THE DEVELOPMENT OF MAXIMUM-ENTROPY (ME) AND BAYESIAN-INFERENCE (BI) ESTIMATION METHODS FOR CALIBRATING TRANSPORT DEMAND MODELS BASED ON LINK VOLUME INFORMATION

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Abstract: Many problems in transportation require an Origin-Destination (O-D) matrix which is usually obtained from a large survey. This survey tends to be costly, labour intensive, and time disruptive. Therefore, more use should be made of low-cost traffic data. One possible way is by applying transport demand models, described as a function of one or more parameters that estimate the number of trips made during a period of time. Two estimation methods have been developed, namely: Non-Linear-Least-Squares (NLLS) and Maximum-Likelihood (ML). The objective is to develop two estimation methods based on Maximum-Entropy (ME) and Bayesian-Inference (BI) approaches. These four estimation methods will be used as methods to estimate the parameters of transport demand models. The work concentrated on the estimation of two types of models, namely: Gravity (GR) and Gravity-Opportunity (GO).

The 1999 Bandung's Urban Traffic Movement Survey (urbanised vehicle movement) has been used to test the developed method. Based on several statistical tests, the estimation methods are found to perform satisfactorily since each calibrated model reproduced the observed matrix fairly closely.

Keywords: O-D matrix, traffic counts, Maximum Entropy, Bayesian Inference, estimation methods

1. INTRODUCTION

Travel is an activity that has become part of our daily life and the demand for it always present problem especially in urban areas such as congestion, delay, air pollution, noise and environment. In order to alleviate these problems, it is necessary to understand the underlying travel pattern. The notion of Origin-Destination (O-D) Matrix has been widely used and accepted by transport planners as an important tool to represent the travel pattern. An O-D matrix gives a very good indication of travel demand, and therefore, it plays a very important role in various types of transport studies, transport planning and management tasks.

Most techniques and methods for solving transportation problems (urban and regional) require O-D matrix information as a fundamental information to represent the transport demand. The conventional method to estimate O-D matrices requires very large surveys such as: home and roadside interviews; which are very expensive, lengthy, labor intensive, subject to large errors, and moreover, time disruptive to trip makers. All of these require an answer. Therefore, the new approach to tackle all of these problems is urgently required.

The need for inexpensive methods, which require low-cost data, less time and less manpower generally called as '**unconventional method**' is therefore obvious due to time and money constraint. Traffic counts, the embodiment and the reflection of the O-D matrix; provide direct information about the sum of all O-D pairs which use those links. Some reasons why traffic counts are so attractive as a data base are: firstly, they are routinely collected by many authorities due to their multiple uses in many transport planning tasks. All of these make them easily available. Secondly, they can be obtained relatively inexpensive in terms of time and manpower, easier in terms of organization and management and also without disrupting the trip makers.

2. METHODS FOR ESTIMATING AN O–D MATRIX

Methods for estimating an O-D matrix can be classified into 2 main groups as shown in **figure 1**. They are as follows: conventional and unconventional methods (**Tamin, 1988**).

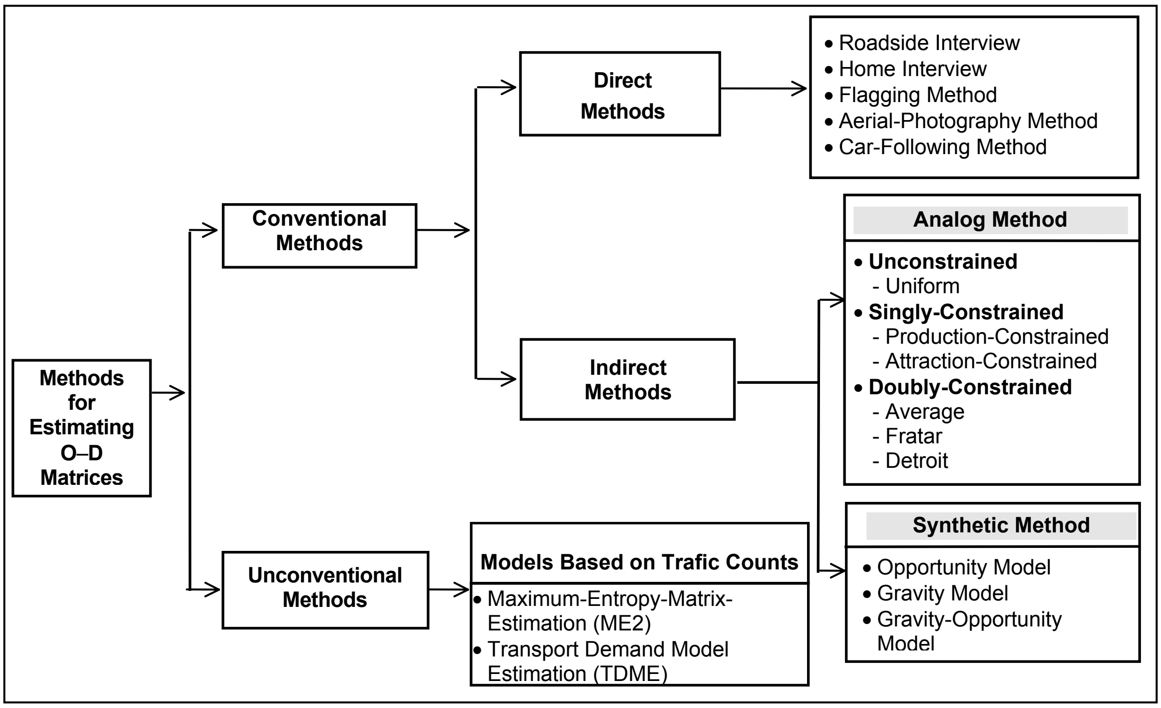

Figure 1. Methods for estimating an O-D matrix (**Tamin, 1988**)

Conventional methods rely heavily on extensive surveys, making them very expensive in terms of manpower and time, disruption to trip makers and most importantly the end products are sometimes short-lived and unreliable. Another important factor is the complications that arise when following each stage of the modelling process. Furthermore, in many cases particularly in small towns and developing countries, planners are confronted with the task of undertaking studies under conditions of time and money constraints, which make the application of the conventional methods almost impossible. The introduction of inexpensive techniques for the estimation of O-D matrices will overcome the problem.

As a result of dissatisfaction expressed by transport planners with conventional methods, other techniques for estimating O-D matrices which based on traffic counts have evolved over the years; these are generally called ‘unconventional methods’. The aim of unconventional methods is to provide a simpler approach to solve the same problem and at a lower cost. Ideally, this simpler approach would treat the four-stage sequential model as a single process. To achieve this economic goal, the data requirements for this new approach should be limited to simple zonal planning data and traffic counts on some links or other low-cost data.

3. TRANSPORT DEMAND MODEL ESTIMATION FROM TRAFFIC COUNTS

The transport demand model estimation approach assumes that the travel pattern behaviour is well represented by a certain transport model, e.g. a gravity model. The main idea is to apply a transport model to represent the travel pattern. It should be noted here that the transport demand models are described as functions of some planning variables like population or employment and some parameters. Whatever the specification and the hypotheses underlying the models, the main task is to estimate their parameters on the basis of traffic counts. Once, the parameters of the postulated transport demand models have been calibrated, they may be used not only for the estimation of the current O-D matrix, but also for predictive purposes. The latter requires the use of future values for the planning variables.

Consider a study area which is divided into **N** zones, each of which is represented by a centroid. All of these zones are inter-connected by a road network which consists of series of links and nodes. Furthermore, the O-D matrix for this study area consists of $\mathbf{N^2}$ trip cells. ($\mathbf{N^2-N}$) trip cells if intrazonal trips can be disregarded. The most important stage is to identify the paths followed by the trips from each origin to each destination. The variable $\mathbf{p_{id}^{lk}}$ is used to define the proportion of trips by mode **k** travelling from zone **i** to zone **d** through link **l**. Thus, the flow on each link is a result of:

- trip interchanges from zone **i** to zone **d** or combination of several types of movement travelling between zones within a study area (=$\mathbf{T_{id}}$); and
- the proportion of trips by mode **k** travelling from zone **i** to zone **d** whose trips use link **l**, which is defined by $\mathbf{p_{id}^{lk}}$ ($\mathbf{0 \le p_{id}^{lk} \le 1}$).

The total volume of flow ($\hat{\mathbf{V}}_{\mathbf{l}}^{\mathbf{k}}$) in a particular link **l** is the summation of the contributions of all trips interchanges by mode **k** between zones within the study area to that link. Mathematically, it can be expressed as follows:

$$\mathbf{V_l^k = \sum_i \sum_d T_{id}^k . p_{id}^{lk}} \qquad (1)$$

Given all the $\mathbf{p_{id}^{lk}}$ and all the observed traffic counts ($\hat{\mathbf{V}}_{\mathbf{l}}^{\mathbf{k}}$), then there will be $\mathbf{N^2}$ unknown $\mathbf{T_{id}^k}$’s to be estimated from a set of **L** simultaneous linear equations (**1**) where **L** is the total number of traffic (passenger) counts. In principle, **N²** independent and consistent traffic counts are required in order to determine uniquely the O-D matrix [$\mathbf{T}_{id}^{k}$]. In practice, the number of observed traffic counts is much less than the number of unknowns $\mathbf{T}_{id}^{k}$'s.

3.1 Fundamental Basis

3.1.1 Gravity model (GR)

Assume that the interzonal movement within the study area can be represented by a certain transport demand model such as gravity (GR) model. Hence, the total number of trips $\mathbf{T_{id}}$ with origin in **i** and destination **d** for all trip purposes or commodities can be expressed as:

$$\mathbf{T_{id}} = \sum_{k} \mathbf{T}_{id}^{k} \tag{2}$$

$\mathbf{T}_{id}^{k}$ is the number of trips for each trip purpose or commodity **k** travelling from zone **i** to zone **d** as expressed by equation (**3**) generally known as a doubly-constrained gravity model (DCGR).

$$\mathbf{T}_{id}^{k} = \mathbf{O}_{i}^{k}.\mathbf{D}_{d}^{k}.\mathbf{A}_{i}^{k}.\mathbf{B}_{d}^{k}.\mathbf{f}_{id}^{k} \tag{3}$$

where:

$\mathbf{A}_{i}^{k}$ and $\mathbf{B}_{d}^{k}$ = balancing factors expressed as:

$$\mathbf{A}_{i}^{k} = \left[\sum_{d}\left(\mathbf{B}_{d}^{k}.\mathbf{D}_{d}^{k}.\mathbf{f}_{id}^{k}\right)\right]^{-1} \text{and } \mathbf{B}_{d}^{k} = \left[\sum_{i}\left(\mathbf{A}_{i}^{k}.\mathbf{O}_{i}^{k}.\mathbf{f}_{id}^{k}\right)\right]^{-1} \tag{4}$$

$$\mathbf{f}_{id}^{k} = \text{the deterrence function} = \mathbf{exp}\left(-\beta.\mathbf{C}_{id}^{k}\right) \tag{5}$$

The equations for $\mathbf{A_i}$ and $\mathbf{B_d}$ are solved iteratively, and it can be easily checked that they ensure that $\mathbf{T_{id}}$ given in equation (**3**) satisfies the constraint equation (**2**). This process is repeated until the values of $\mathbf{A_i}$ and $\mathbf{B_d}$ converge to certain unique values.

3.1.2 Gravity-Opportunity model (GO)

Wills (1978,**1986)** developed a flexible gravity-opportunity (GO) model for trip distribution in which standard forms of the gravity and intervening-opportunity model are obtained as special cases. Hence the question of choice between gravity or intervening-opportunity approaches is decided empirically and statistically by restrictions on parameters which control the global functional form of the trip distribution mechanism.

a. An ordered O-D matrix. Let origins and destinations be numbered consecutively in the usual way, such that **i=1,2,..,I** are origins and **d=1,2,..,J** are destinations, and let $\mathbf{T_{id}}$ be the observed trips from origin **i** to destination **d**. Define now a transformation $\mathbf{d}_{j}^{(i)}$ for each origin **i** such that:

$$\delta_{jd}^{i} = \begin{cases} \mathbf{1} & \text{if destination } \mathbf{d} \text{ is the } \mathbf{j}^{th} \text{ position in ascending order of distance away from } \mathbf{i} \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{6}$$

and then the ordered O-D matrix can be obtained by the following transformation as:

$$\mathbf{Z_{ij}} = \sum_{d} \left\lfloor \delta^{i}_{jd} . \mathbf{T_{id}} \right\rfloor \tag{7}$$

Thus, $\mathbf{Z_{ij}}$ represents the trips from origin **i** to the $\mathbf{j^{th}}$ destination ranked by distance away from **i**. Note that **j** is always defined as a function of **i**, so it is perhaps more correctly designated as **j(i)** but for notational simplicity we omit the **i** as being understood. While the ordering transformation δ^{i}_{jd} produces an ordered O-D matrix, its inverse δ^{i-1}_{jd} allows the observed O-D matrix to be recovered by:

$$\mathbf{T_{id}} = \sum_{d} \left\lfloor \delta^{i-1}_{jd} . \mathbf{Z_{id}} \right\rfloor \tag{8}$$

It should be noted that this part of transformations is applicable to any variable based on the O-D matrix, notably the trip cost matrix, the proportionality factor and the destination balancing factor, in addition to the O-D matrix.

b. Normalization. To achieve the logical consistency such that the sum (over destinations) of the estimated trips for each origin **i** is equal to the observed trips generated at **i** and similarly, for the sum (over origins) of the estimated trips for each destination **j** is equal to the observed trips generated at **j**, then the two following constraints are required.

$$\mathbf{O_i} = \sum_{j} \mathbf{Z_{ij}} \tag{9a}$$

$$\mathbf{D^{i}_{j}} = \sum_{d} \left\lfloor \delta^{i}_{jd} . \mathbf{D_d} \right\rfloor \quad \text{and} \quad \mathbf{D_d} = \sum_{i} \left[\sum_{j} \left(\delta^{i-1}_{jd} . \mathbf{Z_{ij}} \right) \right] \tag{9b}$$

c. Transformations. In order to provide a monotonic scaling of variables in such a manner as to generate families of specific functional forms, the Box-Cox transformations is used. The direct Box-Cox transformation of a variable **y** can be defined as:

$$\mathbf{y^{(\varepsilon)}} = \begin{cases} \dfrac{(\mathbf{y^{\varepsilon}} - 1)}{\varepsilon} & \varepsilon \neq 0 \\ \log_e \mathbf{y} & \varepsilon = 0 \end{cases} \tag{10}$$

and the inverse Box-Cox transformation as:

$$\mathbf{y^{1/\varepsilon}} = \begin{cases} (\mathbf{y}\varepsilon + 1)^{1/\varepsilon} & \varepsilon \neq 0 \\ \exp \mathbf{y} & \varepsilon = 0 \end{cases} \tag{11}$$

These transformations may be combined into a new function which we introduce as a convex combination in $\boldsymbol{\mu}$.

$$\mathbf{y^{(\varepsilon,\mu)}} = \mu . \mathbf{y^{(\varepsilon)}} + (1 - \mu) . \mathbf{y^{(1/\varepsilon)}} \qquad \text{with } 0 \le \mu \le 1 \tag{12}$$

d. Specification of the Opportunity Function

A key step in the integration of both models is the specification of an opportunity function which has as arguments destination-attribute variables such as population, income or some other measures of opportunities and generalized cost or trip impedance variables relating origin and destination. The opportunity function $\mathbf{U_{ip}}$ relates $\underline{\mathbf{i}}$ and the $\mathbf{p^{th}}$ destination away from $\underline{i}$ and is defined generally as:

$$\mathbf{U_{ip} = \exp[(1-\varepsilon).\alpha.D_p.^{i(\Omega)} - \beta.C_{ip}{}^{(\Phi)}}\tag{13}$$

$\mathbf{U_{ip}}$ is defined here as a combined vector of intervening-opportunity factors and impedances. The term (**1-ε**) ensures that, when **ε=1** then the gravity model is obtained and the destination intervening-opportunity effect is removed. These impedances weight the intervening-opportunity by their location to origin and destination, generally the closer the intervening-opportunity to an origin the greater the impact on travel between $\underline{\mathbf{i}}$ and $\underline{\mathbf{j}}$. **Table 1** shows the specification of the opportunity function depending on the value of parameters **Ω** and **Φ**.

Table 1. Specification of the Opportunity Function

Ω	Φ	Intervening-Opportunity	Impedance	U_{ip}
Ω	Φ	$\exp[(1-\varepsilon).\alpha.D_p^{i(\Omega)}]$	$\exp[-\beta.C_{ip}^{(\Phi)}]$	$\exp[(1-\varepsilon).\alpha.D_p^{i(\Omega)} - \beta.C_{ip}^{(\Phi)}]$
1	1	$\exp[(1-\varepsilon).\alpha.D_p^i]$	$\exp[-\beta.C_{ip}]$	$\exp[(1-\varepsilon).\alpha.D_p^i - \beta.C_{ip}]$
0	0	$D_{pi}^{\alpha(1-\varepsilon)}$	$C_{ip}^{-\beta}$	$D_{pi}^{\alpha(1-\varepsilon)}.C_{ip}^{-\beta}$
1	0	$\exp[(1-\varepsilon).\alpha.D_p^i]$	$C_{ip}^{-\beta}$	$\exp[(1-\varepsilon).\alpha.D_p^i - \beta.\log_e C_{ip}]$
0	1	$D_{pi}^{\alpha(1-\varepsilon)}$	$\exp[-\beta.C_{ip}]$	$\exp[(1-\varepsilon).\alpha.\log_e D_p^i - \beta.C_{ip}]$

Source: Wills (1986)

e. Structure of the Proportionality Factor

The opportunity function is incorporated into a general proportionality factor $\mathbf{F_{ij}}$ which is defined by the difference in functions of the cumulative opportunities from **i** to the $\mathbf{j^{th}}$ destination away from **i**, and from **i** to the $\mathbf{(j\text{-}1)^{th}}$ destination away from **i**, and can be defined as:

$$\mathbf{F_{ij} = X_{ij} - X_{ij-1}}\tag{14}$$

The most general form of the cumulative opportunities to be considered here defines $\mathbf{X_{ij}}$ and $\mathbf{X_{ij-1}}$ as:

$$\mathbf{X_{ij} = (\sum_p^j U_{ip})^{(\varepsilon,\mu)}} \quad \text{and} \quad \mathbf{X_{ij-1} = (\sum_p^{j-1} U_{ip})^{(\varepsilon,\mu)}}\tag{15}$$

where (ε,μ) transformation is defined by equations (**10**)-(**12**). Subsitution of equation (**15**) into equation (**12**) leads to the general proportionality factor form as:

$$\mathbf{F_{ij} = (\sum_p^j U_{ip})^{(\varepsilon,\mu)} - (\sum_p^{j-1} U_{ip})^{(\varepsilon,\mu)}}\tag{16}$$

The general proportionality factor is subjected to a convex combination of direct and inverse Box-Cox transformations. The form given by equation (**18**) generates two branches of special cases: the **direct-opportunity (DO)** model, with **μ=1**, and the **inverse-opportunity (IO)** model, with **μ=0**. The DO model is significant because it contains the important special case of the **logarithmic-opportunity (LO)** model, with **ε=0**. That is:

$$\mathbf{F_{ij} = \log_e(\sum_p^{j} U_{ip}) - \log_e(\sum_p^{j-1} U_{ip})} \tag{17}$$

The IO model is particularly important becasue it contains the **exponential-opportunity (EO)** model, again with **ε=0**. That is:

$$\mathbf{F_{ij} = \exp(\sum_p^{j} U_{ip}) - \exp(\sum_p^{j-1} U_{ip})} \tag{18}$$

We can also consider blends of the LO and EO models, without going to the full GO model, by taking a convex combination of equations (**19**) and (**20**) with the mixture depending on values of μ. This blended form, the **blended-opportunity (BO)** model, is given in equation (**19**) as:

$$\mathbf{F_{ij} = \mu[\log_e(\sum_p^{j} U_{ip}) - \log_e(\sum_p^{j-1} U_{ip})] + (1-\mu)[\exp(\sum_p^{j} U_{ip}) - \exp(\sum_p^{j-1} U_{ip})]} \tag{19}$$

Finally, we observe that if **ε=1**, for $\mathbf{0 \le \mu \le 1}$, the gravity (GR) model is revealed as:

$$\mathbf{F_{ij} = (\sum_p^{j} U_{ip}) - (\sum_p^{j-1} U_{ip}) = U_{ij}} \tag{20}$$

showing that the standard GR model can be obtained as a special case of the GO model. As mentioned, different values of the parameters controlling these transformations generate contrasting families of models, notably the exponential-opportunity (EO) model, the logarithmic-opportunity (LO) model and the gravity (GR) model.

Having all the assumptions, the proposed GO model is therefore:

$$\mathbf{T_{id} = \sum_k \left[b_k . O_i^k . D_d^k . A_i^k . B_d^k . f_{id}^k\right]} \quad \text{where:} \tag{21}$$

A_i and **B_d** are defined as equations (**4**)

- $$\mathbf{f_{id}^k = \sum_j \left[\delta_{jd}^{i-1} . F_{ij}^k\right]} \tag{22}$$
- $$\mathbf{F_{ij}^k = (\sum_p^{j} U_{ip}^k)^{(\varepsilon,\mu)} - (\sum_p^{j-1} U_{ip}^k)^{(\varepsilon,\mu)}} \tag{23}$$
- $$\mathbf{U_{ip}^k = \exp\left[(1-\varepsilon).\alpha_k . D_{pk}^{i(\Omega)} - \beta_k . C_{ip}^{(\Phi)}\right]} \tag{24}$$
- $$\mathbf{D_{jk}^i = \sum_d \left[\delta_{jd}^i - D_d^k\right]} \tag{25}$$

- the (Ω,Φ) parameters were chosen, in advance, externally to the main calibration process.

- the (ε,μ) transformation is defined by equations (**10**)–(**12**).

By substituting equation (**3**) to (**1**), the fundamental equation for estimating the transport demand model based on traffic counts is:

$$\mathbf{V_l^k = \sum_i \sum_d \left(O_i^k.D_d^k.A_i^k.B_d^k.f_{id}^k.p_{id}^{lk}\right)} \qquad (26)$$

The fundamental equation (**26**) has been used by many literatures not only to estimate the O–D matrices but also to calibrate the transport demand models from traffic count information (see **Tamin, 1988**; **Tamin and Willumsen, 1988**). Theoretically, having known the values of $\mathbf{\hat{V}_l^k}$ and $\mathbf{p_{id}^{lk}}$, $\mathbf{T_{id}^k}$ can then be estimated. Equation (**6**) is a system of **L** simultaneous equations with only (**K**) unknown parameter β need to be estimated. The problem now is how to estimate the unknown parameters β so that the model reproduces the estimated traffic flows as close as possible to the observed traffic counts.

3.2 Estimation Methods

Tamin (1999) explains several types of estimation methods which have been developed so far by many researchers are:

- Least-Squares estimation method (LLS or NLLS)
- Maximum-Likelihood estimation method (ML)
- Bayes-Inference estimation method (BI)
- Maximum-Entropy estimation method (ME)

3.2.1 Least-Squares estimation method (LS)

Tamin (**1988,2000**) have developed several Least-Squares (LS) estimation methods of which its mathematical problem can be represented as equation (**27**).

$$\textbf{to minimize} \qquad \mathbf{S = \sum_l \left[\left(\hat{V}_l^k - V_l^k\right)^2\right]} \qquad (27)$$

$\mathbf{\hat{V}_l^k}$ = observed traffic flows for mode **k** $\qquad$ $\mathbf{V_l^k}$ = estimated traffic flows for mode **k**

The main idea behind this estimation method is that we try to calibrate the unknown parameters of the postulated model so that to minimize the deviations or differences between the traffic flows estimated by the calibrated model and the observed flows. Having substituted equation (**26**) to (**27**), the following set of equation is required in order to find an unknown parameter β which minimizes equation (**27**):

$$\mathbf{\frac{\delta S}{\delta \beta} = \sum_l \left[\left(2\sum_i \sum_d T_{id}^k.p_{id}^{lk} - \hat{V}_l^k\right).\left(\sum_i \sum_d \left(\frac{\partial T_{id}^k}{\partial \beta}.p_{id}^{lk}\right)\right)\right] = 0} \qquad (28)$$

Equation (**28**) is an equation which has only one (**1**) unknown parameter β need to be estimated. Then it is possible to determine uniquely all the parameters, provided that **L>1**. Newton–Raphson's method combined with the Gauss–Jordan Matrix Elimination technique can then be used to solve equation (**28**) (see **Batty, 1976**; **Wilson and Bennet, 1985**).

The LS estimation method can be classified into two: **Linear-Least-Squares (LLS)** and **Non-Linear-Least-Squares (NLLS)** estimation methods. **Tamin (1988)** has concluded that the NLLS estimation method requires longer processing time for the same amount of parameters. This may due to that the NLLS estimation method contains a more complicated algebra compared to the LLS so that it requires longer time to process. However, the NLLS estimation method allows us to use the more realistic transport demand model in representing the trip-making behaviour. Therefore, in general, the NLLS provides better results compared to the LLS.

3.2.2 Maximum-Likelihood estimation method (ML)

Tamin (**1988,2000**) have also developed an estimation method which tries to maximise the probability as expressed in equation (**29**). The framework of the ML estimation method is that the choice of the hypothesis **H** maximising equation (**9**) subject to a particular constraint, will yield a distribution of $\mathbf{V_l^k}$ giving the best possible fit to the survey data ($\mathbf{\hat{V}_l^k}$). The objective function for this framework is expressed as:

to maximize $$\mathbf{L = c.\prod_l p_l^{\hat{V}_l^k}} \qquad (29)$$

subject to: $$\mathbf{\sum_l V_l^k - \hat{V}_T^k = 0} \qquad (30)$$

where: $\mathbf{\hat{V}_T^k}$ = total observed traffic flows **c** = constant $\mathbf{p_l = \dfrac{V_l^k}{\hat{V}_T^k}}$

By substituting equation (**26**) to (**29**), finally, the objective function of ML estimation method can then be expressed as equation (**11**) with respect to unknown parameters β and θ.

$$\mathbf{max.\ \ L_1 = \sum_l \left[\hat{V}_l^k.\log_e\left(\sum_i \sum_d T_{id}^k.p_{id}^{lk} \right) - \theta.\sum_i \sum_d T_{id}^k.p_{id}^{lk} \right] + \theta.\hat{V}_T^k - \hat{V}_T^k.\log_e \hat{V}_T^k + \log_e c} \qquad (31)$$

The purpose of an additional parameter θ, which appears in equation (**31**), is that to ensure the constraint equation (**30**) should always be satisfied. In order to determine uniquely parameter β of the GR model together with an additional parameter θ**,** which maximizes equation (**31**), the following two sets of equations are then required. They are as follows:

$$\mathbf{\frac{\delta L_1}{\delta\beta} = \sum_l \left[\hat{V}_l^k.\frac{\sum_i \sum_d \frac{\delta T_{id}^k}{\delta\beta}.p_{id}^{lk}}{\sum_i \sum_d T_{id}^k.p_{id}^{lk}} \right] - \left(\theta.\sum_i \sum_d \frac{\delta T_{id}^k}{\delta\beta}.p_{id}^{lk} \right) = 0} \qquad (32a)$$

$$\frac{\delta L_1}{\delta \theta} = -\theta.\left[\sum_i \sum_d T_{id}^k.p_{id}^{lk} - \hat{V}_T^k\right] = 0 \quad (32b)$$

Equation (**32ab**) is in effect a system of two (**2**) simultaneous equations which has two (**2**) unknown parameters β and θ need to be estimated.

3.2.3 Bayes-Inference estimation method (BI)

The main idea behind the Bayes-Inference estimation method is by combining the prior beliefs and observations will produce posterior beliefs. If one has 100% confidence in one's prior belief then no random observations, however remarkable, will change one's opinions and the posterior will be identical to the prior beliefs. If, on the other hand, one has little confidence in the prior beliefs, the observations will then play the dominant role in determining the posterior beliefs. In other words, prior beliefs are modified by observations to produce posterior beliefs; the stronger the prior beliefs, the less influence the observations will have to produce the posterior beliefs.

The objective function of the Bayes-Inference (BI) estimation method can be expressed as:

$$\textbf{to maximize} \qquad BI\ (\tau_l^k V_l^k) = \sum_l \left(\hat{V}_l^k \log_e V_l^k\right) \quad (33)$$

By substituting equation (**26**) to (**33**), the objective function can then be rewritten as:

$$\textbf{to maximize} \qquad BI = \sum_l \left[\hat{V}_l^k.\log_e\left(\sum_i \sum_d T_{id}^k.p_{id}^{lk}\right)\right] \quad (34)$$

In order to determine uniquely parameter β of the GR model, which maximizes equation (**34**), the following two sets of equations are then required. They are as follows:

$$\frac{\partial BI}{\partial \beta} = \sum_l \left[\left(\frac{\hat{V}_l^k}{\sum_i \sum_d \left(T_{id}^k.p_{id}^{lk}\right)}\right).\left(\sum_i \sum_d \left(\frac{\partial T_{id}^k}{\partial \beta}.p_{id}^{lk}\right)\right) = 0\right] \quad (35)$$

Equation (**35**) is an equation which has one (**1**) unknown parameter β need to be estimated.

3.2.4 Maximum-Entropy estimation method (ME)

Tamin (1998) has developed the maximum-entropy approach to calibrate the unknown parameters of gravity model. Now, this approach is used to develop procedure to calibrate the unknown parameters of the transport demand model based on traffic count information. The basic of the method is to accept that all micro states consistent with our information about macro states are equally likely to occur.

Wilson (1970) explains that the number of micro states $W\{V_l^k\}$ associated with the meso state V_l^k is given by:

$$\mathbf{W}\left[\mathbf{V}_l^k\right] = \frac{\mathbf{V}_T^k!}{\prod_l \mathbf{V}_l^k!} \tag{36}$$

As it is assumed that all micro states are equally likely, the most probable meso state would be the one that can be generated in a greater number of ways. Therefore, what is needed is a technique to identify the values $[\mathbf{V}_l^k]$ which maximize **W** in equation (**36**). For convenience, we seek to maximize a monotonic function of **W**, namely $\mathbf{log_e W}$, as both problems have the same maximum. Therefore:

$$\mathbf{log_e\, W = log_e} \frac{\mathbf{V}_T^k!}{\prod_l \mathbf{V}_l^k!} = \mathbf{log_e\, V}_T^k! - \sum_l \mathbf{log_e V}_l^k! \tag{37}$$

Using Stirling's approximation for $\mathbf{log_e X! \approx X log_e X - X}$, equation (**37**) can then be simplified as:

$$\mathbf{log_e\, W = log_e V}_T^k! - \sum_l \left(\mathbf{V}_l^k \mathbf{log_e V}_l^k - \mathbf{V}_l^k\right) \tag{38}$$

Using the term $\mathbf{log_e V}_T^k!$ is a constant; therefore it can be omitted from the optimization problem. The rest of the equation is often referred to as **the entropy function**.

$$\mathbf{log_e\, W' } = -\sum_l \left(\mathbf{V}_l^k \mathbf{log_e V}_l^k - \mathbf{V}_l^k\right) \tag{39}$$

By maximising equation (**39**), subject to constraints corresponding to our knowledge about the macro states, enables us to generate models to estimate the most likely meso states (in this case the most likely $\mathbf{V}_l^k$). The key to this model generation method is, therefore, the identification of suitable micro-, meso- and macro-state descriptions, together with the macro-level constraints that must be met by the solution to the optimisation problem. In some cases, there may be additional information in the form of prior or old values of the meso states, for example observed traffic counts ($\hat{\mathbf{V}}_l^k$). The revised objective function becomes:

$$\mathbf{log_e\, W''} = -\sum_l \left(\mathbf{V}_l^k \mathbf{log_e}\left(\frac{\mathbf{V}_l^k}{\hat{\mathbf{V}}_l^k}\right) - \mathbf{V}_l^k + \hat{\mathbf{V}}_l^k\right) \tag{40}$$

Equation (**40**) is an interesting function in which each element in the summation takes the value zero if $\mathbf{V}_l^k = \hat{\mathbf{V}}_l^k$ and otherwise is a positive value which increases with the difference between $\mathbf{V}_l^k$ and $\hat{\mathbf{V}}_l^k$. The greater the differences, the smaller the value of $\mathbf{log_e\, W''}$. Therefore, $\mathbf{log_e\, W''}$ is a good measure of the difference between $\mathbf{V}_l^k$ and $\hat{\mathbf{V}}_l^k$. Mathematically, the objective function of the ME estimation method can be expressed as:

$$\textbf{to maximise} \qquad \mathbf{E_1 = log_e\, W''} = -\sum_l \left(\mathbf{V}_l^k \mathbf{log_e}\left(\frac{\mathbf{V}_l^k}{\hat{\mathbf{V}}_l^k}\right) - \mathbf{V}_l^k + \hat{\mathbf{V}}_l^k\right) \tag{41}$$

In order to determine uniquely parameter β of the GR model which maximizes the equation (**41**), the following equation is then required. They are as follows:

$$\frac{\partial \mathbf{E}_1}{\partial \beta} = -\sum_l \left[\left(\sum_i \sum_d \frac{\partial \mathbf{T}_{id}^k}{\partial \beta} . \mathbf{p}_{id}^{lk} \right) . \log_e \left(\frac{\sum_i \sum_d \mathbf{T}_{id}^k . \mathbf{p}_{id}^{lk}}{\hat{\mathbf{V}}_l^k} \right) \right] = 0 \qquad (42)$$

Equation (**42**) is an equation which has only one (**1**) unknown parameter β need to be estimated.

3.2.5 Test case with Bandung traffic count data

The real data set of urban traffic movement in Bandung in terms of traffic count information was used to validate the proposed estimation methods. Bandung is a capital of West Java Province and its population is around 6.4 millions in 1998 and expected to increase to 13.8 millions in 2020. The total area of Bandung is around 325,096 Ha and is divided into 66 kecamatans and 590 kelurahans. The study area was divided into 146 zones of which 140 are internal zones and 6 are external. The road network of the study area consisted of 653 nodes and 1,811 road links. There are 95 observed traffic counts ($\hat{\mathbf{V}}_l$), traffic generation and attraction ($\mathbf{O_i}$ and $\mathbf{D_d}$) for each zone, and an observed O-D matrix for comparison purpose. The units used in equation (**26**) are as follows:

$\hat{\mathbf{V}}_l$ = traffic counts in vehicles/hour
$\mathbf{O_i}$, $\mathbf{D_d}$ = trip generation/attraction for each zone in vehicles/hour

In order to establish the strategy for validity tests, it is necessary to introduce at this stage the main issues affecting the accuracy of the estimated O-D matrix produced by the calibrated models. These are as follows:

- the choice of the transport demand model itself to be used in representing the trip behaviour within the study area or, perhaps, a system of the real world;
- the estimation method used to calibrate the parameters of the transport model from traffic count information;
- number of traffic count information;
- the level of errors in traffic counts; and
- the level of resolution of the zoning system and the network definition.

The validity and sensitivity tests can then be established from these five main issues. Two transport demand models, namely gravity (GR) and gravity-opportunity (GO) models, and four estimation methods (NLLS, ML, BI, ME) have been used in the validity tests. The four estimation methods mentioned above have been discussed in detail in **section 3.2**. The value of $\mathbf{R^2}$ statistic as expressed in equation (**43**) is used to compare the observed and estimated O–D matrices to ascertain how close they are.

$$\mathbf{R}^2 = 1 - \frac{\sum_i \sum_d \left(\hat{\mathbf{T}}_{id} - \mathbf{T}_{id} \right)^2}{\sum_i \sum_d \left(\hat{\mathbf{T}}_{id} - \mathbf{T}_1 \right)^2} \qquad \mathbf{T}_1 = \frac{1}{\mathbf{N}.(\mathbf{N}-1)} . \sum_i \sum_d \mathbf{T}_{id} \quad \text{for } i \neq d \quad (43)$$

3.2.6 The best location of traffic counts

It is mentioned that the unconventional method uses traffic count information as the main input for estimating the O-D matrices. Because of that, any process regarding the traffic counts should be clearly and deeply understood in order to obtain the best estimates of O-D matrices; especially those which are related to data collection process e.g. number of traffic counts and their best locations. The data collection process is very important since it is the first action in the whole process of O-D matrix estimation. Some basic analysis used in finding the best location are as follows:

a. Proportion of trip interchanges on a particular link

The total volume of flow in a particular link **l** ($\hat{V}_l$) is the summation of the contributions of all trip interchanges between zones within the study area to that link. Mathematically, it can be expressed as equation (**4**). **Tamin (2000)** stated that the most important stage for the estimation of O-D matrix from traffic counts is to identify the paths followed by the trips from each origin **i** to each destination **d**.

In other words, the proportion of trip interchanges between zone **i** and zone **d** have to be uniquely identified for all those links involved. In this case, the variable $\mathbf{p^l_{id}}$ is used to define the proportion of trip interchanges from origin **i** to destination **d** travelling through link **l**. Therefore, in finding the best location, the traffic counts having many information of the trip interchanges should be chosen. This information can be identified by analysing the total number and value of $\mathbf{p^l_{id}}$ in each link. This information will then be taken as the main criteria in determining choosing the best location of traffic counts.

b. Inter-link relationship

- **Inter-dependence**. **Figure 2** shows that flows on link **5–6** are the summation of flows on link **1–5** and on link **2–5**, then there is no additional information can be extracted by counting flows on link **5–6** because of the **flow continuity condition**, $\bar{V}_{56} = \bar{V}_{15} + \bar{V}_{25}$.

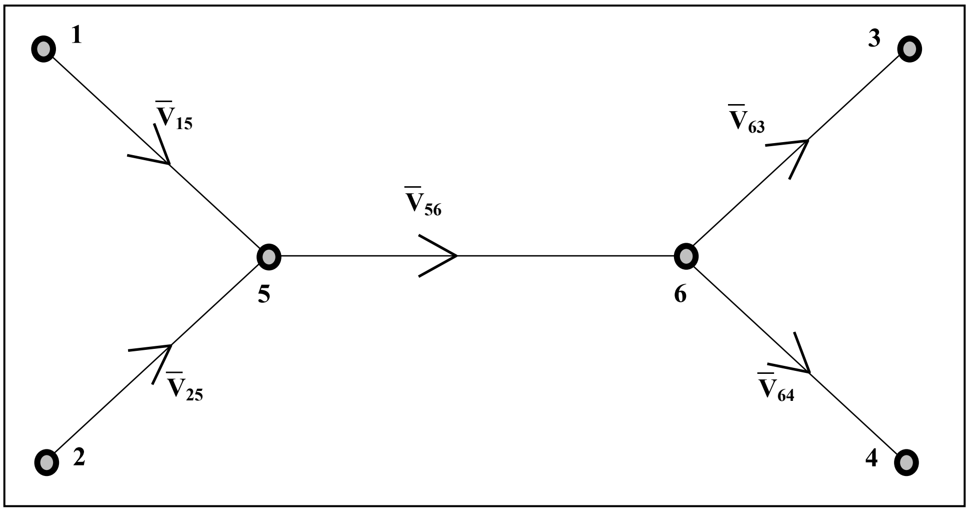

Figure 2. A simple network with link counts (**Tamin, 2000**)

In principle, we need counts for only **3 (three) independent counts** in order to find the flows of all links in **Figure 2**. Therefore, from an economic point of view, some efforts are needed in choosing the appropriate links to be counted.

- **Inconsistency.** In practice, the problem of inconsistency in link counts may arise when the flow continuity conditions are not satisfied by the observed volumes. In the case of **Figure 3**, it may well happen that the observed flows are such that:

$$\overline{\mathbf{V}}_{56} \neq \mathbf{V}_{15} + \overline{\mathbf{V}}_{25} \quad (44)$$

or

$$\overline{\mathbf{V}}_{15} + \overline{\mathbf{V}}_{25} \neq \overline{\mathbf{V}}_{63} + \overline{\mathbf{V}}_{64} \quad (45)$$

This inconsistency in counts may arise due to human or counting errors and counting at different times or dates. As a result of all this, no solution for the OD matrix can be estimated that reproduces all these inconsistent traffic counts. One possible way to remove this problem is by choosing only independent links for counted.

c. Optimum number of traffic counts

Equation (**26**) is the fundamental equation developed for estimating the O-D matrices from traffic counts information. In this model, the parameters $\mathbf{p}^{\mathbf{l}}_{\mathbf{id}}$ are estimated using traffic assgnment technique. Given all the $\mathbf{p}^{\mathbf{l}}_{\mathbf{id}}$ and all the observed traffic counts ($\hat{\mathbf{V}}_{\mathbf{l}}$), then there will be $\mathbf{N}^2$ buah $\mathbf{T}_{\mathbf{id}}$'s to be estimated from a set of **L** simultaneous linear equations (**1**) where **L** is the total number of traffic counts.

In principle, $\mathbf{N}^2$ independent and consistent traffic counts are required in order to determine uniquely the O-D matrix $[\mathbf{T}_{\mathbf{id}}]$; $[\mathbf{N}^2\mathbf{-N}]$ if intrazonal trips can be disregarded. **In practice**, the number of observed traffic counts is much less than the number of unknowns $\mathbf{T}_{\mathbf{id}}$'s. Therefore, it is impossible to determine uniquely the solution.

d. The determination of the optimum number of traffic counts

As mentioned above, the determination of the optimum number of traffic count will be conducted under **1 (one)** condition representing the sensitivity between the number of traffic counts and the link rank to the accuracy of the estimated O-D matrices, namely: **random condition**. In this condition, several combinations of traffic counts will be created based on random selection. Each combination of traffic counts will then be used to estimate the O-D matrices.

In this research, the initial O-D matrix was created by calibrating the **Gravity-Opportunity (GO)** model from traffic counts by using all selected links (**646 links**). **Tamin et al (2001)** reports that the best of values of parameters (ε and μ) are $\boldsymbol{\varepsilon}$**=0,4** and $\boldsymbol{\mu}$**=1,0** for the GO model. The other unknown parameters (α and β) of the GO model were then calibrated using 646 selected traffic counts by using **Non-Linear-Least-Squares (NLLS)** estimation method.

The initial O-D matrix to be used for comparison purposes will then be created using the GO model together with the values of its calibrated parameters. **Figure 3** show the relationship between the level of accuracy of the estimated matrices compared to the initial one and the number of selected traffic counts under **random condition**.

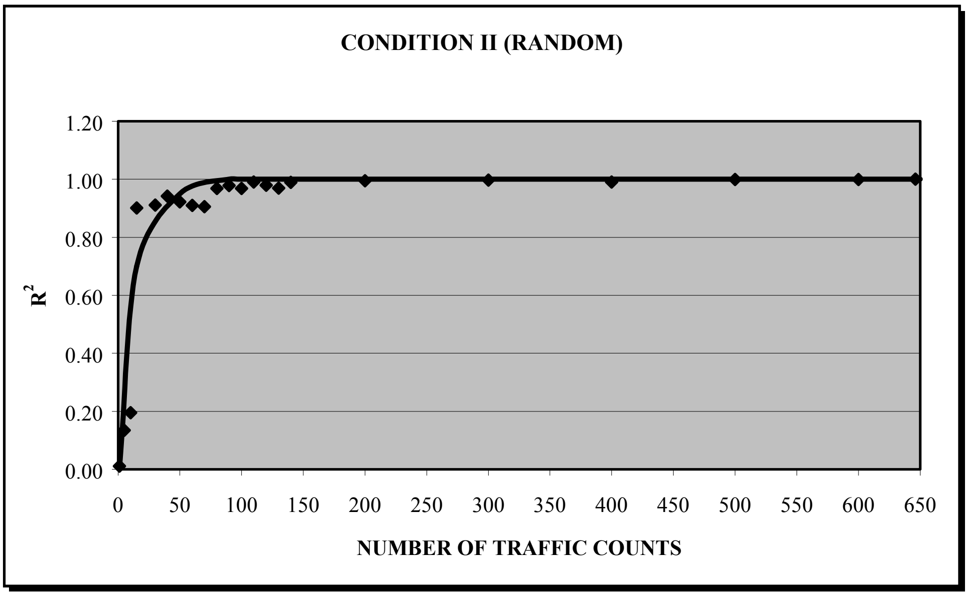

Figure 3: Number of traffic counts and the value of R^2 (random condition)

It can be seen from **Figure 3** that the use of **90** links has reproduced the relatively high accuracy of estimated OD matrices compared to the initial one (in terms of $\mathbf{R^2}$). The use of 90 links has relatively the same accuracy with the use of 646 links. It can be concluded that the optimum number of traffic counts is **90** links (**14%** of **646** selected links or **3.6%** of **2485** available links).

3.2.7 Important findings

Several important findings can be concluded as given in **Table 2**, which shows the performance ranking of model's estimation method according to specified criteria. The purpose of this table is to provide guidance to choose the best overall model's estimation method regarding its behaviour to several criteria such as: accuracy, computer time, sensitivity to errors in traffic counts, sensitivity to zoning level and network solution, and sensitivity to number of traffic counts. The ranking scale ranging from 1 to 8 will be used to see the performance of estimation methods based on the above criteria. Scale **1** shows the worst performance, while scale **8** shows the best performance.

Table 2. Performance ranking of model estimation methods for specified criteria

Model and estimation methods		Criteria				
		Accuracy	**Computer time**	**Sensitivity to errors in traffic counts**	**Sensitivity to zoning level and network resolution**	**Sensitivity to number of traffic counts**
GR	**NLLS**	5	8	7	7	3
	ML	2	6	6	8	2
	BI	1	6	5	5	1
	ME	6	5	8	6	4
GO	**NLLS**	7	4	NA	NA	7
	ML	4	2	NA	NA	6
	BI	3	3	NA	NA	5
	ME	8	2	NA	NA	8

It can be seen from **Table 2** that in terms of accuracy and sensitivity to number of traffic counts criteria, the GO model together with ME estimation performs the best. While, in terms of computer time, sensitivity to errors in traffic counts, sensitivity to zoning level and network resolution, the GR model with NLLS estimation performs the best. In general, it can be concluded that the ME and NLLS estimation methods show the best ranking performance based on several types of criteria.

Table 3 shows the estimated values of parameters of GO model together with their corresponding values of objective functions for each estimation method and **Table 4** shows the values of R^2 statistic of the observed O-D matrix compared with the estimated O-D matrices obtained from traffic counts.

Table 3. The estimated values of GO parameters and their values of objective functions

	Estimation methods	**α**	**β**	**Values of objective function**
1	**NLLS**	0.0041986	1.4796100	56197550.00
2	**ML**	0.0052000	1.7637550	423379.60
3	**BI**	0.0036893	0.979643	478944.40
4	**ME**	0.0050000	1.3867280	-32715.32

Table 4. The value of R^2 for the comparison of the observed and estimated O–D matrices

Transport Model	**Estimation methods**			
	NLLS	**ML**	**BI**	**ME**
GR	0.45123	0.38645	0.37254	0.45434
GO	0.47447	0.40733	0.39445	0.47734

Some conclusions can also be drawn from **Table 4**. They are as follows:

- in terms of O-D matrix level, it was found that the GO model always produced the best estimated matrices. However, these are only marginally better than those obtained by the GR model. Taking into account the results of using other criteria, it can be concluded that the best overall estimation methods are the combination of GO model with ME estimation method.

- with evidence so far, it was found that the estimated models and therefore O-D matrices are only slightly less accurate than those obtained directly from the full O-D surveys. This finding concludes that the transport demand model estimation approach is found encouraging in term of data collection and transport model estimation costs.

4. CONCLUSIONS

The paper explains the development of Maximum-Entropy estimation method to calibrate the parameters of transport demand models from traffic counts information. Some conclusions can be drawn from the results obtained:

- the number of observed traffic counts required are at least as many as the number of parameters. The more traffic counts you have, the more accurate the estimated O-D matrix. From several application, it can be concluded that the optimal number of traffic counts required is between 25–30% of total number of links in the network.

- It is found that in terms of accuracy and sensitivity to number of traffic counts criteria, the GO model together with ME estimation performs the best. While, in terms of computer time, sensitivity to errors in traffic counts, sensitivity to zoning level and network resolution, the GR model with NLLS estimation performs the best. However, in general, it can be concluded that the ME and NLLS estimation methods show the best ranking performance based on several types of criteria.

- The calibrated model can then be used to forecast the future O-D matrices.

- The results are encouraging since the estimated O-D matrices obtained using traffic count information are only marginally worse than those obtained by full O-D survey.

- The level of accuracy of the estimated O-D matrices depends on some following factors:
  a. the transport demand model itself in representing the trip making behaviour within the study area;
  b. the estimation method used to calibrate the model from traffic counts;
  c. trip assignment techniques used in determining the routes taken through the network;
  d. location and number of traffic counts;
  e. errors in traffic count information;
  f. finally, the level of resolution of the zoning system and the network definition.

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