Journal of the Eastern Asia Society for Transportation Studies, Vol. 6, pp. 2060 - 2075, 2005

IMPACT OF ROAD PRICING ON THE NETWORK RELIABILITY

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Abstract: In this paper, the impact of road pricing on the transport network reliability is investigated with the use of a newly developed reliability-based user equilibrium (RUE) model. The RUE model can be adopted to overcome the difficulty in predicting the flow pattern in a congested and unreliable road network. In this RUE model, both the travel time and travel time reliability are taken into account for drivers' route choices by introducing a concept of path preference index (PI) to quantify the attractiveness of each alternative path for travel between a particular origin-destination pair. The PI is a weight sum of the path travel time index (TI) and the path travel time reliability index (RI), which represent the travel time and travel time reliability on a particular path respectively. The RUE model can be applied to investigate the effects of road pricing on travel time and reliability in urban road networks, in which drivers would consider both the travel time and its reliability for their route choices. In this paper, the RUE model is extended to maximize the network reliability by road pricing. The results demonstrated the trade-off between the total network travel time and the network travel time reliability. The effects of the toll on the total network travel time and the network travel time reliability are investigated.

Key Words*:* Network reliability, reliability-based user equilibrium, road pricing.

1. INTRODUCTION

Road pricing is considered to be an effective means of both managing road traffic demand and raising additional revenue for road construction by both transportation researchers and economists. In the past, some practical implementations of road pricing have been carried out in some countries (Holland and Watson, 1978; Dawson and Catling, 1986; Nakamura and Kockelman, 2002). In Hong Kong, the first Electronic Road Pricing (ERP) pilot scheme was conducted in 1983-1985, with economic, financial and technological aspects evaluated as to their feasibilities. However, the ERP demonstration project was shelved due to a variety of reasons such as the privacy problem, economic recession and political reason. Without ERP in Hong Kong at present, more rational road pricing policies would be required in order to eliminate the recurrent traffic congestion problems particularly when there are very few remaining construction solutions to transportation congestion and efficiency in Hong Kong densely populated urban areas. The charging methods of road pricing can be divided into two categories, namely, direct and indirect method. Direct charges are road tolls, and petrol tax, while indirect charges are ALF (annual license fee), FRT (first registration fee), and parking fee.

Currently, there are rapidly increasing renewed interests in electronic road pricing both in terms of theoretical studies and practical implementations. Congestion pricing has been examined extensively in different manners. Ghali and Smith (1992) compared various charge methods through simulation including distanced-based, delay-based charging, and cordon charging. Optimal road pricing in general networks is examined by Lam et al. (1996) and (2002), Yang and Lam (1996), Yang and Huang (1998). In addition, there is also a large bank of work on single or parallel bottleneck congestion pricing analysis (for a review, see Yang and Huang, 1997). In particular, Yang and Meng (1998) extended the static pricing model to including departure time, route choice and congestion tolls in a queuing network with elastic-demand.

In the literature, the effects of road pricing on many different objectives had been well studied, such as reducing congestion, reducing individual and total travel time/cost, raising revenue for infrastructural investment and reducing the air and noise pollution, etc. However, no attention has been given to the effects of road pricing on the reliability of transport network. It is partly due to the difficulty in predicting the flow pattern in a congested and unreliable transport network.

In this paper, a new reliability-based user equilibrium (RUE) model is proposed to overcome this difficulty. In this RUE model, both the travel time and reliability are taken into account for drivers' route choices by introducing a concept of path preference index (PI) to quantify the attractiveness of each alternative path for travel between a particular origin-destination (O-D) pair. The PI is a weight sum of the path travel time index (TI) and path travel time reliability index (RI), which represent the travel time and travel time reliability on a particular path respectively. Thus, the RUE model can be considered as an extension of the conventional user equilibrium (UE) model, in which drivers only consider the path travel time for their route choices.

The RUE model is then used to assess the effects of road pricing on travel time and reliability for network design problem. The variations of traffic flows and system performance with respect to different pricing policies (only toll pricing and distance-based pricing are considered) are investigated in this paper together with the differences between them.

This paper is organized as follows. In section 2, the conventional traffic assignment principle, the user equilibrium (UE) principle, is reviewed and discussed. Then, the reliability-based user equilibrium (RUE) principle, which can be considered as an extension of the UE, is proposed in section 3 together with the proofs of the existence and uniqueness of the RUE solution. Section 4 presents the effects of toll pricing on travel time and reliability in an example network. Finally, in section 5, conclusions are drawn together with recommendations for further study.

2. THE CONVENTIONAL USER EQUILIBRIUM (UE) PRINCIPLE

The earliest widely adopted route choice principle is the user equilibrium (UE) principle (Wardrop, 1952; Dafermos and Sparrow, 1968; Sheffi, 1985), stated as: "*For each origin-destination (O-D) pair, at user equilibrium, the travel time on all used paths is equal, and (also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path.*" This principle can be mathematically expressed by the following nonlinear complementarity conditions:

$$f_j^{rs}(c_j^{rs} - \pi^{rs}) = 0 \qquad \forall j, r, s \tag{1}$$

$$c_j^{rs} - \pi^{rs} \geq 0 \qquad \forall j, r, s \tag{2}$$

$$f_j^{rs} \geq 0 \qquad \forall j, r, s \tag{3}$$

$$\sum_j f_j^{rs} = q^{rs} \qquad \forall r, s \tag{4}$$

where f_j^{rs} is the traffic flow on path j between O-D pair rs, c_j^{rs} is its path travel time, π^{rs} is referred to as the shortest travel time between O-D pair rs, q^{rs} is the demand between O-D pair rs. If path j is used ($f_j^{rs} \geq 0$), then $c_j^{rs} - \pi^{rs} = 0$ or $c_j^{rs} = \pi^{rs}$. On the other hand, if path j is not used ($f_j^{rs} = 0$), there is no restriction on c_j^{rs} other than it is greater or equal to π^{rs}. Therefore, this set of conditions precisely describes the UE principle. Thus, at equilibrium, the paths connecting each O-D pair can be divided into two sets. The first set includes paths that carry traffic flows. The travel time on all these used paths is the same. The other set includes paths that do not carry any flow. The travel time on each of these unused paths is at least as large as the travel time on the paths of the first set.

Eqs. (1)~(4) can be expressed formally as a nonlinear complementarity problem (NCP) or converted to an equivalent variational inequality problem (VIP), which provides a general formulation for the UE route choice principle.

The relationship between link flow, path flow, link choice proportion and O-D demand can be defined as follows:

$$v_a = \sum_{rsj} f_j^{rs} \delta_{aj}^{rs} \qquad \forall a \tag{5}$$

$$P_a^{rs} = \sum_j (f_j^{rs}/q^{rs})\delta_{aj}^{rs} \tag{6}$$

where v_a is the flow on link a and P_a^{rs} is the choice proportion on link a. δ_{aj}^{rs}=1 if link a is a part of path j between O-D pair rs, and =0 otherwise.

Denote $\mathbf{v} = \{\ldots, v_a, \ldots\}$ as the link flows and $\mathbf{T} = \{\ldots, t_a, \ldots\}$ as the link travel times where $t_a(v_a)$ is the travel time function of link a. If the diagonal elements of the Jacobian matrix $\nabla\mathbf{T}_v$ (i.e. $\frac{dt_a}{dv_a}$) are non-negative, whereas all the off-diagonal elements are zero, then Eqs. (1)~(4) can be shown to be equivalent to the first order conditions of the following mathematical program (MP) problem:

$$\min_{v_a} \sum_a \int_0^{v_a} t_a(x)dx \tag{7}$$

$$f_j^{rs} \geq 0 \qquad \forall j, r, s \tag{8}$$

$$\sum_j f_j^{rs} = q^{rs} \qquad \forall r, s \tag{9}$$

$$v_a = \sum_{rsj} f_j^{rs}\delta_{aj}^{rs} \qquad \forall a \tag{10}$$

The traffic assignment solution can be obtained by solving the above NCP or VIP as expressed in Eqs. (1)~(4), or solving the MP problem of Eqs. (7)~(10) if the link travel time function is separable, i.e. the travel time on each link only depends on its own link flow but not on flows of other links. The solution algorithms to solve the equivalent NCP/VIP or MP problems are well established (Nagurney, 1999; Sheffi, 1985).

3. THE RELIABILITY-BASED USER EQUILIBRIUM (RUE) PRINCIPLE

The RUE principle is proposed in this section. Firstly, the path travel time reliability is defined and related to the ratio of mean travel time and the free-flow travel time by path between each O-D pair. By normalization, the path travel time reliability index (RI) and path travel time index (TI) are defined and scaled in the range [0,100] continuously. The RI is positively related to the path travel time reliability and the TI is negatively related to the path travel time. The weighted sum of the RI and TI is referred to as the path preference index (PI). Therefore, a path preference index (PI) is an index that quantifies the attractiveness of each alternative path in an unreliable transport network in which there are temporal variations of travel times due to minute-by-minute fluctuation of traffic flows during the same hourly period. Finally, the mathematical formulation of the RUE model is presented together with proof of the existence of the RUE solution.

3.1 The Path Travel Time Reliability Index

For measuring the variability of travel times encountered by travelers, Asakura (1998) proposed a performance measure using the concept of travel time reliability. It is referred to as the network service level that is the ratio of the random travel time and the free-flow travel time. The travel time reliability is therefore defined as the probability that the current level of network service is less than a threshold. Let π_j^{rs} be the mean travel time on path j from origin r to destination s and π_f^{rs} be the free-flow travel time from origin r to destination s. Then the performance measure proposed in this paper is $\pi_j^{rs}/\pi_f^{rs} (\forall r, s, j)$.

The Highway Capacity Manual (National Research Council, 2000) introduces the concept of "level of service" as a qualitative measure of the composite effects of operating speed and travel time etc. on the highway facilities. Six levels of service are established, A to F. On the basis of this "level of service" concept, the scaled performance measure can be adopted as the Level of Service (LOS) to evaluate alternative paths by O-D pair in the network (Zhang and Lam, 2001) and denoted as $\theta_R \pi_f^{rs}$ where $\theta_R \geq 1.0$ is the scaled parameter that can be related to the LOS. It is assumed in this paper that θ_R = 1.1, 1.3 and 2.0 for LOS A, B and C respectively. However, empirical studies should be carried out to determine the LOS in terms of θ_R and volume/capacity ratio.

Denote c_j^{rs} as the travel time on path j from the origin r to destination s. It can be expressed as

$$c_j^{rs} = \pi_j^{rs} + \varepsilon_j^{rs}, \ \forall r, s, j \tag{11}$$

where ε_j^{rs} is the random item on path j with E[ε_j^{rs}]=0 resulted from the temporal fluctuation of travel flow (Lam and Small, 2001).

The probability of the path travel time is considered as the Normal distribution. Denote x as the random variable with standard Normal distribution and σ_j^{rs} as the standard deviation of travel time. σ_j^{rs} is a function of the mean travel time π_j^{rs} (Shankar and Mannering, 1998), $\sigma_j^{rs} = \sigma_j^{rs}(\pi_j^{rs})$. Hence, the path travel time reliability is defined mathematically as

$$R_j^{rs}(\omega_j^{rs}) = P\left(x \leq \pi_f^{rs}(\theta_R - \omega_j^{rs})/\sigma_j^{rs}(\omega_j^{rs})\right), \ \forall r, s, j \tag{12}$$

where $\omega_j^{rs} = \pi_j^{rs}/\pi_f^{rs}$ is the congestion-performance measure and x = $(c_j^{rs} - \pi_j^{rs})/\sigma_j^{rs}$ is the standard transformation for the deviation of path travel time from the mean. σ_j^{rs} can be rewritten as the form of $\sigma_j^{rs} = \sigma_j^{rs}(\omega_j^{rs})$. The path travel time reliability measures the probability that the actual path travel time falls within the threshold of $\theta_R \pi_f^{rs}$ under a particular LOS. From $\frac{dR_j^{rs}}{d\omega_j^{rs}} < 0$ it can be seen that the path travel time reliability is decreasing when the mean travel time is increasing.

In order to generalize the index for applications, the path travel time reliability index is scaled from 0 to 100 and rewritten as

$$RI_j^{rs}(\omega_j^{rs}) = 100R_j^{rs}(\omega_j^{rs}), \text{and} \frac{dRI_j^{rs}}{d\omega_j^{rs}} < 0, \ \forall r, s, j. \tag{13}$$

Eq. (13) shows that the path travel time reliability is also a monotonously decreasing function of the congestion-performance measure ω_j^{rs}. The path travel time reliability index is 0 when the destination cannot be accessed at the travel time less than the threshold of a given LOS. It approaches to 100 approximately if the path travel time that is less than the threshold of a particular LOS can be attained with certainty to reach the destination from the origin.

3.2 The Path Travel Time Index

Similarly, the path travel time index is defined mathematically as

$$TI_j^{rs}(\omega_j^{rs}) = 100e^{-\theta_T(\omega_j^{rs}-1)} \text{ and } \frac{dTI_j^{rs}}{d\omega_j^{rs}} < 0, \ \forall r, s, j. \tag{14}$$

where θ_T is a scaled parameter. The value of θ_T can be calibrated and considered as a scaled value of time on driver route choice. Eq. (14) implies that the path travel time index is a monotonously decreasing function of the congestion-performance measure. If the mean travel time is equal to the free-flow travel time or the congestion-performance measure ω_j^{rs} is 1.0, then the path travel time index is 100. If the mean travel time approaches to infinite, the path travel time index is then equal to 0.

3.3 The Path Preference Index

The path preference index (PI) is defined as a weighted sum of the RI and TI. Therefore, a path preference index (PI) is an index that quantifies the attractiveness of each alternative path in an unreliable transport network in which there are temporal variations of travel times due to minute-by-minute fluctuation of traffic flows during the same hourly period. The PI is defined as:

$$PI_j^{rs}(\omega_j^{rs}) = \alpha RI_j^{rs}(\omega_j^{rs}) + (1-\alpha)TI_j^{rs}(\omega_j^{rs}) \text{ and } \frac{dPI_j^{rs}}{d\omega_j^{rs}} < 0, \ \forall r,s,j. \tag{15}$$

where $1 \geq \alpha \geq 0$ is a weight allocated to the path travel time reliability index and $100 \geq PI_j^{rs}(\omega_j^{rs}) \geq 0$. If $\alpha = 0$, only the path travel time is considered by drivers for their route choice. If $\alpha = 1$, only the path travel time reliability is taken account. It can be seen that the conventional UE model is a special case of the RUE model when drivers only consider travel time for their route choices, i.e. $\alpha = 0$. In practice, a value of $\alpha = 0.38$ is found on the basis of an empirical study (Lam and Small, 2001). The PI is a monotonously decreasing function of the congestion-performance measure ω_j^{rs}. In this paper, the PI is proposed as an indicator to rank the driver preference to alternative routes.

If the PI is 100 on a path between an O-D pair, it implies that travelers on that path can arrive at their destination in a given speed with 100% reliability under a specified Level of Service (LOS). On the other hand, if the PI on a path between an O-D pair is equal to 0, it means that travelers on that path have no chance of arriving at their destination, as the travel time would be infinitely longer than that under the particular LOS and that its reliability is zero.

3.4 The RUE Model Formulation

Based on the above definitions, the RUE model can be formulated mathematically as below. Describe the road network by a directed graph G(N, A). Let δ_{aj}^{rs} be the link-path incidence matrix, $\delta_{aj}^{rs}=1$ if link a is on path j, and 0 otherwise. Let v_a be the traffic flow on link a together with q^{rs} be the O-D demand from origin r to destination s, f_j^{rs} be the traffic flow on path j and PI^{rs} be the maximal path preference index from r to s. The nonlinear complementary problem (NCP) for the RUE model based on PI is formulated mathematically as follows

$$f_j^{rs}(PI_j^{rs} - PI^{rs}) = 0, \ \forall r,s,j \tag{16}$$

$$PI_j^{rs} - PI^{rs} \leq 0, \ \forall r,s,j \tag{17}$$

$$\sum_j f_j^{rs} = q^{rs}, \ \forall r,s \tag{18}$$

$$\sum_{r,s}\sum_j \delta_{aj}^{rs} f_j^{rs} = v_a, \ \forall a \tag{19}$$

$$f_j^{rs} \geq 0, \ \forall r,s,j. \tag{20}$$

It should be noted that Eqs. (16) and (17) are the complementary slackness conditions: $\mathrm{PI}_j^{rs} - \mathrm{PI}^{rs} < 0,\ f_j^{rs} = 0$ and $\mathrm{PI}_j^{rs} - \mathrm{PI}^{rs} = 0,\ f_j^{rs} \geq 0$. As a result, when the path preference index on route j is smaller than the maximum PI^{rs}, the path flow on route j is equal to zero. If the path preference index on route j is equal to the maximum PI^{rs}, then the path flow on route j is equal to or greater than zero. These RUE conditions are equivalent to that for the user equilibrium principle (Wardrop, 1952) mathematically. Eq. (18) is the constraint of the O-D demand while Eq. (19) is the sum of path flows for O-D pairs via link a that should be equal to the traffic flow on link a. Eq. (20) is the non-negative constraint of the path flow. The proposed RUE model can be solved easily by the method of successive average (MSA) provided that the number of alternative paths by O-D pair is given and fixed (Cascetta, 2001). Cascetta et al. (1996) have tested the path enumeration on a real network with 1784 nodes and 4355 links. The findings showed that it is only necessary to adopt 8 paths for each O-D pair in this real network. Cascetta et al. (1996) have adopted the k-shortest path method to determine the fixed path set for each O-D pair. Therefore, the RUE model can also be solved in practice by using fixed path set for each O-D pair. The new RUE model is aimed for strategic network design problem while both travel time and reliability are taken account in driver route choices.

Let f_j^{rs} be the traffic flow on path j between OD pair $r-s$, q^{rs} be the fixed demand between OD pair $r-s$, PI^{rs} be the maximal path preference index from origin r to destination s. The nonlinear complementary problem (NCP) for the RUE based on PI is formulated mathematically as follows:

$$\sum_{rs}\sum_j f_j^{rs}(\mathrm{PI}^{rs} - \mathrm{PI}_j^{rs}) = 0 \tag{21a}$$

$$f_j^{rs} \geq 0 \qquad \forall r,s,j \tag{21b}$$

$$\mathrm{PI}^{rs} - \mathrm{PI}_j^{rs} \geq 0 \qquad \forall r,s,j \tag{21c}$$

$$\sum_j f_j^{rs} = q^{rs} \qquad \forall r,s \tag{21d}$$

This complementarity problem can be easily transformed into a variational inequality (VI) problem. Let $\mathbf{f}$ be the vector of path flows $\{f_j^{rs}\}$, and $\mathbf{F}(\mathbf{f})$ be the vector of $\{\mathrm{PI}^{rs} - \mathrm{PI}_j^{rs}\}$.

Proposition 1 The NCP (21) is equivalent to the VI problem below:

$$\text{Find } \mathbf{f}^* \in \Omega \text{ such that: } (\mathbf{f}-\mathbf{f}^*)^T \mathbf{F}(\mathbf{f}^*) \geq 0 \quad \forall \mathbf{f} \in \Omega \tag{22}$$

Where Ω is the constraint set of path flows $\Omega = \left\{ f_j^{rs} \geq 0, \forall r,s,j, \sum_j f_j^{rs} = q^{rs}, \forall r,s \right\}$.

Proof See proposition 1.4 in Nagurney (1999).

Theorem 1 the VI problem (22) admits at least one solution $f^* \in \Omega$.

Proof

The VI formulation (22) can be stated as

$$\text{Find } \mathbf{f}^* \in \Omega \text{ such that } \sum_{rs}\sum_{j}(f_j^{rs} - f_j^{rs^*})(\mathrm{PI}^{rs^*} - \mathrm{PI}_j^{rs^*}) \geq 0 \quad \forall \mathbf{f} \in \Omega \tag{23}$$

$$\Leftrightarrow \sum_{rs}\sum_{j}(f_j^{rs} - f_j^{rs^*})\mathrm{PI}^{rs^*} - \sum_{rs}\sum_{j}(f_j^{rs} - f_j^{rs^*})\mathrm{PI}_j^{rs^*} \geq 0 \quad \forall \mathbf{f} \in \Omega$$

$$\Leftrightarrow \sum_{rs}\mathrm{PI}^{rs^*}(\sum_{j} f_j^{rs} - f_j^{rs^*}) - \sum_{rs}\sum_{j}(f_j^{rs} - f_j^{rs^*})\mathrm{PI}_j^{rs^*} \geq 0 \quad \forall \mathbf{f} \in \Omega \tag{24}$$

It follows from $\mathbf{f} \in \Omega, \mathbf{f}^* \in \Omega$

$$\sum_{j} f_j^{rs} = q^{rs}$$

$$\sum_{j} f_j^{rs^*} = q^{rs}$$

Therefore, the first term of the left-hand in equation (24) is zero

$$\sum_{rs}\mathrm{PI}^{rs^*}(\sum_{j} f_j^{rs} - f_j^{rs^*}) = 0 \tag{25}$$

It follows from (25) and (24) that the VI formulation (22) is equivalent to VI below

$$\text{Find } \mathbf{f}^* \in \Omega \text{ such that } \sum_{rs}\sum_{j}(f_j^{rs} - f_j^{rs^*})(-\mathrm{PI}_j^{rs^*}) \geq 0 \quad \forall \mathbf{f} \in \Omega \tag{26}$$

Let $\mathbf{G}(\mathbf{f})$ be the vector of $\left\{-\mathrm{PI}_j^{rs^*}\right\}$, therefore (26) can be stated as

$$\text{Find } \mathbf{f}^* \in \Omega \text{ such that } \left(\mathbf{f} - \mathbf{f}^*\right)^T \mathbf{G}\left(\mathbf{f}^*\right) \geq 0 \quad \forall \mathbf{f} \in \Omega \tag{27}$$

Obviously, the constraint set Ω is a compact convex and $\mathbf{G}(\mathbf{f})$ is continuous on Ω. According to the theorem 1.4 (Nagurney 1999) the existence of RUE solution is verified.

However, we can only guarantee the solution existence on the VI formulation. Multiple solutions may exist due to the non-monotonicty of the VI. Unique solution of link flow can be guaranteed when the RUE problem reduces to the UE problem (set $\alpha = 0$).

3.5 Example to illustrate the RUE Model

Figure 1 shows a small example network tot illustrate RUE model. There are one O-D pair and two paths in this network. Table 1 presents the mean travel times for these two paths together with the standard deviation, TI, RI and PI.

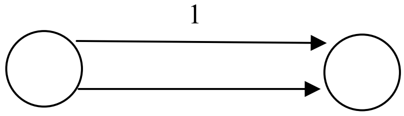

Figure 1. Example Network to Illustrate RUE

Table 1. Example to illustrate RUE

	Path 1	Path 2
Mean Travel Time (min)	30	45
Standard Deviation (min)	25	5
TI	69.51	35.15
RI	77.64	97.72
PI (0.5*TI+0.5*RI)	**73.58**	**64.61**
PI (0.7*TI+0.3*RI)	**71.95**	**52.83**
PI (0.3*TI+0.7*RI)	**75.20**	**76.39**

(Note: $\mathrm{TI}_j^{rs}(\omega_j^{rs}) = 100e^{-\theta_T(\omega_j^{rs}-1)}$ and $\dfrac{d\mathrm{TI}_j^{rs}}{d\omega_j^{rs}} < 0, \ \forall r,s,j.,$

$$\mathrm{RI}_j^{rs}(\omega_j^{rs}) = 100\mathrm{R}_j^{rs}(\omega_j^{rs}), \text{and} \ \frac{d\mathrm{RI}_j^{rs}}{d\omega_j^{rs}} < 0, \ \forall r,s,j.,$$

$$\mathrm{R}_j^{rs}(\omega_j^{rs}) = \mathrm{P}\left(x \le \pi_f^{rs}(\theta_R - \omega_j^{rs})/\sigma_j^{rs}(\omega_j^{rs})\right), \ \forall r,s,j)$$

When the drivers only consider the travel time, they will choose the path 1 with higher TI. When they consider the reliability, they choose path 2 with higher RI. However, both the travel time and reliability are taken into account for drivers' route choices which is represented by PI as the weighted sum of TI and RI. The choice between paths 1 and 2 depends on the weighting put in TI and RI. When the weighting is 0.5 for both TI and RI, the PI for path 1 is slightly greater than PI for path 2. When higher weighting is on TI such as 0.7, path 1 is chosen as its PI is greater. On the other hand, higher weighting on RI (0.7) will lead to the route choice on path 2.

4. NUMERICAL EXAMPLE

The purposes of the numerical example are to illustrate that: (i) the effects of the RUE when drivers do consider travel time reliability for their route choices; (ii) the impacts of road pricing on network performance, particularly on traffic flow, travel time and reliability.

The example network used by Yang and Lam (1996) and Yang et al. (2000) is shown in Figure 2. This network includes seven links and six nodes, of which one and two are origin nodes and three and four are destination nodes. Link 2 is the toll link in this network. Table 2 and Table 3 show the link travel time characteristics and the O-D matrix. Link travel time function is $t_a(v_a)=t_a^0\{1.0+0.15(v_a/C_a)^4\}$where t_a^0 is the free-flow travel time and C_a: is capacity. In this example, the standard deviation of link travel time is written as: $\sigma_a = \beta_{a0} + \beta_{a1} \ln \bar{t}_a$, where $\beta_{a0} \ge 0$ and $\beta_{a1} \ge 0$ are the coefficients related to the link a only. The coefficients of the links in this example network are given in Table 2.

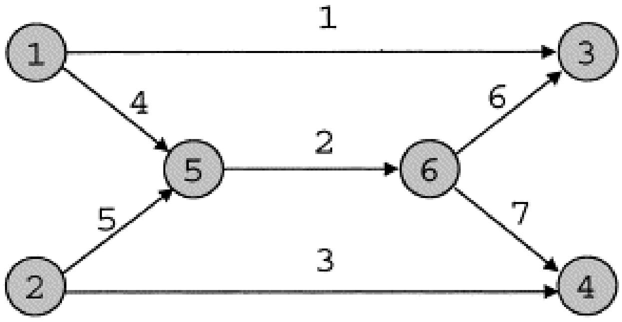

Figure 2. The Example Network

Table 2. Link Travel Time Data

Link	1	2	3	4	5	6	7
t_a^0	15	4	15	5	5	5	5
C_a	100	120	100	80	80	80	80
β_{a0}	8.0	4.0	8.0	3.0	3.0	4.0	4.0
β_{a1}	0.6	0.3	0.6	0.2	0.2	0.3	0.3

Table 3. The O–D Matrix

		Destination	zones
		3	4
Origin	1	D	20
zones	2	20	D

Firstly, we investigate the RUE effects on travel time and reliability in comparison to that of the conventional UE by employing the example network. Table 4 shows that the path travel times are 15.0 (min) equally and the path travel time reliabilities are 91.0% and 91.0% respectively on path 1 and 2 under the UE condition (α=0.0). When drivers do consider both travel time and reliability for their route choices and account the reliability with α=0.5, more drivers choose the path 2, which is more reliable than path 1, and the path travel time reliability on path 3 is increased to 93%. It can be improved to 95% if the drivers only consider travel time reliability for route choices (i.e. α=1.0). It illustrates that the higher the α, the more drivers choose the more reliable path. From Table 5, it can be seen that the total PI is increased from 33650 to 38919. The total RI is increased in similar manner from 33694 to 38919. However, it can be seen in Table 5 that the total network travel time is increased from 2706 to 3209 (veh-min) when α is increased from 0.0 to 1.0. Therefore, higher total network travel time and higher total network reliability occur when α=1.0 which reflects the trade-off between the travel time and travel time reliability.

Table 4. The Equilibrium Path Flow (veh/hr), Path Travel Time (min) and Path Travel Time Reliability (%) by Path (D =60 veh/hr)

		Path 1 [a]	Path 2
α=0.0 (**UE**)	Path Flow (veh/hr)	32	28
	Path Time (min)	**15**	**15**
	Path TTR (%) [b]	**91**	**91**
α=0.25 (**RUE**)	Flow	28	32
	Time	15	15
	TTR	92	92
α=0.5 (**RUE**)	Flow	22	38
	Time	15	16
	TTR	**93**	**93**
α=0.75 (**RUE**)	Flow	16	44
	Time	15	17
	TTR	94	94
α=1.0 (**RUE**)	Flow	8	52
	Time	15	18
	TTR	**95**	**95**

Note: [a] Path 1: Link 1; Path 2: Links 4-2-6

[b] Path Travel Time Reliability TTR (on path j) = $P\left(x \le (\theta_R \pi_f^{rs} - \pi_j^{rs})/\sigma_{rsj}\right)$

Table 5. The Total Network Travel Time (veh-min), Total PI and Total RI (D =60 veh/hr)

	Total Network Travel Time (veh-min)	Total PI [c]	Total RI [d]
α=0.0 (**UE**)	**2706**	**33650**[c]	**33694**[d]
α=0.25 (**RUE**)	2752	34557	34842
α=0.5 (**RUE**)	2828	35799	36269
α=0.75 (**RUE**)	2968	37146	37613
α=1.0 (**RUE**)	**3209**	**38919**	**38919**

Note:

[c] Total PI= $\sum f_j PI_j$, [d] Total RI= $\sum f_j RI_j$

The effects of the toll on the flow pattern are investigated. Figure 3 shows the variations of flow on link 1 with respect to the toll (in HK$) on link 2 when O-D demand D=30, 60 (veh/hr) and α=0.5. It can be seen that the flow on link 1 is 22 (veh/hr) when no toll on link 2 is introduced. When the toll is introduced and increased, the flow on link 1 is increased since path 2 (links 4-2-6) includes the toll link. On the contrary, the flow on link 2 is decreased when the toll on link 2 is increased. Figure 3 implies that no vehicle use link 2 from origin 1 to destination 3 when the toll on link 2 is equal to or larger than 3 equivalent minutes.

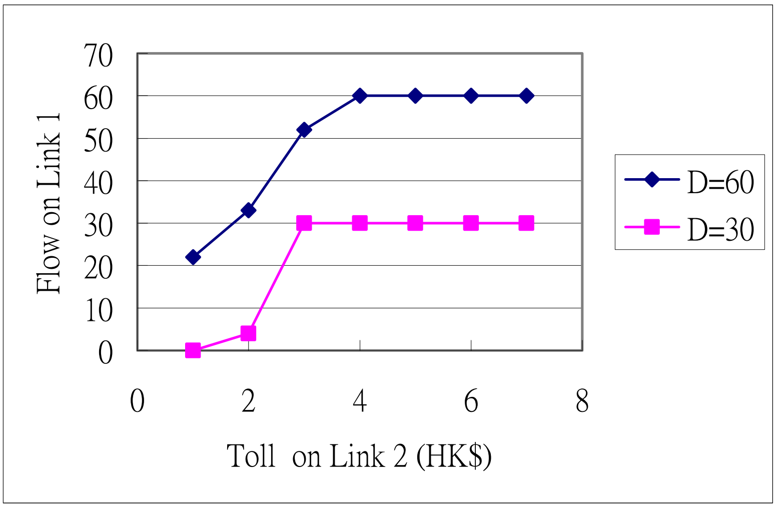

Figure 3. Variations of Flow on Link 1 (α=0.5)

The effects of total network travel time and the total PI which represent the network reliability can be illustrated by investigating Figures 4 and 5. Figure 4 show that the minimum network travel times are 2671 and 1749 for D=60 and 30 respectively when toll=HK$ 2. Figure 5 displays that the total PI decrease as the toll on link 2 increases. It is because the increase of toll on link 2 leads to fewer drivers choose the path 2 which is via link 2. However, the reliability of path 2 is higher than that of path 1. As such, the total PI decreases due to the smaller flow on path 2 which lower the network reliability.

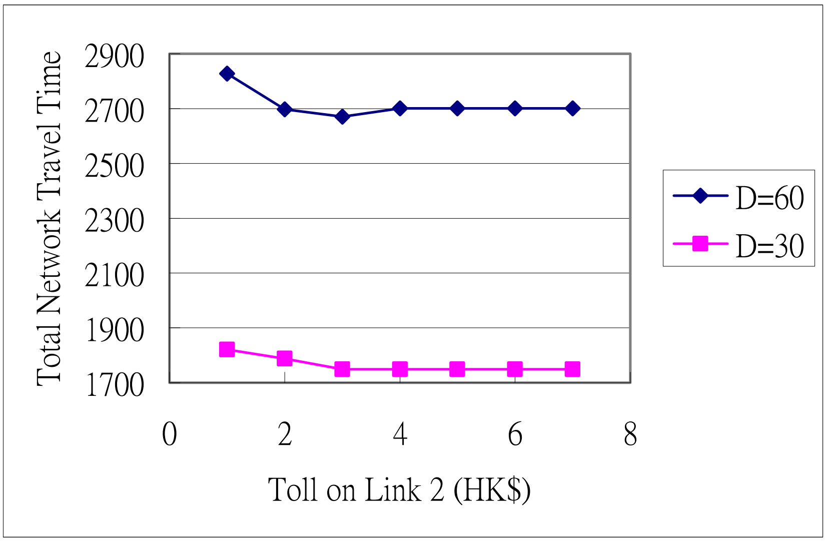

Figure 4. Variations of Total Network Travel Time (α=0.5)

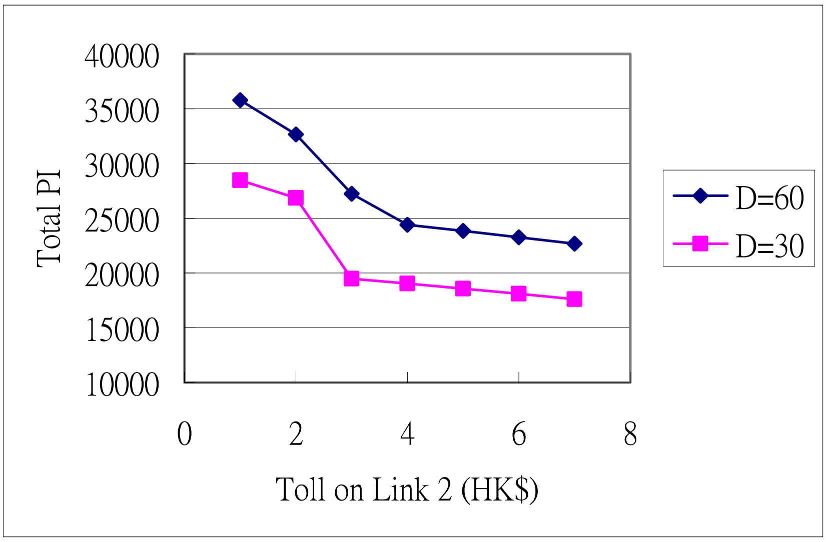

Figure 5. Variations of Total PI (α=0.5)

5. CONCLUSIONS

A new reliability-based user equilibrium (RUE) model was proposed in this paper to assess the effects of travel time and reliability for network design problem. Both the travel time and reliability are taken into account in the proposed RUE model for drivers' route choices. The unreliability on travel time is due to the minute-by-minute fluctuation of traffic flow during the peak hour period. It was shown that the conventional user equilibrium (UE) model can be considered as a special case of the proposed RUE model when drivers only consider travel time for their route choices.

The mathematical formulation of the RUE principle was presented together with the proof of the existence of RUE solution. An example network was used for investigating the effects of the travel time reliability on route choice. The results demonstrated the trade-off between the total network travel time and the network travel time reliability. The effect of the toll on the total network travel time and the network travel time reliability are investigated.

Some future extensions of research work are suggested. Further model development will be considered to incorporate the effect of O-D demand distribution on the travel time variability. A case study will be carried out for the application of the proposed model in practice. Efficient method will be investigated for searching the alternative feasible reliable paths by O-D pair in congested and unreliable road network. The RUE formulation can be extended for multi-modal transport networks together with activity-based travel and/or departure time choice problems. The network design problem on the basis of the RUE principle will be further studied. The proposed RUE model can also be extended to a dynamic framework.

ACKNOWLEDGEMENTS

The work described in this paper was jointly supported by research grants from the Research Committee of The Hong Kong Polytechnic University (Project No. G-YX24 and 1-ZE10) and from the Research Grants Council of the Hong Kong Special Administrative Region (Project No. N_PolyU 515/01).

REFERENCES

Asakura, Y. (1998) Reliability measures of an origin and destination pair in a deteriorated road network with variable flow. **Transportation Networks: Recent Methodological Advances** (Bell, M.G.H., ed.), Pergamon Press, Oxford, pp. 273-287.

Cascetta, E. (2001) **Transportation Systems Engineering: Theory and Methods.** Kluwer Academic Publisher, Dordrecht, The Netherland.

Chen, A., Ji, Z. and Recker W. (2003) Effect of route choice models on estimation of travel time reliability under demand and supply variations. **Network Reliability of Transport.** Bell, M.G.H. and Iida, Y. (ed.), Kyoto, Japan, 93-118.

Dafermos, S.C. and Sparrow F.T. (1968) The traffic assignment problem for a general network. **Journal of Research of the National Bureau of Standards, 73B**, 91-118.

Jackson, W. B. and Jucker, J. V. (1982) An empirical study of travel time variability and travel choice behavior. **Transportation Science 16(4)**, 460-475.

Herman, R. and Lam, T. (1974) Trip characteristics of journeys to and from work. **Transportation and Traffic Theory**. Buckley, D.J. (ed.), Sydney, 57-85.

Holland, E.P. and Watson, P.L. (1978) Traffic restraint in Singapore. **Traffic Engineering and Control, 19**, 14-22.

Lam, T.C. and Small, K.A. (2001) The value of time and reliability: measurement from a value of pricing experiment. **Transportation Research, 37E**, 231-251.

Mirchandani, P. and Soroush, H. (1987) Generalized traffic equilibrium with probabilistic travel times and perceptions. **Transportation Science, 21(3)**, 133-152.

Nagurney, A. (1999) **Network Economics: A Variational Inequality Approach.** Kluwer Academic Publishers, Boston.

National Research Council (2000) Highway Capacity Manual. Washington, D.C., U.S.A.

Shankar, V., Mannering, F., 1998. Modeling the endogeneity of lane-mean speeds and lane-speed deviations: A structural equations approach. **Transportation Research, 32A**, 311-322.

Smeed, R. J. and Jeffcoate, G.O. (1971) The variability of car journey times on a particular route. **Traffic Engineering and Control, 13(6)**, 238-243.

Sheffi, Y. (1985) **Urban Transportation Networks**. Prentice Hall, Englewood Cliff.

Tatineni, M., Boyce, D.E., and Mirchandani, P. (1997) Comparisons of deterministic and stochastic traffic loading models. **Transportation Research Record, 1607**, 16-23

Taylor, M.A.P. (1982) Travel time variability-the case of two public modes. **Transportation Science, 16(4)**, 507-521.

Turner, J.K. and Wardrop, J.G. (1951) **The Variation of Journey Times in Central London.** Road Research Laboratory. Note No. RN/1511/JKT.JGW.

Uchida, T. and Iida, Y. (1993) Risk assignment : a new traffic assignment model considering risk of travel time variation. **Proceedings of the 12th International Symposium on Transportation and Traffic Theory**. Elservier, Amsterdam, 89-105

Wardrop, J. G. (1952) Some theoretical aspects of road traffic research. **Proceedings of the Institution of Civil Engineers, II(1)**, pp. 325-378.

Yang, H., Bell, M.G.H. and Meng, Q. (2000) Modeling the capacity and level of service of urban transportation networks. **Transportation Research, 34B**, 255-275.

Yang, H. and Lam, W.H.K. (1996) Optimal road tolls under the conditions of queueing and congestion. **Transportation Research, 30A**, 319-332.

Yin, Y., Lam, W.H.K. and Ieda, H. (2002) Modeling Risk-taking Behavior in Queuing Networks with Advanced Traveler Information Systems". **Transportation and Traffic Theory in the 21st Century**, Edited by Taylor, M.A.P., 2002, Elsevier, Oxford, pp. 309-328.

Zhang, N. and Lam, W.H.K. (2001) An alternative to the road network reliability by travel demand satisfaction ratio. **Proceedings of the 4th International Conference on Management**, Xian Jiaotong University, China, May 5-7, pp. 47-53.