VEHICLE ROUTING PROBLEM FOR DISTRIBUTING REFRIGERATED FOOD

Chaug-Ing Hsu
Professor
Department of Transportation Technology and Management
National Chiao Tung University
1001 Ta Hsueh Road, Hsinchu, Taiwan
30050 R.O.C
Fax: 886-3-5720844
E-mail: cihsu@cc.nctu.edu.tw

Sheng-Feng Hung
Graduate Research Assistant
Department of Transportation Technology and Management
National Chiao Tung University
1001 Ta Hsueh Road, Hsinchu, Taiwan
30050 R.O.C
Fax: 886-3-5720844
E-mail: roger.tem90g@nctu.edu.tw

Abstract: This research extends Vehicle Routing Problem with Time Windows (VRPTW) by considering randomness in the refrigerated food distributing process and constructs a VRPTW model to solve the optimal shipping route. The objective function of the model minimizes the sum of transportation cost, inventory cost, and energy cost. Among them, transportation cost depends on vehicle routing distance, while inventory cost accounts for the deterioration of fresh food due to the vehicle routing around many customer demand locations. For preservation of refrigerated food on the distribution, cold storage equipments on vehicles must consume extra energy and incur extra time-dependent energy cost. Given customer demand, this paper introduces a strategy, which loads extra food in refrigerated truck before shipping, to prevent food delivery from route failure, defined as incapability for delivering the required amount of food at time-windows due to food perishing. Finally, the study develops algorithms to solve the optimal vehicle route model. Results of this research provide guidance such as the required fleet, vehicle departure time, load, and distribution route for carriers on making the optimal shipping decisions.

Key Words: VRPTW, refrigerated food distribution, inventory cost

1. INTRODUCTION

As life styles of people living in most of developed countries are busier and women labor rates continuously increase, household food purchasing period gets longer and family eating outside becomes popular. As a result, refrigerated food becomes the major source of diet. According to the investigation of American Frozen Food Institute (AFFI) in 1999, average annual growth rate of total retail sales on frozen food in the U.S. reached more than 20 percent and 16 percent of people said they would continue to increase consumption. Moreover, as more countries join World Trade Organization (WTO), the barriers of tariff and importing restriction on international trade remove gradually. The increasing amount of imported refrigerated food brings logistics carriers enormous business on distributing refrigerated food.

The weather in Taiwan is hot and humid all around the year, therefore refrigerated food must be stored in standard cold storage equipments to keep proper temperature in the process of production, storage, transportation and retail sale, which is called "Cold Chain". Refrigerated food is shipped to retails in cold storage trucks, which are expensive and consume fuels more than general trucks. On the other hand, in practice, short refrigerated food life cycle also means short sale time. The efficiency of refrigerated food distribution not only significantly affects the cost of the carriers but also the revenues of retailers. To enhance the level of service, carriers usually must satisfy time-window constraints of customers. For instances, many convenience stores contract with distributors for delivering refrigerated food in specific time-window. So, considering time-window constraint well in advance on planning shipping routes can lower the carrier's shipping cost and match customers' requests to raise level of service.

Refrigerated food belongs to perishable product, which deteriorates uncritically on the shipping process. Shipping time downgrades food quality due to the organic function. Short life cycle food, such as milk or lunch box, etc., may also be sold with low probability due to increased delay of shipping. The loss in retailers' revenues is usually transferred to the distributors in terms of penalty cost to take the responsibility due to shipping delay.

Past literature about vehicle routing problems (VRP) focused on shipping normal products (e.g., Bodin *et al.*, 1983; Solomon and Desrosiers, 1988; Daganzo, 1987a; Daganzo, 1987b; Fisher and Jaikumar, 1981). Some studies investigated perishable products, but most of them dealt with perishable product inventory models and discussed the best economic order quantity (EOQ) (e.g., Covert and Philip, 1973; Philip, 1974; Charkrabarty *et al.*, 1998; Giri and Chaudhuri, 1998; Hariga, 1996; Ghare and Schrader, 1963). Only few studies discussed the distribution of refrigerated food (e.g., Tarantilis and Kiranoudis, 2002). Nevertheless, inventory cost due to the deterioration of refrigerated food and energy cost of equipments of cold storage in trucks, which are important characteristics of distributing refrigerated food, are seldom considered. This research extends Vehicle Routing Problem with Time Windows (VRPTW) by considering randomness in refrigerated food distributing process and constructs a VRPTW model to solve the optimal shipping route. The objective function of the model aims at minimizing the sum of transportation cost, inventory cost, and energy cost. Among them, transportation cost depends on vehicle routing distance, while inventory cost accounts for the deterioration of fresh food due to the vehicle routing around many customer demand locations. Energy cost is due to extra energy consumption of freezing equipments on the vehicles. Then, the study formulates a time-dependent fresh food deteriorating function, and calculates the probability of deterioration occurrences and evaluates how much loss it causes. Given customer food demand, this paper introduces a strategy, which loads extra food in refrigerated truck before shipping, to prevent food delivery from route failure, defined as incapability for delivering the required amount of food at time-windows due to food perishing. This model can be used as a reference for distributors to make decisions, such as the required fleet, vehicles departure time, load, and distribution route.

The rest of this paper is organized as follows. Section 2 formulates inventory and energy costs to construct a refrigerated food shipping route model. In section 3, this study develops an algorithm to solve the model. An example is performed to illustrate the application of the model in Section 4. Section 5 presents concluding remarks.

2. REFRIGERATED FOOD DISTRIBUTION MODEL

The vehicle routing problem for distributing refrigerated food in this paper is defined as follows: given customers' locations, demands, and time-windows, one depot serves many customers for single refrigerated food by vehicles with refrigerating equipments. First, we define a complete symmetric graph $G=(V,A)$, where $V=\{v_0,v_1,...,v_n\}$ is the set of nodes and $A=\{(v_i,v_j): v_i,v_j \in V, i \neq j\}$ is the set of links. Node v_0 is the depot, and nodes v_i, $i=1,2,...,n$ are the locations of customers i, and d_i, $i=1,2,...,n$ stands for customer i's demand. Superscript l, $l=1,2,...,m$ stands for vehicles l, where m is total number of vehicles and is a decision variable.

There arises uncertainty on the arrival time of food delivering to each customer because of the randomness of vehicle running time due to congestion on urban streets. On the other hand, the perishing of refrigerated food on distribution process is also stochastic, while food spoilage may reduce the amount of food available and causes the problem of 'route failure'. Route failure is defined herein as the condition that the customer can't receive the enough amount of food within the appointed time-window, and it usually incur extra cost to the carrier in a way that the carrier has to pay for the penalty or loss of the customer. Therefore, this study incorporates the penalty cost due to route failure into the total cost of food distribution. Moreover, due to the perishing of refrigerated food, this study develops a shipping strategy that the carrier adds extra food to prevent routing from route failure before vehicles depart from the depot. This study explores the characteristics of vehicle routing problem specific to distributing refrigerated food so as to formulate pipeline inventory cost and energy cost. The fixed vehicle cost and transportation cost about traditional VRP is also included. Then, the study constructs a mathematical programming model to decide the optimal routing strategy by minimizing the sum of all of the costs subject to satisfying customers' demands and time-window.

The total costs involving vehicle routing for distributing refrigerated food include fixed cost for dispatching each vehicle, transportation cost related to traveled distance, inventory cost about food perishing, and energy cost consumed by refrigerated equipments in the vehicle. Fixed cost for dispatching each vehicle includes driver salary, loading/unloading and handling cost, and vehicle depreciation or rental cost. If there are m vehicles serving n customers and fixed cost of vehicle l is f^l, then, the total fixed cost can be expressed as:

$$\sum_{l=1}^{m} f^l \tag{1}$$

The vehicle transportation cost is related to travel time and includes gasoline and maintenance cost. This study defines the vehicle transportation problem as:

$$\varepsilon \sum_{l=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} t_{ij}^l x_{ij}^l \tag{2}$$

where t_{ij}^l is the travel time of vehicle l running between node v_i and v_j, and ε is transportation cost per unit travel time. And x_{ij}^l is an indicator variable, if vehicle l runs through link (v_i, v_j), $x_{ij}^l = 1$; otherwise, $x_{ij}^l = 0$.

Refrigerated food belongs to perishables. Generally speaking, influencing factors of food perishing include: preserving temperature, water activity, PH value of food, existence of oxygen, and metabolites from perishing process. (Liu and Wang, 1997) This study assumes food can be stored and distributed in fixed temperature by vehicles with refrigerated equipments and ignores other influencing factors. Therefore, in the distribution process, the perishing of food is merely the function of distribution time. However, opening carriage door for serving customers makes temperature higher and perishing faster. (Hsu, 1989) Therefore, inventory costs related to the deterioration of food can be divided into two types. One is due to time accumulation on routing process. The other is due to loading/unloading that hot air invades while opening vehicle carriage. Let $\tilde{b}_j$ denote the amount of inventory loss during the duration between finishing time of serving customer j and finishing time of serving the preceding customer. Because of randomness of perishing, $\tilde{b}_j$ is defined as a random variable in this paper, thus the expected total inventory cost is:

$$p \sum_{l=1}^{m} \sum_{j=1}^{n} z_j^l \bar{b}_j \tag{3}$$

where p is the product unit cost and z_j^l is an indicator variable, if vehicle l serves customer j, $z_j^l = 1$; otherwise, $z_j^l = 0$. Moreover, $\bar{b}_j$ is the expectation of $\tilde{b}$, and its derivation can be depicted as Figure 1.

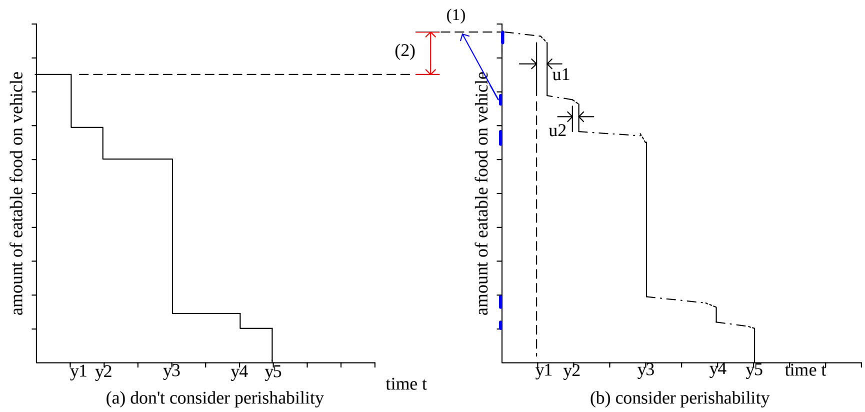

Figure 1. Refrigerated food perishing profile

As shown in Figure 1, (1) is the loss of food during the duration between finishing serving the customer and the preceding customer, (2) is the total loss in the amount of food during the entire routing process. Furthermore, u_j and y_j, $j = 1,2,...,n$, are the duration and starting time of serving customer j. Refrigerated food is classified as perishable products. Several probability distribution functions have been considered to describe the deterioration functions of perishable products, such as "Exponential distribution", "Weibull distribution", and "Linear distribution". For different refrigerated food, the probability density function of deterioration rate is different. Due to the opening of carriage, temperature usually rises, and results in the steep decay of food quality. When opening carriage, cool air flows outside from the bottom of carriage door while hot air flows inside from the top. The amount of heat interchange relates to air speed, size of carriage door, difference in the temperature between carriage inside and outside, and duration of opening door. This study considers air speed, size of carriage door, difference in the temperature between carriage inside and outside as exogenous variables. Thus, the inventory loss for opening vehicle carriage door only relates to duration of opening door, which relates to the customer demand. Therefore, the inventory loss is defined in this study as the function of customer demand.

Let $G(d_j)$ be the probability of inventory deterioration due to opening vehicle carriage door for serving customer j and d_j is the demand of customer j. Variable d_0 could be treated as the total amount of food loaded into vehicles from the distribution center (depot), and $G(d_0) = 0$, if there is no deteriorated food loaded from the depot. Let $F(\cdot)$ be the cumulative probability density function of food deterioration due to time accumulation on routing process, then the expected amount of deteriorated food for vehicle departing from the depot to serve customer i, $\bar{b}_i$, is:

$$\bar{b}_i = L^l \times \left[F\left(y_i - y_s^l + u_i\right) + G(d_i)\right] \tag{4}$$

where L^l is the load of vehicle l, y_i is the starting time to serve customer i, and y_s^l is the departure time of vehicle l from the depot. If vehicle l first serves customer i and then serves customer j, the available amount of food after finishing serving customer i is $\underline{L}_i^l = L^l - b_i - d_i$. And the expected amount of deteriorated food after serving customer j, $\bar{b}_j$, is therefore:

$$\bar{b}_j = L_i^l \times \left[F\left(y_j - y_s^l + u_j\right) - F\left(y_i - y_s^l + u_i\right) + G\left(d_j\right)\right] \tag{5}$$

Multiplying x_{0i}^l on both sides of Eqn. (4) can obtain the expected amount of deteriorated food after serving the first customer. Those after serving other customers along the route can be found by multiplying x_{ij}^l on both sides of Eqn. (5).

For energy cost, thermal load origins from radiation of sun and burning ground, invading hot air, and heat conduction is mainly due to temperature difference between inner and outer carriage (MOEA-IDB, 2001).

From law of thermal conduction, $Q = \frac{k}{x} \times A \times \Delta T$, where Q is conductivity; k is coefficient of conduction; x is insulation thickness; A is contacting area; and ΔT is the difference of temperature. Vehicles shipping refrigerated food usually have thick walls to insulate hot air outside. However, opening carriage door makes a great quantity of hot air enter into carriage while unloading food for serving customers. In practice, thermal load due to opening carriage door can be calculated as follow (MOEA-IDB, 2001):

$$Q_s = (0.54V_i + 3.22)(T_o - T_i) \times \beta \tag{6}$$

where Q_s: Thermal load

V_i: Carriage volume

T_o: Outer temperature

T_i: Inner temperature

β: Indicator of opening frequency

The value of β is related to the frequency of opening the door, as show in Table 1.

Table 1. β vs. opening frequency (MOEA-IDB, 2001)

Classification	Frequency	β
A	Zero	0.25
B	Half of C	0.5
C	Two to three times in a hour	1
D	One and half times of C	1.5
E	Double of C	2

Note that Q_s is the thermal load per unit time. From Eqn. (6), we farther know thermal load also relates to the volume of carriage, carriage inner and outer temperature, and door opening frequency. This study assumes homogeneous vehicles ship the same kind of food, so carriage volume and inner temperature are the same for all vehicles. Symbol β is the frequency of door opening and relates to demand and spatial pattern of customers. Furthermore, the study assumes the carrier plans the distribution routing in advance, so as to satisfy time-constraints appointed with customers. Therefore, the frequency of carriage door opening for each vehicle can be represented by its expected value, $\overline{\beta}$. Then, Eqn. (6) can be simplified as:

$$Q_s = \alpha_s \overline{\beta}(T_o - T_i) \tag{7}$$

where α_s ($= 0.54V_i + 3.22$) is a constant. Eqn. (7) reveals that thermal load of door opening is simplified to be merely proportional to the difference of temperature between inner and outer carriages. Furthermore, if we assume outer temperature is known; then thermal load can further be simplified to be a constant, i.e. the energy loss of opening carriage door per unit time is fixed. For carriers with specific customers, if demand pattern is fixed, then the energy cost of each vehicle due to opening carriage door is merely the function of total travel time on routing and serving the customers.

On the other hand, in practice, thermal conduction due to the difference between inner and outer carriage temperature can be estimated as:

$$Q_T = U\sqrt{A_i A_o}\left(T_o - T_i\right)\left(1+\mu\right) \tag{8}$$

where Q_T : Thermal load of carriage
U : Conductivity of carriage
A_i : Surface area of inner carriage
A_o : Surface area of outer carriage
μ : degree of spoil of carriage

Notably, Q_T is the thermal load of carriage per unit time. Eqn. (8) shows that Q_T relates to the difference of temperature between inner and outer carriages, material and size of carriage, and degree of spoil of carriage. This study doesn't aim to investigate material, size and degree of spoil of carriage, so supposes these factors are exogenous. Therefore, thermal conduction due to the difference of temperature between inner and outer carriages can be simplified as:

$$Q_T = \alpha_T\left(T_o - T_i\right) \tag{9}$$

Following the assumption discussed above, the difference of temperature between inner and outer carriage is fixed, so vehicle energy cost due to thermal conduction resulted from the temperature difference can be simplified to be merely related to the total travel time on routing and serving the customers. Therefore, the total energy cost of each vehicle is proportional to its travel time. The total energy cost for all vehicles can then be expressed as:

$$\sum_{l=1}^{m}\left[\alpha\left(y_f^l - y_s^l\right)\right] \tag{10}$$

where y_f^l is the returning time of vehicle l to the depot, and α $\left(=\left(\alpha_s\overline{\beta}+\alpha_T\right)\left(T_o - T_i\right)\right)$ is energy cost per unit time.

Based on the discussions above, the study constructs a mathematical programming model to formulate VRPTW for distributing refrigerated food. The model is formulated as Eqns. (11a)-(11k):

$$Min \qquad \sum_{l=1}^{m} f^l + \varepsilon\sum_{l=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n} t_{ij}^l x_{ij}^l + p\sum_{l=1}^{m}\sum_{j=1}^{n} z_j^l \overline{b}_j + \sum_{l=1}^{m}\left[\alpha\left(y_f^l - y_s^l\right)\right] \tag{11a}$$

st.

$$\sum_{l=1}^{m} z_i^l = \begin{cases} m & i = 0 \\ 1 & i = 1,\ldots,n \end{cases} \tag{11b}$$

$$\sum_{i=0}^{n} x_{ij}^l = z_j^l \qquad j = 0,\ldots,n \quad l = 1,\ldots,m \tag{11c}$$

$$\sum_{j=0}^{n} x_{ij}^l = z_i^l \qquad i = 0,\ldots,n \quad l = 1,\ldots,m \tag{11d}$$

$$y_j \geq y_i + u_i + t_{ij}^l - \left(1 - x_{ij}^l\right)M \qquad i = 1,\ldots,n \quad j = 1,\ldots,n \quad l = 1,\ldots,m \tag{11e}$$

$$y_i \geq y_s^l + t_{0i}^l - \left(1 - x_{0i}^l\right)M \qquad i = 1,\ldots,n \quad l = 1,\ldots,m \tag{11f}$$

$$y_f^l \geq y_j + u_j + t_{j0}^l - \left(1 - x_{j0}^l\right)M \qquad j = 1,\ldots,n \quad l = 1,\ldots,m \tag{11g}$$

$$r_i \leq y_i \leq s_i \tag{11h}$$

$$L^l = \sum_{i=1}^{n} z_i^l d_i + v^l \leq K^l \quad l = 1,\ldots,m \tag{11i}$$

$$x_{0i}^l \bar{b}_i = x_{0i}^l L^l \times \left[F\left(y_i - y_s^l + u_i\right) + G\left(d_i\right)\right] \tag{11j}$$

$$\bar{b}_j = L_i^l \times \left[F\left(y_j - y_s^l + u_j\right) - F\left(y_i - y_s^l + u_i\right) + G\left(d_j\right)\right] \tag{11k}$$

where s_i and r_i are the upper and lower bounds of customer i time-window; M is a very large positive number; K^l is the capacity of vehicle l; and v^l is the extra load of food of vehicle l.

Due to perishing characteristics of refrigerated food, the distribution model formulated in this study is not following the network flow equilibrium within the delivery routing strategy. Eqn. (11a) is an objective function that minimizes the carrier's transportation cost, inventory cost, and transportation cost. Eqn. (11b) represents each customer is served by one vehicle and each route starts and ends at the depot. Eqns. (11c) and (11d) are flow conservation constraints. Eqns. (11e), (11f) and (11g) are time-order constraints that confine the arrival times of any two customers wouldn't conflict with each other. Eqn. (11h) is the time-window constraint; Eqn. (11i) is the vehicle-capacity constraint; and Eqns. (11j) and (11k) count the expected amount of perishing food. The decision variables are x_{ij}^l, y_i, y_s^l, y_f^l, v^l, z_i^l, and m. That is, the carrier can apply the model to optimally decide the vehicle shipping route, arrival time of each vehicle on serving each customer, departure and returning time of vehicles, the extra load of vehicles, customers served by each vehicle, and the size of the shipping fleet. The larger fleet will incur the higher fixed cost of vehicles, but the less serving customers and routing time of each vehicle, thereby resulting in the less inventory loss and extra load. Thus there exists a trade-off between transportation cost and inventory cost.

Considering the shipping route of vehicle l by solving the model, the shipping order is $v_{i_0}, v_{i_1}, \ldots, v_{i_h}, v_{i_0}$ and $v_{i_0} = v_0$. When vehicle l finishes serving its qth customer, its expected amount of available food is $L^l - \sum_{k=1}^{q}\left[\bar{b}_{i_k} + d_{i_{k-1}}\right]$. If the probability of satisfying customers' demand by each vehicle must be greater than the level of service θ, the chance constraint of route failure of vehicle l is $P\left\{\left[L^l - \sum_{k=1}^{q}\left[\bar{b}_{i_k} + d_{i_{k-1}}\right]\right] < d_{i_q}\right\} \leq 1 - \theta$, $q = 1,\ldots,h$. Let w_{i_q} be the insufficient amount of food demanded by qth customer on route l, $w_{i_q} = d_{i_q} - \left\{L^l - \sum_{k=1}^{q}\left[\bar{b}_{i_k} + d_{i_{k-1}}\right]\right\}$, then w_{i_q} is a greater-or-equal-to-0 integer and can be counted by the following three constraints:

$$L^l - \sum_{k=1}^{q}\left[\bar{b}_{i_k} + d_{i_{k-1}}\right] - d_{i_q} + w_{i_q} \geq 0 \tag{12}$$

$$w_{i_q} \geq 0 \tag{13}$$

$$w_{i_q} \in Integer \tag{14}$$

3. ALGORITHM

VRP inherently belongs to NP-Hard Problem. Using exact algorithms to solve the problem usually requires considerable amount of time and can just solve small problems. This study adopts a heuristic method, which extends "Time-Oriented Nearest-Neighbor Heuristic" by Solomon (1983). The details of the heuristic algorithm are described as follows:

Step 1. Input the basic data about refrigerated food and network, $G=(V,A)$, including (1) basic customer and network data: v_i, n, c_{ij}^l, d_j, u_j, s_i, and r_i; (2) data of vehicle and refrigerated food characteristics: f^l, k^l p, α, $F(\cdot)$, and $G(d_j)$.

Step 2. From the depot, search for and add the "nearest customer" as the first served customer of the first route. The method to search for the "nearest customer" is as follow:

Suppose i is the latest added customer. Let customer j be one of any unrouted customers, and then j must satisfy the following conditions to be added to the route.

(1) An unrouted customer
(2) Satisfying time-window and vehicle-capacity constraints

Factors determining the nearest customer are:

(1) Distance between customers i and j: c_{ij}
(2) Difference in time between finishing serving customer i and starting to serve customer j: ΔT_{ij}
(3) Difference in time between customer j time-window upper bound and the earliest time available for starting to serve customer j: a_{ij}

And they could be defined as follows:

$$\Delta T_{ij} = y_j - (y_i + u_i) \tag{15}$$

$$a_{ij} = s_j - (y_i + u_i + t_{ij}) \tag{16}$$

$$y_j = \max\{r_j, y_i + u_i + t_{ij}\} \tag{17}$$

C_{ij} is the cost function that determines if customer is the nearest one and can be represented as:

$$C_{ij} = \delta_1 c_{ij} + \delta_2 \Delta T_{ij} + \delta_3 a_{ij} \tag{18}$$

where δ_1, δ_2, δ_3 are the weights of these factors, which represent, respectively, the marginal cost of the constructed model with respect to additional one unit of c_{ij}, ΔT_{ij} and

a_{ij}. And $\delta_1 + \delta_2 + \delta_3 = 1$, $\delta_1 \geq 0$, $\delta_2 \geq 0$, and $\delta_3 \geq 0$ must be satisfied. Here i is the distribution center or depot, and the customer with the smallest C_{ij} is the nearest one. Factor c_{ij} is transportation cost on link (v_i, v_j) in the shipping route model, and represents the space distance factor. On the other hand, ΔT_{ij} and a_{ij} are time distance factors, where ΔT_{ij} relates to the inventory and energy cost of the model, and a_{ij} is available time left if customer j join this route.

The weights of δ_1, δ_2, and δ_3 can be further evaluated. Increasing one unit of c_{ij} means customers j and i are one more unit distant apart, and adding customer j will make the total cost increase δ units. Suppose there is one increased unit of ΔT_{ij}, which means customers j and i are one more unit time- distant apart, then the energy cost will increase α units. Moreover, the increase of inventory cost is the total value of deteriorated food due to one more unit of shipping time. We estimate the average amount of food on in-transit vehicles by $K^l/2$ and average deterioration rate per unit time is $1/\mu$, where μ is the life cycle of refrigerated food in proper temperature. Then the increase on inventory cost is $pK^l/2\mu$. So the total cost will be increased by $\alpha + pK^l/2\mu$ due to increasing one unit of ΔT_{ij}. Factor a_{ij} is available time left if customer j is added in the route, and represents the influence regarding the order of time-windows of customers on the shipping route. Due to the order of time window, adding customer j into the route may make those unrouted customers cannot join the route. The influenced customers can be identified as follows:

The value of a_{ij} must be greater to u_j and can be divided into two conditions:

1. If $u_j \leq a_{ij} \leq s_j - r_j$, $r_j \leq y_i + u_i + t_{ij}$, and $y_j = y_i + u_i + t_{ij}$, then adding next customer $j^{'}$ in the route must satisfy not only vehicle-capacity constraint but also $a_{jj^{'}} = s_{j^{'}} - \left(y_j + u_j + t_{jj^{'}}\right) \geq u_{j^{'}}$. Rewrite the constraint, we can get

$$s_{j^{'}} - y_j \geq u_j + u_{j^{'}} + t_{jj^{'}} \tag{19}$$

2. If $a_{ij} \geq s_j - r_j$, $r_j \geq y_i + u_i + t_{ij}$, and $y_j = r_j$, then, similar to condition above, adding next customer $j^{'}$ in the route must satisfy not only vehicle-capacity constraint but also

$$s_{j^{'}} - r_j \geq u_j + u_{j^{'}} + t_{jj^{'}} \tag{20}$$

Step 3. Let i be the last added customer of this route, and repeat Step 2, we can get the nearest customer j and add j into the route in order. If all unrouted customers cannot satisfy time-window and vehicle-capacity constraints, then start another route until all customers are routed.

4. NUMERICAL EXAMPLE

A numerical example is presented herein to demonstrate the application of the proposed model for bread distribution. The network comprises a random extraction of 35 customers' characteristics including customers' locations, time-window, and demand from international test problems. Suppose bread unit price is 15 NT dollars and bread life cycle is 1 day or 24 hours. Then μ=1440 min. and $F(\cdot)$=0.000694y. Values of other parameters are shown in Table 2.

Table 2. Initial values of parameters

Parameters	Initial Value	Unit
f^l	1000	NT dollars per dispatch
α	0.5	NT dollars/minutes
K^l	300	
$G(d_j)$	0.0001d_j	

Moreover, $\alpha + pK^l/2\mu$=2.0625, so $\delta_1 : \delta_2 = 1 : 2.0625$. Furthermore, this study uses three sets of weights, i.e., $(\delta_1, \delta_2, \delta_3)$=(0.33, 0.67, 0), (0.25, 0.5, 0.25), (0.2, 0.4, 0.4) in numerical example to explore the changes in the optional solution due to variants in key parameter values. Then, we solve the problem using the model and the algorithm, described in Sections 2 and 3, respectively. The results are shown in Tables 3 and 4.

This study also solves general product shipping route problem using traditional VRPTW for comparison. That is, it determines the optimal shipping strategy without consideration of inventory and extra energy cost. Shipping cost thus merely includes vehicle dispatching and transportation cost. All other data and parameter values are the same as those for shipping perishable product. However, due to there is no derivation on weights of traditional VRPTW, this study refers to Solomon (1987) and assumes $(\delta_1, \delta_2, \delta_3)$ to be (0.5, 0.5, 0), (0.4, 0.4, 0.2), (0.3, 0.3, 0.4) as node selection criteria in the case study. The results are shown in Tables 5.

Table 3. Results for shipping perishable food for $(\delta_1, \delta_2, \delta_3)$ =(0.33, 0.67, 0)

Route Number	Order of service	v^l	L^l	y_s^l (min.)	y_f^l (min.)
1	$0 \to 21 \to 6 \to 20 \to 1 \to 31 \to 0$	12	142	378.5	525
2	$0 \to 2 \to 25 \to 10 \to 33 \to 32 \to 19 \to 0$	18	150	281.5	490.5
3	$0 \to 7 \to 23 \to 30 \to 22 \to 3 \to 13 \to 18 \to 0$	13	259	130.5	315.5
4	$0 \to 16 \to 29 \to 15 \to 11 \to 17 \to 24 \to 28 \to 12 \to 0$	17	299	200.5	405.5
5	$0 \to 5 \to 9 \to 4 \to 26 \to 34 \to 14 \to 0$	19	179	165.5	411.5
6	$0 \to 35 \to 8 \to 27$	7	59	105.5	211

Table 4. Results for shipping perishable food

$(\delta_1, \delta_2, \delta_3)$	m	Extra load	Vehicles dispatching cost (NT dollars)	Transportation cost (NT dollars)	Inventory cost (NT dollars)	Energy cost (NT dollars)	Total cost (NT dollars)
(0.33, 0.67, 0)	6	87	6000 (68.74%)	1168 (13.38%)	1098 (12.58%)	462 (5.3%)	8728
(0.25, 0.5, 0.25)	5	100	5000 (56.92%)	1610 (18.33%)	1642 (18.7%)	532 (6.05%)	8784
(0.2, 0.4, 0.4)	5	118	5000 (55.15%)	1622 (17.89%)	1816 (20.03%)	628 (6.93%)	9066

Note: Values in parenthesis is the percentage of total cost

Table 5. Results for shipping general product

$(\delta_1, \delta_2, \delta_3)$	m	Vehicles dispatching cost (NT dollars)	Transportation cost (NT dollars)	Total cost (NT dollars)
(0.5, 0.5, 0)	8	8000(84.16%)	1506(15.84%)	9506
(0.4, 0.4, 0.2)	4	4000(75.84%)	1274(24.16%)	5274
(0.3, 0.3, 0.4)	5	5000(76.55%)	1532(23.45%)	6532

Note: Values in parenthesis are the percentages of total cost

From the result of comparing Tables 4 and 5, it is apparent that when considering energy and inventory cost for shipping refrigerated food, the total cost will be markedly increased by 65.5% ((8728-5274)/5274 $\times$ 100%). It suggests that energy and inventory cost are not negligible, and ought to be considered in VRPTW for distributing refrigerated food. And from Table 4, energy cost accounts for about 6% of total cost and inventory cost accounts for about 17% of total cost. Furthermore, the more the shipping vehicles, the less the energy and inventory cost will be. There exist a trade-off relationship between transportation and inventory cost.

From the results of Table 4 and 5, it is apparent that without considering order of time-window ($\delta_3 = 0$) will cause more dispatching of vehicles. Although more dispatching of vehicles raises dispatching cost, it will reduce the inventory and energy cost for shipping perishable food. But for shipping general product, proper considering order of time-window will always lower down the total shipping cost due to fewer dispatches.

5. CONCLUSIONS

This research extends VRPTW by considering randomness in refrigerated food distributing process and constructs a SVRPTW model to solve the optimal shipping route, vehicle load, fleet, and departure time. A case study is provided to demonstrate the results and the feasibility of applying the proposed models. Results of this research not only help understand how the perishing randomness and properties of food storage affects the vehicle routing and resulting costs, but also provides guidance such as the required fleet, vehicles departure time, load and distribution route for carriers on making the optimal shipping decisions.

This study adopts a heuristic method, which extends “Time-Oriented Nearest-Neighbor Heuristic” by Solomon (1983). However, the quality of the results, using a heuristic method, depends on the number of customers in the problem. The result of the example, which uses the number of customers as many as Solomon (1983), sufficiently reflects the key characteristics of the refrigerated food distribution model. Future study can discuss how environment changes such as time-varying travel time and temperature affect the optimal shipping decision. On the other hand, the time-window constraint could be also modified as “soft time-window”.

REFERENCES

Bodin, L., Golden, B., Assad, A. and Ball, M. (1983) Routing and Scheduling of Vehicles and Crews: The State of The Art, **Computers Ops. Res., Vol. 10, No. 2**, 63-211.

Charkrabarty, T., Giri, B.C. and Chaudhuri, K.S. (1998) An EOQ Model for Items with Weibull Distribution Deterioration, Shortages and Trended Demand: An Extension of Philip's Model, **Computers Ops. Res., Vol. 25, No. 7/8**, 649-657.

Covert, R.P. and Philip, G.C. (1973) An EOQ Model for Items with Weibull Distribution Deterioration, **AIIE Transactions, Vol. 5, No. 4**, 323-326.

Daganzo, C.F. (1987a) Modeling Distribution Problems with Time Windows. Part I, **Transportation Science, Vol. 21, No. 3**, 171-179.

Daganzo, C.F. (1987b) Modeling Distribution Problems with Time Windows. Part II: Two Customer Types, **Transportation Science, Vol. 21, No. 3**, 180-187.

Fisher, M. and Jaikumar R. (1981) A Generalized Assignment Heuristic for the Vehicle Routing Problem, **Networks, Vol. 11**, 109-124.

Ghare, P.M. and Schrader, G.F. (1963) A Model for Exponentially Decaying Inventories, **Journal of Industrial Engineering, Vol. 14**, 238-243.

Giri, B.C. and Chaudhuri, K.S. (1998) Deterministic Models of Perishable Inventory with Stock-Dependent Demand Rate and Nonlinear Holding Cost, **European Journal of Operational Research, Vol. 105**, 467-474.

Hariga, M. (1996) Optimal EOQ Models for Deteriorating Items with Time-Varying Demand, **Journal of the Operational Research Society, Vol. 47, No. 10**, 1228-1246.

Hsu, C.T. (1989) **Science of Food Refrigeration**. Fu-Wen Publisher, Taipei, Taiwan.

Liu, I.J., Wang, J.Z. (1987) **Process and Storage of Food**, Central Publisher, Taipei, Taiwan.

MOEA-IDB (2001) **A Handout of Training Class for Industry Technology Talent in 2001**. Taiwan.

Philip, G.C. (1974) A Generalized EOQ Model for Items with Weibull Distribution Deterioration, **AIIE Transactions, Vol. 6, No. 2**, 159-162.

Solomon, M.M. (1983) Vehicle Routing and Scheduling with Time Windows Constraints: Models and Algorithms, ***Ph.D. Dissertation***, Dept. of Decision Science, University of Pennsylvania.

Solomon, M.M. and Desrosiers, J. (1988) Time Window Constrained Routing and Scheduling Problems, **Transportation Science, Vol. 22, No. 1**, 1-13.

Tarantilis, C.D. and Kiranoudis, C.T. (2002) Distribution of Fresh Meat, **Journal of Food Engineering, Vol. 51**, 85-91.