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CAR-FOLLOWING MODELS: AN EXPERIMENT BASED BENCHMARKING

Prakash RANJITKAR
Research Fellow
Transportation and Traffic Systems
Graduate School of Engineering
Hokkaido University
Kita-13, Nishi-8, Kita-ku,
Sapporo 060-8628, Japan
E-mail pranjitkar@yahoo.com

Takashi NAKATSUJI
Associate Professor
Transportation and Traffic Systems
Graduate School of Engineering
Hokkaido University
Kita-13, Nishi-8, Kita-ku,
Sapporo 060-8628, Japan
E-mail naka@eng.hokudai.ac.jp

Akira KAWAMUA
Professor
Department of Civil Engineering
Kitami Institute of Technology
165 Koen-cho, Kitami 090-8507, Japan
E-mail kawamura@vortex.civil.kitami-it.ac.jp

Abstract: Car-following model has important applications in traffic and safety engineering e.g. for traffic simulation, Advance Vehicle Control and Safety System etc. A great deal of investigation works were conducted in the last five decades to model the longitudinal interaction between adjacent vehicles as a result numerous models are available now. A performance based benchmarking of these models might be useful to evaluate their capabilities in representing real driving behavior. Data precision is the key factor to make any such evaluation meaningful. RTK GPS is the latest technology in data acquisition that makes it possible to acquire high resolution vehicular movement data at an outstanding level of accuracy. Several car-following models were evaluated based on test track experiment data using a GA based optimization method. It was interesting to see a simple linear model performing better than some sophisticated models.

Key Words: car-following, RTK GPS, genetic algorithm

1. INTRODUCTION

Since the early investigation on car-following dynamics started in mid 50's, numerous models have been developed (for review see Brackstone et al. (1999)). The model is an essential component in microscopic traffic simulation programs that are widely preferred for applications in traffic and safety engineering e.g. for capacity analysis, traffic impact studies, junction design, accident analysis, network analysis etc. The model has been given more importance in recent years for its scope in some intelligent transportation systems.

With nearly hundred models proposed so far, not much works were done towards evaluating their capabilities and limitations. A few models were calibrated and validated also with limited scope in terms of data used and models dealt. It might be because there were limitations on data acquisition and calibration techniques as well. The data quality was not satisfactory or it is not mentioned clearly in the most of previous investigations. This suggests a need for benchmarking these models using precise vehicular movement data that will be helpful to evaluate their strengths and weakness, hence providing a ground for further improvements. For any benchmarking to be meaningful, a reasonable quality of data is a prerequisite.

The data acquisition techniques have witnessed significant advancements in the last few decades. The real time kinematic (RTK) GPS is the latest advancement that is capable of measuring vehicular movement data at an outstanding level of accuracy and much conveniently than ever before. A few years ago, we have conducted extensive car-following experiments in a test track located in Hokkaido, Japan. The RTK GPS receivers were used to measure position and speed of moving vehicles. These receivers measure position based on differential GPS technique while speed based on Doppler's principle. These data particular suitable for this study for two reasons: first, its outstanding accuracy features and second, it is well recognized in different research communities, for example those from USA, Germany, Spain, South Korea, Thailand, Sri Lanka and Japan. Some published works include Gurushinghe et al. (2001, 2002), Suzuki et al. (2002), Ranjitkar et al. (2003), Brockfeld et al. (2004) etc.

In our previous attempt (Ranjitkar et al. (2004)) we investigated six car-following models. Here we expand our research further to include some other models in this benchmarking process. Previously, the models were compared based on how well they fit with spacing (between vehicles) and speed data, while in this study acceleration will also be used in parallel to evaluate the models. A genetic algorithm based optimization method is adapted to calibrate the models. The optimized performance of the models will be compared vis-à-vis using percentile error as performance index.

The car-following experiments conducted in a test track is explained in the next section that includes information about the test track, experimental arrangements, drivers' characteristics, speed patterns and data accuracy. The models investigated in this study are described under section three. This includes the conceptual backgrounds of the models, their formulations and parameters to be optimized. The model evaluation set up is described in section four. The analysis results are presented in section five under two different subheadings: model calibration and validation. Finally, the outcomes of this study will be summarized in the last section.

2. DATA SETS

A test track was chosen for these experiments mainly to ensure simple driving conditions as assumed in car-following theories. The purpose was to avoid complications that exist in real highways due to several influencing factors such as traffic signs, vehicles on adjacent lanes etc. The test track consists of two 1.2 km long straight sections connected by two semicircular curves 150 m each. A schematic layout is shown in **Figure 1**. Ten drivers/passenger cars participated in these experiments, each car equipped with a RTK GPS receiver. The receiver outputs position and speed data at every 0.1 second interval with a position accuracy of 10 mm + 2ppm and speed accuracy of less than 0.2 km/h.

All drivers were young college students between 22 to 30 years, except the lead vehicle's driver who was at his late 50's. They were queued up in a row before driving and instructed not to overtake the vehicle in front, while the leader was instructed to follow some predetermined speed patterns as shown in **Figure 2**. The first four are sine wave speed patterns with wave lengths vary from 267 to 1600 m, while other two are random and constant speed patterns. The drivers were arranged in two different order patterns A and B. In patterns A, the drivers were arranged in an order with D1 followed by D2, D3, D4, D5, D6, D7, D8, D9 and D10, while in pattern B, they were arranged as D1 followed by D8, D7, D6, D5, D4, D3, D2, D9 and D10.

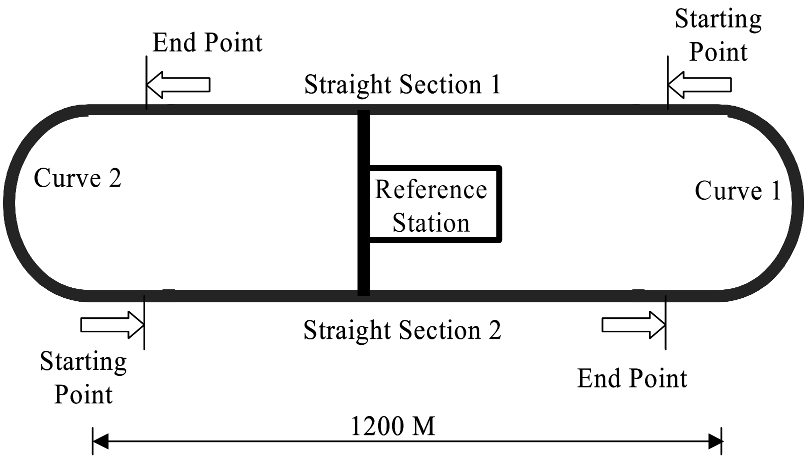


Figure 1. Schematic layout of the test track

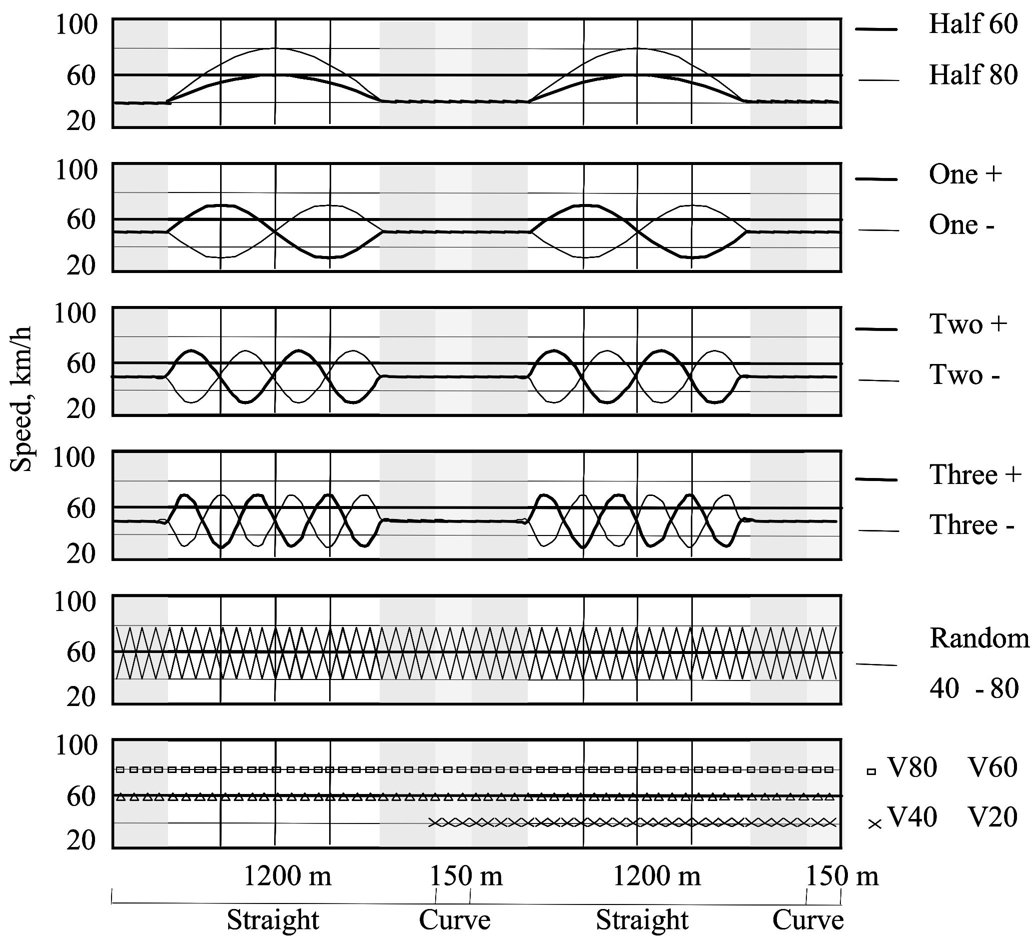


Figure 2. Speed patterns tested for the leader vehicle

The position and speed data measured by the receivers were processed to computer other car-following variables such as spacing (space headway), relative speed and acceleration. Acceleration was computed from speed measurements by polynomial fitting technique. Gurushinghe et al. (2003) have confirmed the accuracy of these data after comparing them with those taken by distance meter, speedometer and accelerometer.

These data represents wide range of uninterrupted driving conditions with different level of disturbances. The data shows that the drivers were mostly preferred to remain in car-following situations maintaining close to comfortable following distance from the vehicle in front that typically vary from 20 to 50 m with speed ranging from 30 km/h to 80 km/h. **Table 1** presents the number of data sets analyzed from each speed patterns. In total 47 data sets were analyzed 20 from pattern A and 27 from pattern B, each representing a single run in a straight section. The data from curves were not analyzed for possible effects of curves in driving behavior. For further details on these experiments please refer Ranjitkar (2004).

Table 1. The data sets used in this study

P.N.	Speed patterns	Drivers' order pattern	
		A	B
1	Half wave	4	6
2	One wave	4	4
3	Two wave	2	6
4	Three wave	4	5
5	Random	4	4
6	Constant speed	2	2
Total		20	27

3. THE MODELS

Table 2 presents a list of the models investigated that can be classified into the following groups based on the concept behind the model:

- Stimulus response model (Chandler model (1958), generalized GM model (1961))
- Safe distance model (Gipps model (1981), Krauss model (1997))
- Psychophysical model (Leutzbach model (1986))
- Cell based model (cellular automata model (Nagel (1992))
- Optimum velocity model (Bando et al (1995))
- Trajectory based model (Newell model (2002))

Table 2. A list of car-following models investigated

S.N.	Models	Parameters to be optimized
1	**Chandler Model** $a_n(t+T)=\lambda\,\Delta v(t)$	T and λ
2	**Generalized GM (GGM) Model** $a_n(t+T)=\alpha\,\dfrac{v_n(t+T)^m}{\Delta x(t)^{\ell}}\Delta v(t)$	T, α, m and l
3	**Gipps Model** $v_n(t+T)=\min\left\{\begin{array}{l} v_n(t)+2.5aT(1-v_n(t)/V)\sqrt{0.025+v_n(t)/V}, \\ bT+\left[b^2T^2-b\left\{\begin{array}{l}2[x_{n-1}(t)-x_n(t)-s]-v_n(t)T \\ -v_{n-1}(t)^2/b^*\end{array}\right\}\right]^{1/2}\end{array}\right\}$	T, b, V and b^*
4	**Krauss Model** $v_{safe}=v_{n-1}+\dfrac{g_n(t)-v_{n-1}(t)T}{(v_n(t)+v_{n-1}(t))/2b+T}$ Where, $g_n(t)=x_{n-1}(t)-x_n(t)-s$ $v_{des}=\min\{v_n(t)+a\Delta t, v_{safe}, V\}$ $v_n(t+\Delta t)=\max\{0, v_{des}-\varepsilon a\eta\}$ Where, simulation time step $\Delta t=0.1$ and the stochastic error term $\varepsilon a\eta$ is set to zero.	T, b and V
5	**Leutzbach Model** $a_n(t+T)=\dfrac{\Delta v(t)^2}{2[S-\Delta x(t)]}+a_{n-1}(t)$	T and S
6	**Cellular Automata** $v_n(t+\Delta t)=\min\{g_n(t)/T, v_n(t)+a, V\}$ Where, $g_n(t)=x_{n-1}(t)-x_n(t)-s$, and simulation time step $\Delta t=0.1$	T and V
7	**Optimum Velocity Model (a modified model)** $a_n(t+T)=\alpha\{V_o-v_n(t)\}$ Where $V_o=\sqrt{2b(x_{n-1}(t)-x_n(t))}$	T and α
8	**Newell Model** $x_n(t+\tau)=x_{n-1}(t)-D_n$	τ and D_n

3.1 Stimulus Response Model

Chandler et al. (1958) was first to propose a linear model based on stimulus response concept. It was stated that the response of a driver is proportional to the stimulus he perceives. The relative speed was defined as the only stimulus. The response of the following driver comes at a time delayed by the response time T. The proportionality factor λ was called sensitivity factor. A series of investigations on this model by researchers associated with General Motor's laboratory incorporated spacing and speed into the sensitivity term giving the model a nonlinear form. Later, Gazis et al. (1961) generalized the nonlinear form of the model that is termed here as generalized GM (GGM) model. The parameters α, m and l are termed as sensitivity parameters. The Chandler model have only two parameters to be optimized i.e. response time T and sensitivity factor λ, while for GGM there are sensitivity parameters also to be optimized. Previously, Ranjitkar et al. (2003) have investigated the stability of traffic flow based on the GM model.

3.2 Safe Distance Model

The first model based on safe distance concept came from Kometani and Sasaki (1959). It was stated that the driver of following vehicle chooses his speed based on a safe following distance to avoid possible collision with the vehicle ahead. Later, Gipps (1981) proposed a modified model that can be calibrated using common sense assumptions about driving behavior such as acceleration, deceleration, maximum speed etc. The model is widely preferred for simulation purpose. In this model, 2.5 and 0.025 are arbitrarily chosen parameters as proposed by its developer, while acceleration rate a, jammed spacing s can be assigned with some fixed values but the parameters like response time T, braking rate b, maximum desired speed V and assumed braking rate need to be optimized. Recently, Kruass (1997) proposed a model which is a variant of the Gipps model. This is a stochastic model as it includes a stochastic term that was set to zero to unify the comparison. All the parameters to be optimized are same as of Gipps model i.e. response time T, braking rate b and maximum desired speed V.

3.3 Psychophysical Model

Leutzbach et al. (1986) proposed a model that considers psychophysical aspects of driving behavior. The model is well recognized especially for simulation purpose. It has two parameters to be optimized i.e. response time T and desired spacing S. What different from other models is that this model considers acceleration $a_{n-1}(t)$ of the vehicle ahead as a stimulus for the following vehicle, in addition to the difference between the current spacing and desired following distance S.

3.4 Cell Based Model

This type of model was first introduced by Nagel-Schereckenberg (1992). It is commonly known as cellular automata. Some simulation software developed recently has preferred this model. The model has two parameters to be optimized i.e. acceleration a, and desired maximum speed V, while other parameters can be assigned with some fixed values (presented in the next section).

3.5 Optimum Velocity Model

Bando et al. (1995) was first to propose a model based on the optimum velocity concept. It was stated that the following driver's response is proportional to the difference between his optimum speed (for the given spacing) and his speed at that time. We have modified the optimum speed term that depends on the spacing from the vehicle ahead. The modified model is presented in **Table 2**. It has two parameters to be optimized i.e. the response time T and a sensitivity term α.

3.6 Trajectory Based Model

Newell (2002) proposed a simple model based on the concept that the driver of following vehicle drives as a shifted space trajectory of the vehicle ahead. It was stated that the space trajectory of the following vehicle is same as that of the vehicle ahead except for a translation in space and in time. This model also has two parameters to be optimized i.e. the time lag τ and a distance lag D_n.

4. EVALUATION SET UP

The models will be evaluated based on their optimized performance i.e. after calibrating them against the given data sets. Calibration is the optimization of model parameters so that the model can make better predictions. Among several methods available for this purpose, genetic algorithm (GA) is well-recognized for its effectiveness in escaping local minima particularly when the objective function has a lot of peaks (a common problem faced by many optimization methods). It is based on the mechanism of natural selection of natural genetics. GENECOP III is the latest version of a GA based optimization program proposed by Z. Michalewicz (1992). We have modified this program to implement in car-following models. Its major features include operation by floating-point number, flexibility in dealing with constraints and boundary conditions, and systematic application of mutations and crossovers.

A percentile error function is used as the objective function to be minimized.

$$J(e) = \mathrm{Min}\,\frac{\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right|}{\sum_{i=1}^{n}\left|y_i\right|} \tag{1}$$

Where, y is the objective variable. y_i and $\hat{y}_i$ are the measured data and predicted values for the objective variable. n is the total number of data points in the data set. For car-following models either of spacing, speed or acceleration can be the objective variable. The vehicular motion data (including spacing, speed, acceleration) of a vehicle is predicted using the respective model formulation, where the motion data of the vehicle in front is supplied as input. For a particular set of parameters, the percentile error in the prediction of the objective variable is computed as given in equation (1). The parameter set that produce the lowest percentile error is the optimal one. We have calibrated the models using three different objective variables i.e. spacing, speed and acceleration one by one. Once the parameters are optimized for a given data set, the respective percentile error values are computed that will be used to compare of the performances of the models.

The following are some general boundary conditions set to optimize the model parameters,

- Response time (T) = 0.5 to 3 sec
- Maximum acceleration (a) =1.5 m/sec^2
- Maximum deceleration (b and b*) = - 3 to -4.5 m/sec^2
- Maximum desired speed (V) = 20 to 25 m/sec
- Jam headway (s) = 7.5 m
- Minimum desired spacing (S) = 10 to 50 m

The constraint used to avoid collision is:

$x_{n-1} \geq x_n$ + minimum gap

5. SIMULATION RESULTS

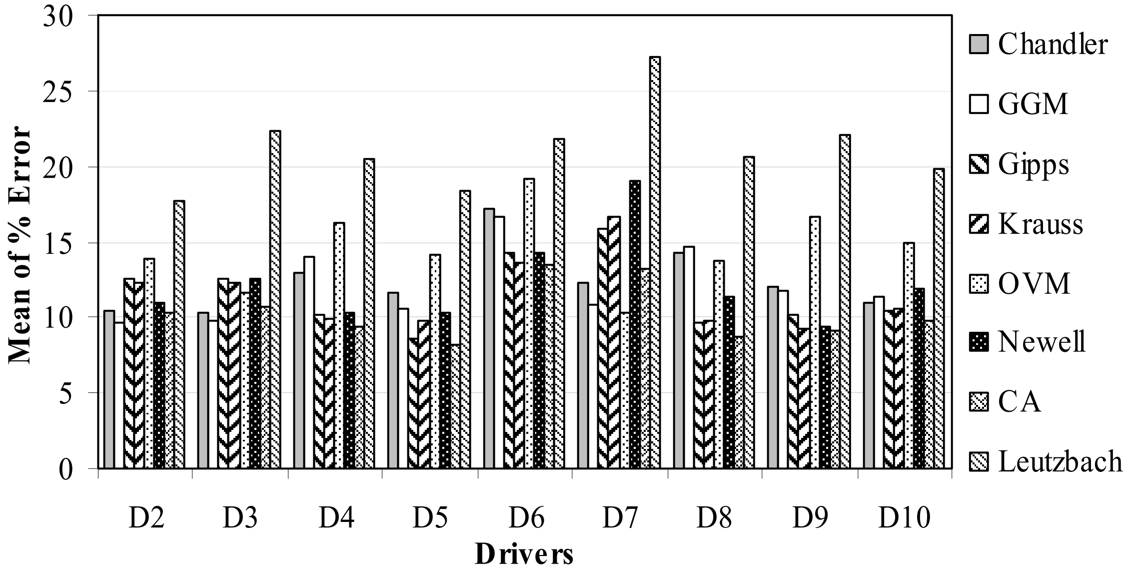
5.1 Model Calibration

Table 3 presents mean values and the respective standard deviation (SD) and coefficient of variation (COV) for the percentile error in the prediction of spacing, speed and acceleration using the same as objective variable. All models produce relatively lower percentile error in speed prediction and higher for acceleration prediction. It might be because the acceleration data have a lot more fluctuations than the speed data. The mean percentile errors in spacing prediction vary from 10.93% for CA to 21.39% for Leutzbach model with COV vary from 35 to 56%. For speed prediction, the mean percentile errors vary from 3.46% for Chandler model to 4.71% for Newell model with COV in the range of 25 to 33%. While for acceleration prediction, the mean values vary from 52.22% for Chandler model to 64.89% for Leutzbach model with relatively low COV that vary from 20 to 26%.

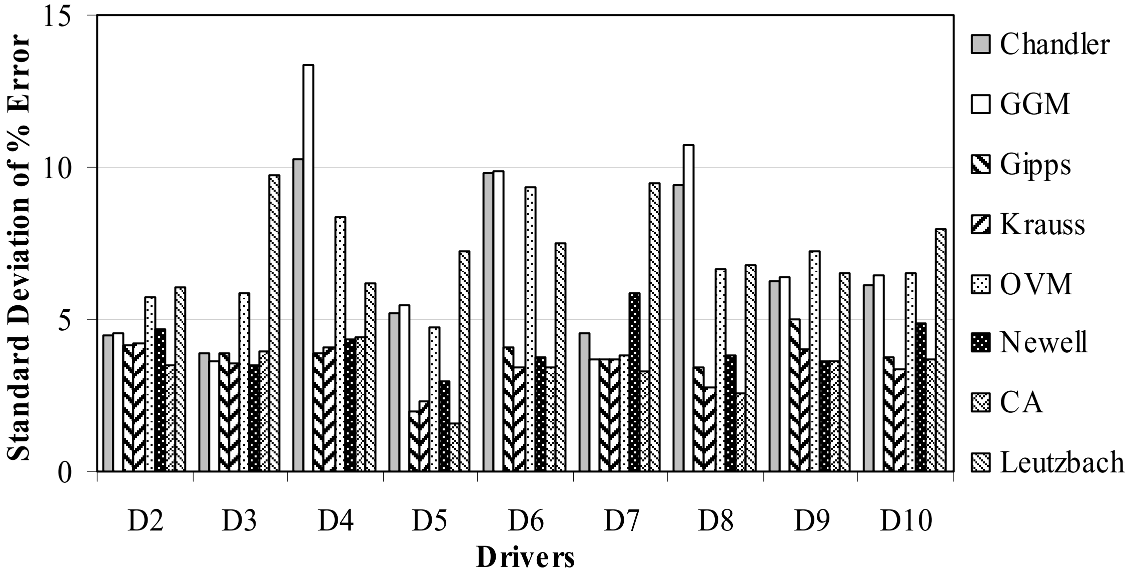
Table 3. Mean, standard deviation and coefficient of variation of the percentile errors

Data Base	**Descriptor**	**Models**							
		Chandler	GGM	Gipps	Krauss	OVM	Newell	CA	Leutzbach
Spacing	Mean	12.73	12.13	12.20	12.20	14.52	12.91	10.93	21.39
	SD	6.35	6.84	4.65	4.65	6.38	5.13	4.08	7.51
	COV	50%	56%	38%	38%	44%	40%	37%	35%
Speed	Mean	3.46	3.52	4.30	4.30	3.66	4.71	4.06	4.70
	SD	0.95	1.03	1.09	1.09	1.00	1.54	1.08	1.25
	COV	27%	29%	25%	25%	27%	33%	27%	27%
Acceleration	Mean	52.22	52.96	64.49	64.49	55.53	63.55	63.46	64.89
	SD	10.97	10.88	14.11	14.11	11.00	16.40	14.25	14.97
	COV	21%	21%	22%	22%	20%	26%	22%	23%

Figure 3a presents breakdown of the mean percentile error in spacing prediction for each driver and model separately, while **Figure 3b** presents the respective standard deviations. There are significant interpersonal variations in the models' performance. For example, Chandler model and GGM perform better than others for the drivers D2 and D3 while for D4, D5, D6, D8, D9 and D10 CA, Gipps and Krauss models perform better than others.



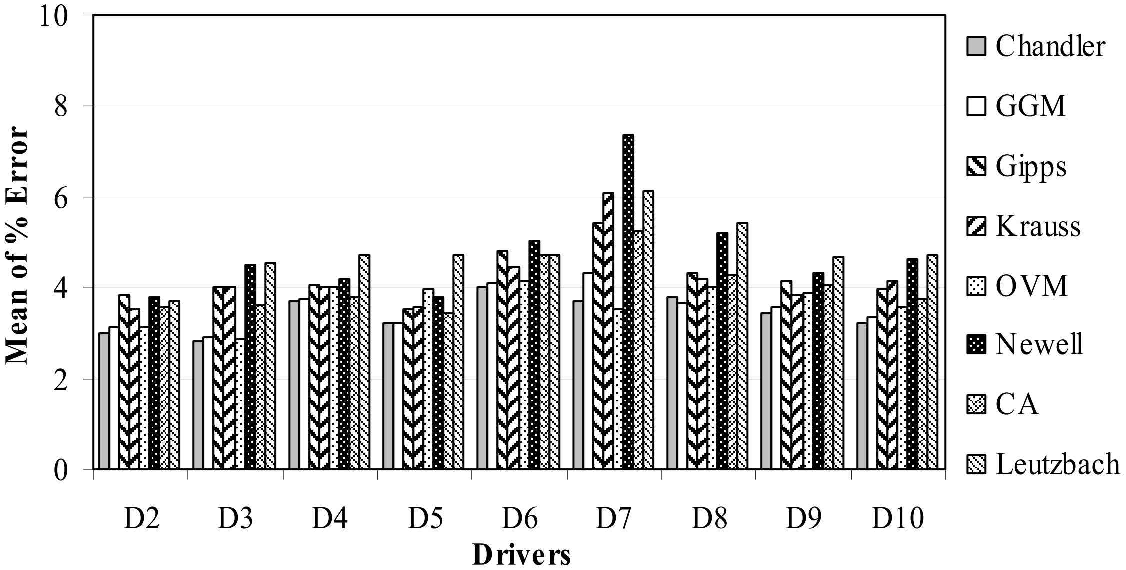
a) Mean of % error



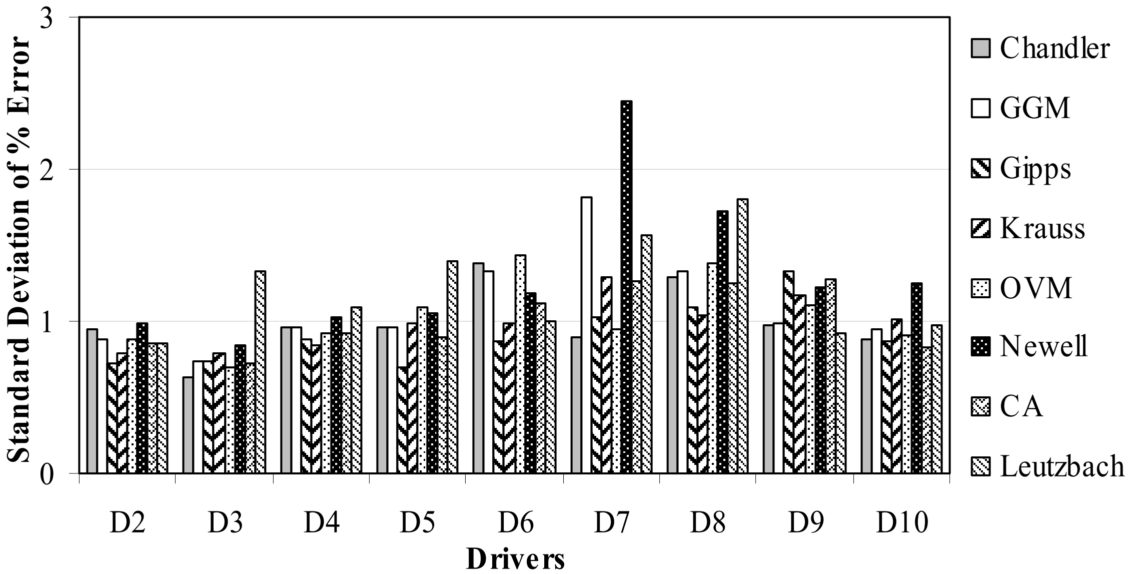
b) Standard deviation of % error

Figure 3. Percentile error in the estimation of spacing between adjacent vehicles

Figure 4 presents the mean and standard deviation of the percentile error in speed prediction for each driver and model separately using the same as objective variable. The interpersonal variations are dominant here also for an example almost all model gives higher percentile error for the driver D7. This is an indication of individual's influence in car-following process. In general, the models are more completive than the previous case, while Chandler model and GGM perform better than other models for almost all drivers.



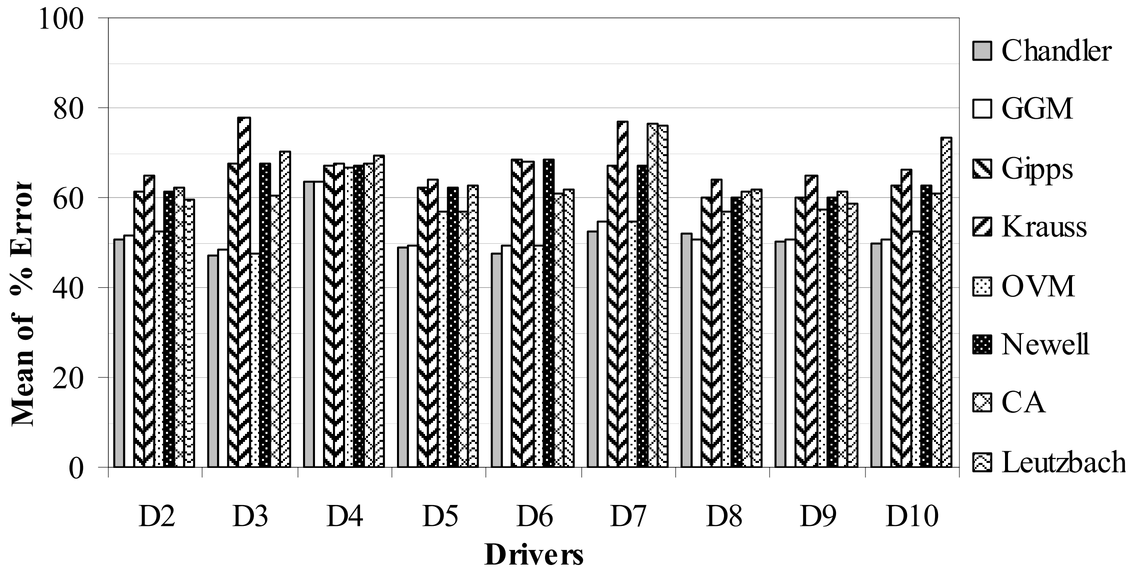
a) Mean of % error



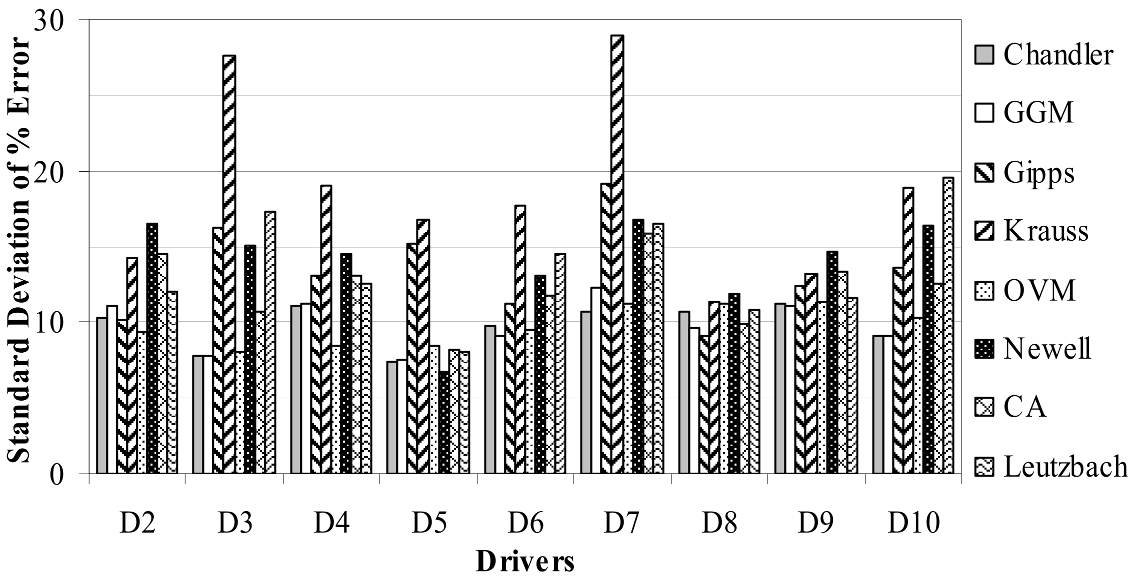
b) Standard deviation of % error

Figure 4. Percentile error in the estimation of speed

Figure 5 presents the mean and standard deviation of the percentile error in acceleration prediction for each driver and model separately using the same as objective variable. Here also Chandler model and GGM produce relatively lower percentile error than other models. While for other models the performance varies case by case. Different from the previous case, it is difficult to claim significant influence from individual drivers.



a) Mean of % error



b) Standard deviation of % error

Figure 5. Percentile error in the estimation of acceleration

5.2 Statistical Verification

The differences seen in the models' performances are further analyzed to verify them statistically. The F-test is conducted to see the difference in the variances, while T- test is conducted to see the difference in the means.

Table 4 presents F- test and T- test results for the percentile error data using spacing as the objective variable. The shaded cells represent the case where F- value or T- value exceed the F- critical or T- critical values respectively. In this case, the variances of the percentile errors computed in the case of Leutzbach model are different from those for all other models, while for those cellular automata model are not different from other models. The results of T- test show that the mean percentile error estimated for chandler model, generalized GM, Gipps model and Krauss model are statistically same, while those for other models are different from these models in most of the cases. In other words, these results validate better performances of these four models compared with other models in this case.

Table 5 presents F- test and T- test results for the percentile error data using speed as objective variable. The F-test results show the variances of the estimated percentile errors are different in the many cases. While the results from T- test show the mean values also different in the most of the cases. That means though the differences in mean values are not much statistically the differences are valid.

Table 6 presents F- test and T- test results for the percentile error data using acceleration as objective variable. Several shaded cells in F-test results show the variances are different in those cases. Except a few cases, t-values have exceeded the critical value, showing differences in the mean values estimated for the percentile error using acceleration data. That means the models can be evaluated based on the mean and standard deviations estimated in this case.

Table 4. F- test and T- test results for percentile error estimated using spacing data

		F-test Results							
		Chandler	GGM	Gipps	Krauss	OVM	Newell	CA	Leutzbach
T-test Results	Chandler		1.16	0.54	0.51	1.01	0.65	0.41	1.40
	GGM	1.31		0.46	0.44	0.87	0.56	0.36	1.21
	Gipps	1.36	0.18		0.95	1.89	1.22	0.77	2.61
	Krauss	1.79	0.21	0.50		1.99	1.28	0.81	2.75
	OVM	4.06	5.21	5.99	6.45		0.65	0.41	1.39
	Newell	0.46	1.87	2.10	2.59	4.00		0.63	2.15
	CA	4.85	3.06	4.18	3.71	9.66	6.16		3.40
	Leutzbach	17.94	18.57	21.19	21.71	14.19	18.99	24.93	

Table 5. F- test and T- test results for percentile error estimated using speed data

		F-test Results							
		Chandler	GGM	Gipps	Krauss	OVM	Newell	CA	Leutzbach
T-test Results	Chandler		1.19	1.32	1.56	1.13	2.66	1.31	1.73
	GGM	0.99		1.11	1.31	0.95	2.24	1.11	1.46
	Gipps	11.87	10.49		1.18	0.86	2.02	1.00	1.32
	Krauss	9.83	8.61	1.38		0.72	1.71	0.84	1.11
	OVM	2.99	1.90	8.77	6.94		2.36	1.17	1.54
	Newell	14.11	13.02	4.49	5.50	11.64		0.49	0.65
	CA	8.53	7.28	3.14	1.62	5.52	7.04		1.32
	Leutzbach	16.13	14.76	4.93	6.04	13.20	0.16	7.86	

Table 6. F- test and T- test results for percentile error estimated using acceleration data

		F-test Results							
		Chandler	GGM	Gipps	Krauss	OVM	Newell	CA	Leutzbach
T-test Results	Chandler		0.98	1.65	2.89	1.01	2.24	1.69	1.86
	GGM	0.97		1.68	2.94	1.02	2.27	1.72	1.89
	Gipps	13.99	13.19		1.75	0.61	1.35	1.02	1.13
	Krauss	14.77	14.11	2.98		0.35	0.77	0.58	0.64
	OVM	4.34	3.39	10.20	11.65		2.22	1.68	1.85
	Newell	11.70	10.97	0.88	3.58	8.27		0.75	0.83
	CA	12.73	11.94	1.05	3.86	8.97	0.08		1.10
	Leutzbach	13.90	13.14	0.39	2.58	10.26	1.22	1.40	

6. SUMMARY AND DISCUSSION

With rapid growth of computer technology, the processing speed of computers has increased significantly in the last few years. The microscopic traffic flow simulation models have more important role to play in traffic and safety engineering. It is possible now to acquire a network level representation of traffic movements without compromising on its indigenous driving behavior such as car following and lane changing behavior. Recognizing the needs of next generation traffic simulation, Federal Highway Administration (FHA) has recently proposed a Next Generation Simulation (NGSIM) program (2003). The efforts are to develop core research in behavioral algorithms to support traffic simulation with a primary focus on microscopic modeling.

This paper has evaluated the performance of several car-following models based on how well they represent real driving behavior. The data were taken from car-following experiments conducted in a test track where the driving conditions were kept as simple as considered in the car-following theories. A genetic algorithm based optimization method is adapted to calibrate the models based on three different objective variables i.e. spacing, speed and acceleration. These three cases are analyzed separately to benchmark the models' performances using percentile error value as performance index.

Among the eight models investigated, in general speaking Chandler model and generalized GM model performed better than others producing lower percentile errors for speed and acceleration predictions. However, cellular automata performed better than others for the prediction of spacing. Some sudden drops were seen in the acceleration data predicted by this model. The Gipps model and Krauss model have performed well behind the leading models described earlier for spacing and speed prediction, while for acceleration the percentile error were relatively higher. The modified optimum velocity model performed well with speed and acceleration predictions, while for spacing the percentile error was relatively higher. The Newell model produced competitive percentile error values for spacing prediction, while the same for speed and acceleration prediction were relatively higher. The percentile error for Leutzbach model was generally higher, while for some drivers the model was competitive with others e.g. for the driver D2, D6 and D9 in the case of acceleration prediction. These differences in the performances of the models were verified statistically using F-test and T-test. The results show clear difference in the mean percentile errors in many cases.

All models produce relatively lower percentile error for speed predictions and higher values for spacing and acceleration predictions. The higher error values for acceleration prediction might be due to the fact that one can expect a lot more fluctuations with acceleration data than the speed or spacing data, making it difficult for the models to predict in a close range. The spacing data is computed from the position measurements of the adjacent vehicles. This gives possibility of increase in error size due to addition effect. While for the speed data, the measurements taken by the receivers is used directly without any computations. The higher values of standard deviation and coefficient of variations in the case of spacing predictions shall be noted in this regard.

Besides the observations discussed above, it is important to note that the interpersonal variations are influential particularly in the case of speed predictions. The same models performed differently from driver to driver. Such interpersonal variations are much higher than inter-model variations. While in the case of acceleration predictions, these variations are not that dominant.

As for the application of this study in real world is concerned, we would like to emphasize that these results are based on some particular diving conditions tested in the test track that might not necessarily represent real world driving behavior. In fact, the driving conditions in real world are much more complicated than the one analyzed here.

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