

## A LOCATION MODEL FOR THE ALLOCATION OF THE OFF-STREET PARKING FACILITIES

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**Abstract:** In this study, an application of the location theory on the allocation of the off-street parking facilities is explored. A multi-objective linear Integer Programming model is developed for this problem. Due to its linearity, the proposed compromise programming method for the model can be solved by the LINDO compute software. Based on the results of the numerical experiment, the proposed model can provide reasonable solution to the problem addressed. The following information can be provided by the proposed solution procedure for the decision makers to determine the locations of the off-street parking facilities: the optimal locations of facilities, the demands served by each facility, and the optimal type for each of the selected parking facilities.

**Key Words:** Compromising programming, Location model, Multi-objective programming, and Parking allocation

### 1. INTRODUCTION

With the rapid increase in the car ownership per household and the urban population, in the newly developed industrial country, the mass use of the private car in the metropolitan area results in a serious parking problem with limited land resource. The more the city is urbanized, the more serious the parking situation is for the city. To develop an effective procedure to allocate appropriate space for the parking facility is a crucial task for the urban planner and the associated traffic agencies. With the scarce land resource in the urban area, the use the non-parking public facilities as the potential candidates of the parking facilities is a promising alternative for the metropolitan decision maker. With the relative high land cost in the urban area, it will be a waste for the public investment, if the public facilities do not used for multiple purposes. The use of the existing public facility as a parking facility, it will not only reduce the public resistance for buying their land, but also decrease the additional investment for the land cost. Therefore, the purpose of this study is to develop an efficient procedure to allocate the off-street parking facilities in the existing public facilities as the reference basis for the decision making process.

In the current parking practice, the allocation of the off-street parking facilities is often based on the ratio between forecast parking demand and supply. The facility location theory has seldom been applied to this type of problem. In this study, we explore the application of the facility location theory on the off-street parking facility locations. A suitable location model is developed to deal with the allocation of the public off-street parking facilities.

In the previous studies dealt with the parking problem, Kanafani (1972) treated the behavior of the parking system as a distribution process. He developed a gravity-type model to forecast the potential parking demand for each parking facility. The allocation of the parking facility is

determined by the economic analysis of each facility. If it is profitable, then the associated facility is built. Drickx and Jennergren (1975) applied the location theory to determine the optimal use of the existing available parking space under different types of parking demand. Goyal and Gomes (1984) applied the linear programming to solve the allocation of the campus parking facilities with different priorities. Huang (1991) used the Location III decision support system to identify the optimal location based on three criteria, economy, equity and efficiencies. Hsu (1994) applied the location theory on the allocation of the parking facilities in a metropolitan area. Liao (2001) dealt with the dynamic parking demand in the allocation of the parking facilities. A mathematical software AMPL was used to determine the location of these facilities. Based on the review of these works cited above, it indicates that various objective functions have been applied to the parking facility location problem. In general, the focus of the public sector is its equity and efficiency. As for the private sector, the efficiency of the facility is its major concern. In the existing studies, most studies deal with a single goal. Therefore, in this study, we explore the potential of the application of the multi-objective programming on the parking facility location problem. The remainder of the paper is organized as follows: In section 2, we present a statement of the problem and the proposed multi-objective linear integer programming model for this location problem. Section 3 discusses the solution methodology for the problem. In section 4, we choose the Ta-an District of Taipei city as the study area to test the applicability the proposed methodology. Finally, in Section 5, we summarize the paper and suggest possible future research directions.

## **2. PROBLEM STATEMENT**

With limited land resource and restriction of the off-street parking facilities, it is impossible for the supply of the off-street parking facility to meet the growing parking requirement. Therefore, in the allocation of the off-street parking facilities, the major concern will be the efficiency of the facility. To reflect the critical decision criteria for the allocation of the off-street parking facilities, there are two types of goals are considered in this study. The first goal is to maximize the parking demand served by the parking facilities. The second goal is to minimize the total social cost, The major components of the social cost are: the construction cost, the operating cost, maintenance cost for the operators, the walking cost for the users, the anti-pollution cost of the noise reduction cost and die non-users, and penalty cost for the unsatisfied demand.

The problem dealt in this paper is as follow: there are  $n$  public facilities in the study area. Each can provide some space for the off-street parking with limited capacity. The problem is to determine the appropriate locations for the  $N$  off-street parking facilities. There is preset demand at each point, which is the maximum number of cars looking for the parking in each demand point; these demands are need not be fully satisfied. A fixed construction cost is incurred for each unit parking space supplied to the demand. The capacity of the each type of parking facilities is the maximum number of cars can be parked in the facilities. There are fixed charge of the unit operation cost, maintenance cost, air pollution cost and noise reduction cost for the parking unit. In addition, a penalty cost is incurred for each unit of unsatisfied parking demand. The outputs of the problem are: locations of the  $N$  off-street parking facilities, the type of parking facility for each selected location, the maximum number of parking space amount of demand served for each demand point, and the assignment of the demand to off-street parking facilities. There are two types of objectives in this problem, i.e., the maximum parking demand served and the minimum total social cost.

The notations for this problem are defined as follows:

I: The set of parking demand point i

J: The set of parking facility candidate j

J': The set of parking candidate j, if there are two or more types of parking facilities to be considered.

P : The set of parking facilities type p

$D_i$  : The demand in point I, i.e., number of cars looking for parking.

$B_{jp}$  : The capacity in candidate j if type p is selected, i.e., the maximum number of cars that can be parked in this type of facilities.

$CC_{jp}$  : The annual unit construction cost for candidate j if type p is selected.

$OC_{jp}$  : The annual unit operating cost for candidate j if type p is selected.

$MC_{jp}$  : The annual unit maintenance cost for candidate j if type p is selected.

$PC_j$  : The annual air pollution cost per car in candidate j

$NC_j$  : The annual noise reduction cost per car in candidate j

$PENC_i$ : The annual penalty cost per unsatisfied demand for point i

T : The annual number of parking

R : The average number of persons in a car

Tv: The unit lime cost for the driver

$Wt_{jp}$  : The walking time between point i and candidate j if type p is selected

N : The number of off-street parking facilities to be built

S: The maximum walking distance for the driver

$d_{ji}$  : The distance between point i and candidate j if type p is selected

$N_i$  : The candidate set of j which is within maximum walking distance

The decision variables used in this study is ad follows:

$X_{i,j-p}$  : The amount of demand in point i served by parking facility candidate with type p facility (parking space per perk hour).

$Q_i$  : The unsatisfied demand in point i due to the limitation of the packing space.

$$Y_{jp} = \begin{cases} 1 & \text{if the type } p \text{ is selected for candidate } j \\ 0 & \text{Otherwise} \end{cases}$$

The mathematical model for the off-street location problem is presented next.

$$\begin{aligned} \text{MAX. } Z_1 &= \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} X_{ijp} \quad (1) \\ Z_2 &= \sum_{j \in J} \sum_{p \in P} CC_{jp} \times Y_{jp} + \sum_{j \in J} \sum_{p \in P} OC_{jp} \times B_{jp} \times Y_{jp} + \sum_{j \in J} \sum_{p \in P} MC_{jp} \times B_{jp} \times Y_{jp} \\ \text{MIN. } &+ \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} T \times R \times T_v \times 2 \times W_{t_{jp}} \times X_{ijp} + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} PC_j \times X_{ijp} \\ &+ \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} NC_j \times X_{ijp} + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} PEN \times Q \quad (2) \end{aligned}$$

$$\sum_{j \in N} \sum_{p \in P} X_{ijp} + Q_i = D_i, \forall i \in I \quad (3)$$

$$\sum_{i \in I} X_{ijp} \leq B_{jp} \times Y_{jp}, \forall j \in J; \forall p \in P \quad (4)$$

$$\text{s.t. } \sum_p Y_{jp} \leq 1, \forall j \in J \quad (5)$$

$$\sum_{j \in N} \sum_{p \in P} Y_{jp} = N \quad (6)$$

$$X_{ijp}, Q_i \geq 0, \forall i \in I; \forall j \in J; \forall p \in P \quad (7)$$

$$Y_{jp} = 0, 1, \forall j \in J; \forall p \in P \quad (8)$$

The first objective of the problem is to maximize the total parking demand it served as shown in Equation (1). The second objective is to minimize the total social cost as defined in Equation (2). The first three cost components of the social cost are the construction, operating, and maintenances cost for the operator of the parking facilities, the fourth component is the user cost, the air pollution cost and noise reduction cost for the non-users are the fifth and sixth component respectively, the final component is the penalty cost of unsatisfied demand for the users. Constraint (3) ensures that the demand in each point is either served by facility or as part of unsatisfied demand. Constraint (4) limits the total demand assigned to each facility to the available capacity of that facility. Constraint (5) indicates that only one type of parking facility can be built in each location. Constraint (6) states the number of off-street parking facilities can be built.

### 3. THE SOLUTION PROCEDURE

A three-phase procedure is applied in this study in dealing the off-street parking facility location problem. First, the potential locations is selected as the facilities candidates, the criteria of selection used in this study are the minimum available land use, and the length and the width of the nearby roads around the potential location. The available types of parking facilities are then identified in the second phase, based on the following assigning principles: rule number one, the multiple stories parking facilities is first choice, either by ramp or by elevator. Unless this selection is impossible, the underground facilities will then be considered. Rule number two, if the width of the location is less than 35 meters and its space is less than 1500 square meters, then the underground facilities type is selected. If the width is greater than 35 meters and its space is between 1500 and 4000 square meters, elevator and ramp-type

can be considered. If its width is greater than 35 meters and its space is greater than 4000 square meters, than the ramp-type facilities is used. After the locations and types of candidate parking facilities are identified, the problem is then formulated as a multiobjective programming problem in phase three. A number of successful solution methods have been developed for this type problem. Since we do not have the preference of the decision makers in this study, therefore, the compromise programming is selected as the solution for the problem under consideration. The proposed solution method will put all the objectives in the problem into consideration with no preference on either objective. The best solution is one with the closest to the optimal solution of the system under each objective.

The Compromise programming is an appropriate method in the multiple objectives programming when the preference of the decision maker is unknown. The Compromise programming is an iterative method appropriately used in a multiple linear problem. However, it have also been used in analysis of discrete objective problems (Duckstein and Opricvic, 1980). The method identifies solutions that are closest to the ideal solutions as determined by some measure of distance. The solutions identified as being closest to the ideal solution are called compromising solutions and constitute the compromise set. To apply this methodology, we first must define the ideal solution and specify the particular distance measure to be used in the problem. The ideal solution is defined as the optimal solution of the problem under each objective of the multiple objectives as a single objective problem. The procedure for evaluation of the set of nondominated point is to measure how close these points come to the ideal solution. One of the most frequently used measures of closeness is as the follows:

$$\text{MIN. } d_\alpha = \left[ \sum_{i \in n} \left| 1 - Z_i(x) / Z_i^* \right|^\alpha \right]^{1/\alpha} \quad (9)$$

s.t.  $x \in X$

When  $1 \leq \alpha \leq \infty$  ; when  $\alpha = 1$ , the Equation (9) can be expresses as follows:

$$\text{MIN. } d_1 = \sum_{i \in n} \left| 1 - Z_i(x) / Z_i^* \right| \quad (10)$$

When  $\alpha = \infty$ , it indicates the one with maximum distance has the dominated impact, Equation (10) can be expressed as follows:

$$\text{MIN. } d_\infty = \text{MAX} \sum_{i \in n} \left| 1 - Z_i(x) / Z_i^* \right|, \quad i=1,2,\dots,n \quad (11)$$

s.t.  $x = X$

#### 4. NUMERICAL EXPERIMENT

A district of the City of Taipei, the Ta-An District, is chosen as the study area. The parking zoning is based on the "Annual Survey of the parking supplies and demands for the Taipei City". There are 100 parking zones in the study area. The demands in the study area are far greater than the supplies of parking based on the annul survey. Therefore, it is impossible to provide enough space for the entire demands. With the 2003 as the basic year, the target year is set to be 2008 in this study. The unsatisfied demand in that year is computer as follow:

$$TS_{pi} = D_{bi} \frac{C_p}{C_i} * 0.85 - TS_{bi}$$

$TS_{pi}$  =The project unsatisfied demand in zone i in target p year

$TS_{bi}$  =The existing supply in zone i in basic year

$D_{bi}$  =The demand in zone i in basic year

$C_p$  =The project demand in Taipei city in target p year

$C_b$  =The demand in Taipei city in basic year

In this study, we first obtain the available land use for the public facilities in the study area from the City of Taipei. The public facilities under consideration are park, public market, and public school. Based on the criteria discussed in Section 3, 23 potential locations are selected as the candidates for the off-street parking facilities as shown in figure 1. The appropriate type of parking facilities for each location is identified based on the criteria discussed in Section 3. All the cost parameters are derived from the following reports:

Statistics of tire Engineering Practice in Taipei City (1981)

Planning Handbook for the Parking Facilities (1986)

Planning of the Multiobjective Parking Facilities in San-chung (1991)

All the cost parameters are converted to the target year present worth value by the estimated annual inflation rate, which are listed in Table 1.

Table 1: The Units Cost Components for the Candidate Locations

Candidate location	1	2	3	4	5	6	7	8
Type	4	3	4	4	3	3	4	4
Capacity	2400	600	960	2400	320	720	1280	720
Demand served	1929	4071	4363	2472	2242	1857	2535	2196
Annual construction cost	273653	53129	109461	273653	28336	63755	145948	82096
Maintenance and operation	30.6	32.3	30.6	30.6	32.3	32.3	30.6	30.6
Pollution and Noise cost	15.47	15,47	25.47	15.47	15.47	15.47	25.47	25.47

Table 1: The Units Cost Components for the Candidate Locations (continuous)

Candidate location	9	10	11	12	13	13	14	14	15
Type	4	3	1	3	3	4	3	4	1
Capacity	800	320	400	1600	920	500	680	360	680
Demand served	1678	2231	3626	1942	3902	3902	3032	3032	3889
Annual construction cost	91218	28336	18932	141678	81446	57011	60213	41050	32169
Maintenance and operation	30.6	32.3	25.3	32.3	32.3	30.6	32.3	30.6	25.3
Pollution and Noise cost	15.47	15,47	25.47	25.47	15.47	15.47	25.47	25.47	15.47

Table 1: The Units Cost Components for the Candidate Locations (continuous)

Candidate location	16	17	18	19	20	20	21	22	23
Type	4	4	4	3	3	4	3	4	4
Capacity	960	1280	1120	600	576	290	760	1045	1280
Demand served	1992	3551	2953	4666	4883	4883	4331	1845	2430
Annual construction cost	109461	145948	127705	53129	51005	33077	67298	118585	145950
Maintenance and operation	30.6	30.6	30.6	32.3	32.3	30.6	32.3	30.6	30.6
Pollution and Noise cost	25.47	25.47	25.47	15.47	15.47	15.47	15.47	15.47	25.47

Following the compromising programming requirement, we need to obtain the ideal solution for each objective, i.e., maximum parking demand (Table2) and minimum total social cost (Table 3). We then obtain compromise solutions with respect to different values of  $\alpha$  (1 and  $\infty$ ), as shown in Table 4 and 5. Based on the results of the numerical experimental as shown in Table 2, 3, 4, and 5, the following conclusions can be drawn:

Table 2 Results under the Objective of the Maximum Demand It Served

Service number	n=5				
Location number	3	7	13	17	22
Type	4	4	3	4	4
Capacity	1200	1192	1150	1600	1246
Demand served	1200	1192	1150	1600	1246
Total social cost	2477084				
Total demand served	6388				

Table 3 Results under the Objective of the Minimum Total Social Cost

Unit penalty cost	0				
Location number	5	10	11	15	20
Type	3	3	1	1	4
Capacity	400	400	500	850	360
Demand served	0	0	0	0	0
Total social cost	246765				
Total demand served	0				

Table 3 Results under the Objective of the Minimum Total Social Cost (continued)

Unit penalty cost	40				
Location number	5	10	11	15	20
Type	3	3	1	1	4
Capacity	400	400	500	850	360
Demand served	400	400	370	850	360
Total social cost	813176				
Total demand served	2380				

Table 3 Results under the Objective of the Minimum Total Social Cost (continued)

Unit penalty cost	150				
Location number	10	11	15	20	21
Type	3	2	1	1	4
Capacity	400	500	850	720	950
Demand served	400	500	850	720	950
Total social cost	2189759				
Total demand served	3420				

Table 4 Results under the Multiple Objectives ( $\alpha = 1$ )

Unit penalty cost	0				
Location number	5	10	11	15	20
Type	3	3	1	3	1
Capacity	400	400	500	795	850
Demand served	400	400	500	795	850
Total social cost	380025				
Total demand served	2945				

Table 4 Results under the Multiple Objectives ( $\alpha = 1$ ) (continued)

Unit penalty cost	40				
Location number	11	12	13	15	21
Type	1	3	3	1	3
Capacity	500	1055	1150	850	950
Demand served	500	1055	1150	850	950
Total social cost	1051027				
Total demand served	4505				

Table 4 Results under the Multiple Objectives ( $\alpha = 1$ ) (continued)

Unit penalty cost	150				
Location number	3	15	17	21	22
Type	4	1	4	3	4
Capacity	1200	850	1600	950	1245
Demand served	1200	850	1600	950	1245
Total social cost	2367536				
Total demand served	5845				

Table 5 Results under the Multiple Objectives ( $\alpha = \infty$ )

Unit penalty cost	0				
Location number	5	10	11	15	20
Type	3	3	1	3	1
Capacity	400	400	500	850	360
Demand served	400	400	370	855	360
Total social cost	303256				
Total demand served	2380				

Table 5 Results under the Multiple Objectives ( $\alpha = \infty$ ) (continued)

Unit penalty cost	40				
Location number	11	13	15	20	21
Type	1	3	1	3	3
Capacity	500	1155	850	720	950
Demand served	500	1155	850	720	950
Total social cost	995363				
Total demand served	4170				

Table 5 Results under the Multiple Objectives ( $\alpha = \infty$ ) (continued)

Unit penalty cost	150				
Location number	3	7	13	17	22
Type	4	4	3	4	4
Capacity	1200	1192	1150	1650	1245
Demand served	1200	1192	1150	1650	1245
Total social cost	2470224				
Total demand served	6387				

1. For the objective of the maximum parking demand it served as shown in Table 2, the locations it selected tends to be those with larger capacities. The types of parking facilities it selected with the same pattern, i.e., type with larger capacities.
2. For the objective of the minimum total social cost as shown in Table 3, the penalty cost is the dominated factor in determining the amount of the parking space it provides. If the penalty cost is relative small to other cost components, then the parking space it provided is often small. When the penalty cost is less than the total variable cost, there are always unused spaces in the locations it selected. The locations it selected are those with smaller construction cost. It means under this situation, the construction cost is dominated factor in the allocation procedure. When the penalty cost is grater the total variable cost, there is no unused space in each location it selected. Furthermore, computational results indicate that as the penalty cost increases, the locations selected are those with larger capacity. When the penalty cost is relative larger, the solution is similar with those under maximum parking demand it served.
3. Under multiple objectives, the solutions provide compromise solutions that are between two ideal solutions. Comparing the solutions with those form minimum total social cost, it tends to provide more parking spaces and wider service area for each location selected. While, comparison between these solutions with those form the maximum parking it provided, we can find it tends to assign demand to the nearest locations which is ignored by those solutions under maximum parking demand it provided.
4. The value of  $\alpha$  plays an important role in the compromise solution. When the penalty cost is less than the total variable cost, the tendency of selecting locations with larger capacity is more obvious when its value is  $\infty$  than those when its values is 1. However, when the penalty cost exceeds the total variable cost, the trend is then reverse. Furthermore, when the  $\alpha$  value is  $\infty$ , it sometime will select the locations that do not serve the demand in its parking zone. This situation never occurs when its value is 1. Therefore, the results of the  $\alpha = 1$  is more realist than those of the  $\alpha = \infty$ .

## 5. CONCLUSIONS AND RECOMMENDATION

A solution procedure for the allocation of the off-street parking facilities among the available land use of the public facilities was developed in this paper. A multiobjective linear programming model was derived from the traditional facility location theory. Several measures of effectiveness (MOEs) from different aspects on the parking demand are incorporated into the proposed model, i.e., the operators, users and nonusers. In addition, we also consider the cost on the unsatisfied demand. A three-phase solution procedure is proposed for this study. In phase one, a series of selection criteria was developed for the selection of the potential locations. The principles in assigning type of parking facility for each potential location were identified in phase two. Finally, the compromising programming method is applied in phase three to the multiobjective programming model developed in this study. Due to its linearity, the proposed compromise programming method can be solved using the LINDO compute software. The results of the numerical experiments indicate the ability of the proposed model in providing of the reasonable solutions to the problem. The solution of this model can provide following information for the decision makers to determine the locations of the off-street parking facilities: the optimal locations of facilities, the demands served, the optimal types of the parking facilities. One area that deserved for the devotion of the future research is the estimation of the penalty cost. Due to the significant impact of this factor on the result of the problem, a suitable method to estimate its cost is critical to the success of the proposed methodology. Another interest area for the future study is to the incorporation of the nonlinear cost function to reflect the real situation. This would result in a complex nonlinear multiobjective programming problem, which will require significant effort to develop an effective solution method.

## REFERENCES

- Dirickx, M.I. and Jennergren, L.P. (1975), "An Analysis of the Parking Situation in the Downtown Area of West Berlin", **Transportation Research**, Vol. 9, pp 1-11
- Duckstein, L., and Opricovic, S. (1980), "Multiobjective Optimization in River Basin Development", **Water Resources Research**, 16(1), pp 14-20.
- Goyal, S. K., and Gomes, L.F.A.M., "A model for Allocating Car Parking Space in Universities", **Transportation Research Vol.18B, No.3**, pp.267-269 (1984).
- Huang, J.M. (1991) " A Study on the Location of the Off-street Parking, facilities", Master Thesis, Feng-Chia University.
- Hsu, C.W., (1994). "A Study on the Location of the Off-street Parking Facilities", Master Thesis, Tamkang University.
- Institution of Transportation (1983), Survey of the parking supplies and demands for the Taipei City in 1993, Report for Department of Parking, City of Taipei..
- Kanafani, A.K. (1972), "Location Model for Parking Facilities", **Transportation Engineering Journal**, Vol. 98, pp 117-129.
- Liao, J.Y. (2002), "A Study on the Dynamic Parking Location Problem", Master Thesis, Cha-young Institution of Technology.