,2 in Fig.5. Because the link free travel time of truck travelers is longer than passenger car travelers (the speed of trucks is less than that of passenger cars under un-congested conditions). Truck travelers choose earlier departure than passenger car travelers In order to avoid more schedule delay costs.

Fig.6 presents the link inflow rate and queue length of trucks and passenger cars on links 3 and 4. And the queue lengths of passenger cars and trucks are given on Figs.5 and 6, respectively. We can find the queue length of passenger cars and trucks on links 1,2,3 are same. Because there is high exit capacity (in *pcu*) on link 4, the queue lengths of passenger cars and trucks are zero on link 4.

7. CONCLUSIONS

A new model for multimode dynamic traffic assignment in general networks with queues is proposed in this paper. Single mode deterministic point queue model is extended to multimode deterministic point queue model. Multimode stochastic dynamic simultaneous route/departure time equilibrium problem is formulated as a route-based VI problem. In order to avoid route enumeration, we extend DYNASTOCH method to calculate stochastic dynamic simultaneous route/departure time network loading based on the logit flow assignment, and prove the new stochastic network loading algorithm does generate the logit flow assignment. Finally, the model and algorithm are tested in a simple network.

REFERENCES

Bliemer, M.C.J., and P.H.L Bovy (2003) Quasi-Variational Inequality Formulation of the Multiclass Dynamic Traffic Assignment Problem. **Transportation Research 37B**, 501-519

Bliemer, M.C.J., R.J. Castenmiller, and P.H.L. Bovy. (2001) Analytical Multiclass Dynamic Traffic Assignment using a Dynamic Network Loading Procedure. **Proceedings of the 9th Meeting of the EURO Working Group on Transportation, Bari.**

Dafermos, S.C (1972) The traffic assignment problem for multiclass-user transportation networks. **Transportation Science 6**, 73-87.

Han, S, (2003) Dynamic traffic modeling and dynamic stochastic user equilibrium assignment for general road networks. **Transportation Research 37B** 225-249.

Huang,H.J and William,H.K (2002) Modeling and solving the dynamic user equilibrium route and departure time choice problem in network with queues, **Transportation Research 36B** 253-273

Jun.Li, O.Fujiwura (2000) A reactive dynamic user equilibrium model in network with queues, **Transportation Research B34**, 605-624.

Kuwahara, M. and Akamatsu, T. (1993) Dynamic equilibrium assignment with queues for a one-to-many OD pattern. **Proceedings of the 12th International Symposium on Transportation and Traffic Theory,** ed. C. F. Daganzo, pp. 185-204, Berkeley, CA. Elsevier, New York.

Kuwahara, M., Akamatsu, T., (1997) Decomposition of the reactive dynamic assignments with queues for a many-to-many origin-destination pattern. **Transportation Research 31B**(1), 1-10.

Kuwahara, M. and Akamatsu, T. (2001) Dynamic user optimal assignment with physical queues for a many-to-many OD pattern. **Transportation Research 35B**, 461-479.

Mahmassani H.S , (1988) some numerical results on the diagonalization algorithm for network assignment with asymmetric interactions between cars and trucks. **Transportation Research 22B** (4),275-290.

T.L. Friesz, D. Bernstein, T.E. Smith,R.L. Tobin and B.W. Wie, (1993) A Variational Inequality

Formulation of the Dynamic Network User Equilibrium Problem, **Operations Research**, Vol. 41,No. 1, pp. 179-191.
Ran, B., Boyce, D.E., (1996) Modeling Dynamic Transportation Networks: An Intelligent Transportation System Oriented Approach. Springer, New York.
Wynter, L.M., (1995) Advances in the theory and application of the multiclass traffic assignment problem. Ph.D. thesis,**Ecole National des Ponts et Chauss_ees**, France.
William H.K.Lam, Huang H.J, (1988) a combined trip distribution and assignment model for multiple user classes. **Transportation Research 26B**(4),275-287.
Wie, B.W., Tobin, R.L., Friesz, T.L.,Bernstein, D., (1995) A discrete time, nested cost operator approach to the dynamic network user equilibrium problem.**Transportation Science** 29 (1),79-92.
Wie, B.W., Tobin, R.L.,M.Carey, (2002) The existence, uniqueness and computation of an arc-based dynamic network user equilibrium formulation, **Transportation Research 36B**, 897-918.
Wu, J.H., Chen, T., Florian, M., (1998) The continuous dynamic network-loading problem: a mathematical formulation and solution method. **Transportation Research 32B**, 173-187.
Xu, Y.W., Wu, J.H., Florian, M., Marcotte, P., Zhu, D.L., (1999) Advances in the continuous dynamic network loading problem. **Transportation Science 33**, 341-353.

Appendix:

Proof of the algorithm

We now prove that the algorithm does generate logit-based flow independent ideal stochastic dynamic simultaneous route and departure time choices between each OD pair. We note that each link likelihood $L_{(i,j)}(k)$ is proportional to the logit probability that link a=(*i,j*) is used during time interval *k* by a traveler chosen at random from among the population of trip-makers between origin *r* and destination *s*, given that the traveler is at node *i* during time interval *k*. The probability that a given route will be used is proportional to the product of all the likelihood of the links comprising this route. Suppose route *p* consists of nodes (*r; 1,2…n, s*) and links (*1,2…h)*. Sub-route p1 includes (*1,2…n, s*) and links (*2…h)*. The probability of traveler choosing route *p* and departure time *k* between origin *r* and destination *s* is $P_p^{rs}(k)$.

$$P_p^{rs}(k) = G \cdot \prod_{a \in p} \left\{ L_{(i,j)}(k) \right\}^{\delta_{ap}^{rs}} \qquad \text{(a1)}$$

Where *G* is proportionality constant for each OD pair and the product is taken over all links in the networks. Here, $t = k + t_p^{ri}(k)$. The incidence variable δ_{ap}^{rs} ensures that $P_p^{rs}(k)$ include only those links in the *p*th route between origin *r* and destination *s*. Substituting the expression for the likelihood $L_{(i,j)}(k)$ in the above equation, the probability of choosing a particular efficient route-departure time pair becomes

$$\begin{aligned} P_p^{rs}(k) &= G \cdot \exp\left\{\theta \cdot \left[\pi_{rs} - \pi_{js}(k + t_{(r,j)}(k)) - c_{(r,j)}(k)\right]\right\} \\ &\qquad \cdot \prod_{a \in p_1} \exp\left\{\theta \cdot \left[\pi_{is}(t) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)\right] \cdot \delta_{ap_1}^{rs}\right\} \\ &= G \cdot \exp\left\{\theta \cdot \left[\pi_{rs} - \pi_{js}(k + t_{(r,j)}(k)) - c_{(r,j)}(k)\right]\right\} \\ &\qquad \cdot \exp\left\{\theta \cdot \sum_{a \in p_1} \left[\pi_{is}(t) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)\right] \cdot \delta_{ap_1}^{rs}\right\} \\ &= G \cdot \exp\left\{\theta \cdot (\pi_{rs} - c_p^{rs}(k))\right\} \end{aligned} \qquad \text{(a2)}$$

The last equality results from the following summations:

$$\begin{aligned}
&\pi_{rs} - \pi_{1s}(k + t_{(r,1)}(k)) + \sum_{a \in p_1} \left[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k))\right] \cdot \delta_{ap}^{rs} \\
&\quad = \pi_{rs} - \pi_{1s}(k + t_p^{r1}(k)) + \pi_{1s}(k + t_p^{r1}(k)) - \pi_{2s}(k + t_p^{r2}(k)) \\
&\quad + \mathrm{M} \\
&\quad + \pi_{ns}(k + t_p^{rn}(k)) - \pi_{ss}(k + t_p^{rn}(k)) \\
&\quad = \pi_{rs}
\end{aligned} \qquad \text{(a3)}$$

and

$$\sum_{a \in p} c_a(k)\delta_{ap}^{rs} = c_1(k) + c_2(k + t_p^{r1}(k)) + \Lambda + c_h(k + t_p^{rn}(k)) = c_p^{rs}(k) \qquad \text{(a4)}$$

Since $\sum_p \sum_k P_p^{rs}(k) = 1$

The proportionality constant must equal

$$G = \frac{1}{\sum_p \sum_k \exp\{\theta \cdot [\pi_{rs} - c_p^{rs}(k)]\}} \qquad \text{(a5)}$$

Thus

$$P_p^{rs}(k) = \frac{\exp\{\theta \cdot [\pi_{rs} - c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{\theta \cdot [\pi_{rs} - c_p^{rs}(k)]\}} = \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} \qquad \text{(a6)}$$

Above equation depicts a stochastic dynamic simultaneous route/departure time choice among the efficient routes connecting OD pair *rs*. The algorithm does generate a stochastic dynamic simultaneous route/departure time choice probability using actual route travel costs.

Now we try to prove the forward pass of the algorithm does generate the results of the logit flow assignment for simultaneous route/departure time choice. Firstly, we transform equation (a6) to the following equation.

$$\begin{aligned}
f_p^{rs}(k) &= q^{rs} \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} = q^{rs} \cdot \frac{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} \cdot \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}} \\
&= q^{rs}(k) \cdot \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}}
\end{aligned} \qquad \text{(a7)}$$

Where

$$q^{rs}(k) = q^{rs} \cdot \frac{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} = q^{rs} \cdot \frac{\sum_{(i,k) \in A(i)} w_{(i,k)}(k)}{\sum_k \sum_{(i,k) \in A(i)} w_{(i,k)}(k)} \quad i \in r, \forall rs, k \qquad \text{(a8)}$$

The demand between OD pair *rs* during time interval *k* assigns to the network according to the DYNASTOCH algorithm. The equation (a8) is substituted into the forward pass of the DYNASTOCH algorithm; the equation (35) can be obtained. The major difference (one is $\pi^{rs}(k)$,other is π^{rs}) between the origin link 's link likelihood of the DYNASTOCH algorithm and this algorithm does not influence the calculation results.