A TRANSIT ASSIGNMENT MODEL ON STOCHASTIC NETWORKS

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Abstract: In this paper, a transit assignment model has been developed on stochastic networks, with considering vehicle capacity and scheduled timetable. The stochastic network based transit assignment model is appropriate to apply for bus and metro networks altogether since it can distinguish between uncertain and reliable travel times characteristics by imposing stochastic parameters on links or lines. The vehicle capacity constraint based model is useful to evaluate some alternative operational plans and policies in transit networks. On the other hand, the scheduled timetable based model is necessary to evaluate detailed transit operations by calculating personal route choice based on the exact timetables of the transit operations. As a result of this we can assign the passengers in more realistic representation.

Keyword: timetable-based transit assignment, stochastic networks, genetic algorithm

1. INTRODUCTION

These days public transportation oriented transport policies have been increasingly important in order to relieve serious traffic congestion problems due to rapid increasing of car ownership. However, there are small amounts of researches on public transportation policy and planning evaluation or management tools by comparison with those of automobiles on roads. Even worse the existing methodologies on the evaluation for public transport policies and operations cannot represent realistic important features on public transport systems. The important public transport characteristics differed from automobiles are scheduled timetable information, transfer, capacity of vehicle, common lines among others. These kinds of distinguished characteristics should be modeled in order to reflect the realism on the public transport systems.

In this paper, a transit assignment model has been developed on stochastic networks, with considering vehicle capacity and scheduled timetable. The stochastic network based transit assignment model is appropriate to apply for bus and metro networks altogether since it can distinguish between uncertain and reliable travel times characteristics by imposing stochastic parameters on links or lines owing to boarding and alighting. The vehicle capacity constraint based model is useful to evaluate some alternative operational plans and policies in transit networks. On the other hand, the timetable-based model is necessary to evaluate detailed transit operations by calculating personal route choice based on the exact timetables of the transit operations. As a result of this we can assign the passengers in more realistic representation.
In our studies, a genetic algorithm has been developed in order to obtain transit assignment results under the stochastic networks, and vehicle capacity and scheduled timetable. The genetic algorithm (GA) is a method for an approximate optimization by simulating the process of a nature evolution. In the GA, candidate solution sets for a particular problem are represented by a set of symbolic or numeric arrays, which are called chromosome. Gene is a component for consisting of chromosome. The population is generated by a set of chromosomes. Evolution process of generating population through iterations can lead to find a global solution of a particular problem. The process of each generation has been continued until it reaches to maximum generation by initialization, selection, crossover and mutation processes. In each generation, the same size of population has been maintained.

These characteristics in the GA imply that in the shortest path finding problem of transit assignment, alternative shortest path set can be obtained. In this paper, the GA has been developed for the shortest paths finding problem under the stochastic networks with vehicle capacity and scheduled timetable. After the shortest paths have been obtained in the genetic algorithm, the multinomial logit model has been used to obtain the assigned volumes on the transit lines and networks. In the utility of the alternative paths is composed of a sequence of link travel times from origin to destination.

2. TIMETABLE-BASED TRANSIT ASSIGNMENT MODEL ON STOCHASTIC NETWORKS

The traditional transit assignment is based on the headway such as in Spiess and Florian, 1989. Headway-based transit assignment needs only average headways and trips defined by their origin and destination. Therefore, headway-based transit assignment is easily applied for the analysis of public transport. However, the disadvantage of the headway-based transit assignment is that it calculates the various attributes based on only average values. In order to analyze the detailed operational plans in public transport, the exact characteristics of public transport should be produced in the transit assignment model. Recently, several researches have considered the property of public transport that is sensitive to scheduled timetable and stochastic transit service characteristics. (e.g., Hall(1986), Hickman and Bernstein(1997)). Tong and Wong(1999) highlighted the difference between frequency-based(headway-based) and scheduled-based(timetable-based)models. Nuzzolo and Russo(1998) used a stochastic network loading model to assess results of their path choice model.

The timetable-based transit assignment can calculate real vehicle arrival/departure times, and hence all the values of level of service attributes can be evaluated explicitly. This approach allows us to take into account the dynamical evolution of both supply and demand. The stochastic network represents the variability of transit travel time. This variation of transit travel time can be caused by the effects of automobile as well as by irregular passenger boarding and alighting characteristics.

2.1. PATH SEARCH ALGORITHM

In this paper, a new approach, namely path based representation, is proposed to create an initial population by focusing attention on paths consisting a network. The approach is to use the path to be represented as the chromosome. Each component of the chromosome, namely
gene, represents nodes for forming a path connecting an origin and a destination. The chromosome assigns all paths in the original network. Because the gene (node) values have been changed, the chromosome creates another path of a new structure from the original path. Therefore this encoding method is capable of equally representing all possible paths (candidate solutions) for the original network. These multiple alternative shortest paths can be used to load the demand stochastically in stochastic network loading procedure in transit assignment. But, if the chromosome were only represented by set of nodes, a chromosome would correspond to several paths because of common line problem of public transport. Therefore, in case of public transport, the method of chromosome representation is designed specially.

1) Method of chromosome representation

As stated above, all candidate solutions (paths) are represented by chromosome in Genetic Algorithm. While the chromosomes of automobile are only represented by set of nodes, the chromosomes of public transport are represented by both set of nodes and service lines. The service line is added to chromosome between node and node for the purpose of one-to-one matching path and chromosome.

\[ A = (g[i] \text{ (line[1]) } g[2] \text{ (line[2]) } g[3] \text{ (line[3]) } \ldots g[•] \text{ (line[•]) } \ldots g[j]) \]

Where, \( A = \) chromosome
\( g[•] = \) gene (node), \( g[i]=\)origin node, \( g[j]=\)destination node
\( \text{line[•]} = \) service line

This method of chromosome representation solves not only the common line problem but also the transfer problem. For example, if the first service line “line[1]” is different from the second service line “line[2]” in the chromosome A, the transfer will occur at gene(node) “g[2]”. Frequently, the existing studies of public transport have used the dummy links to analyze the transfer. But, if we used the method of the chromosome representation, we could analyze the transfer without expanding a network.

2) A genetic algorithm for the shortest path search

(1) Initialization

In the procedure of the initialization, the chromosomes are created and the travel time of the chromosome is calculated. The total travel time is the summation of the waiting time, the travel time, the transfer time, and the variable time by the stochastic network.

\[ c_i = w_i + c_{al} + t_m + \varepsilon_{al} \]  

(1)

where, \( c_i = \) total travel time
\( w_i = \) waiting time at node \( i \)
\( c_{al} = \) travel time of line \( l \) on link \( a \)
\( t_m = \) transfer time at node \( m \)
\( \varepsilon_{al} = \) variable time
In order to consider the vehicle capacity, the travel time \( c_{al} \) is formulated by the travel cost function that is non-decreasing as a volume.

\[
c_{al} = c_{0l} (1 + 0.15 \cdot \left( \frac{v_{al}}{C_l} \right)^4)
\]  

where, \( c_{0l} \) = fixed travel time of line \( l \) on link \( a \)
\( \text{distant of link } a \) = speed of line \( l \)
\( C_l \) = vehicle capacity of line \( l \)
\( v_{al} \) = volume of line \( l \) on link \( a \)

In addition, the waiting time is calculated by the arrival time of trips and timetables of lines.

\[
w_i = st_i - at_i
\]  

where, \( w_i \) = waiting time at node \( i \)
\( st_i \) = starting time of line \( l \) according to timetables at node \( i \)
\( at_i \) = arrival time of trip at node \( i \)

The steps of the initialization are as follows:

- **Step 0**: Preparation
  \( z = l, m = l, \text{popsize} = 10, g[1] = i, \)
  \( i(\text{Origin node}) \in I, j(\text{Destination node}) \in J, \)
  \( r(\text{Start node}) \in R, s(\text{Arrival node}) \in S, \)
  \( a(\text{Link}) \in A, l(\text{Line}) \in L, a'(\text{Line Segment}) \in A \)

- **Step 1**: Determination of Gene
  If \( z > \text{popsize} \), goto Selection Step,
  Otherwise,
  If \( r = g[m] \) and \( a = (r,s), s \in X \) is determined,
  Randomly select \( s \in X \) and then set \( g[m+1] = s \)

- **Step 2**: Determination of Line
  In \( a = a' \), \( l \in Y \) is determined,
  Randomly select \( l \in Y \) and then set \( \text{line}[m] = l \)

- **Step 3**: Iterations
  If \( s = j \), goto step 4,
  Otherwise, \( m = m + 1 \), goto step 1
Step 4 : Calculation of Travel Time

Calculate travel time in each generated chromosomes,
Set \( z = z + 1 \), goto step 1

In step 4, we can calculate the travel time as the following sub-steps.

- **Step 4-1 : Preparation**
  \[ m = 1, \quad c_k = 0 \]

- **Step 4-2 : Calculate the line travel time**
  \[ c[m] = c_0 \cdot [\text{line}[m]] \cdot (1 + \alpha \cdot \beta) + \varepsilon[\text{line}[m]], \quad \alpha = 0.15, \quad \beta = 4 \]

- **Step 4-3 : Calculate the transfer time**
  If \( \text{line}[m] \neq \text{line}[m + 1] \),
  \[ c[m] = c[m] + t_m \]

(2) **Selection Procedure**

In the selection procedure, the excellent chromosome is selected according to the total travel time. In this study, both elitist preserving rule and the fitness ratio selection rule are used in order to maximize the chance of obtaining an optimal solution. The steps of selection are as follows:

- **Step 1 : Calculate the fitness values**
  In the initialization, calculate the fitness values in the generated chromosomes.

- **Step 2 : Preserving the good chromosomes**
  According to the fitness values, preserve the chromosomes in top \( x \% \) and these are sent to the next generation.

- **Step 3 : Reorganization of the chromosomes**
  The number of the preserving chromosomes in step 2 are included in the next generation, insufficient numbers of chromosomes, \( N = (pop\_size - (pop\_size \cdot x\%)) \) are generated randomly.
(3) Crossover Procedure

The crossover procedure generates new generated chromosomes by the one-point crossover procedure with the crossover probability \((p_c)\). The steps of the crossover are as follows:

- **Step 1**: Select the chromosome pairs with a predetermined probability
  
  \[
  A = (g_s[1] \text{ line}_s[1]) \quad g_s[2] \text{ line}_s[2] \quad \ldots
  \]
  
  \[
  g_s[m] \quad \text{ line}_s[m] \quad g_s[m+1] \text{ line}_s[m+1] \quad \ldots
  \]
  
  \[
  B = (g_b[1] \text{ line}_b[1]) \quad g_b[2] \text{ line}_b[2] \quad \ldots
  \]
  
  \[
  g_b[m] \quad \text{ line}_b[m] \quad g_b[m+1] \text{ line}_b[m+1] \quad \ldots
  \]

- **Step 2**: Select a common gene
  
  If \( g_s[m] = g_b[m] \),
  
  then common gene is set to \( g[m] = g_s[m] = g_b[m] \)

- **Step 3**: Swapping Procedure
  
  Swap the genes after the common gene as
  
  \[
  A' = (g_s[1] \text{ line}_s[1]) \quad g_s[2] \text{ line}_s[2] \quad \ldots
  \]
  
  \[
  g[m] \quad \text{ line}_s[m] \quad g_b[m+1] \text{ line}_b[m+1] \quad \ldots
  \]
  
  \[
  B' = (g_b[1] \text{ line}_b[1]) \quad g_b[2] \text{ line}_b[2] \quad \ldots
  \]
  
  \[
  g[m] \quad \text{ line}_s[m] \quad g_s[m+1] \text{ line}_s[m+1] \quad \ldots
  \]

- **Step 4**: Calculate the Travel Time
  
  Calculate the travel times of the chromosomes, \( A' \), \( B' \) as the initialization step.

(4) Mutation Procedure

The mutation procedure generates entire new chromosomes with a mutation probability \((p_m)\). The steps of mutation are as follows:

- **Step 1**: Determination of the location of the mutation
  
  Determine the location of the mutation in the gene of the chromosome.
  
  \[
  A' = (g_s[1] \text{ line}_s[1]) \quad g_s[2] \text{ line}_s[2] \quad \ldots
  \]
  
  \[
  g[m] \quad (\text{ line}_s[m] \quad g_s[m+1] \text{ line}_s[m+1] \quad \ldots
  \]
  
  If the mutation gene is determined as \( g[m] = g[m] * \)

- **Step 2**: Determine a next mutated gene
  
  If \( r = g[m] \) and \( a = (r, s) \), \( s \in X \) is determined,
  
  Select randomly \( s \in X \) and set \( g[m+1] = s \)

- **Step 3**: Determination of Line
  
  In \( a = a' \), \( l \in Y \) is determined,
  
  Randomly select \( l \in Y \) and then set \( \text{ line}_s[m] = l \)
• Step 4 : Iterations
  If  \( s = j \), then goto step 5,
  Otherwise,  set \( m = m + 1 \), and goto step 2

• Step 5 : Calculation of Travel Time
  Calculate the travel times of the generated chromosomes.

2.2. STOCHATIC NETWORK LOADING PROCEDURE

After obtaining multiple alternative shortest paths from origin to destination pairs, the transit demand is loaded on multiple shortest paths using a logit based network loading. The utility function of multiple shortest paths consists of only total travel time.

\[
V_k = \theta \times c_k
\]  

where, \( V_k \) = measured utility of path \( k \)
\( \theta \) = parameter
\( c_k \) = total travel time of path \( k \)

3. NUMERICAL EXAMPLE

The evaluation of the GA approach is conducted using a small example network shown in Figure 1. Table 1 gives the itinerary information, travel time, travel time variation and vehicle capacity and its speed in transit lines. Table 2 shows some timetable information in transit lines. As the Figure 1 shows, a small network that we used in this study has 25 nodes, 40 links and 4 lines. The transit lines have the shuttle service from origin to destination, and have 20 passengers demand. While line 1 and line 3 have high speeds, line 2 and line 4 have low speeds.

![Figure 1. Example network](image)
<table>
<thead>
<tr>
<th>Line</th>
<th>Itinerary (circulation)</th>
<th>Travel time (min)</th>
<th>Variation of Travel time</th>
<th>Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1</td>
<td>101 ↔ 106</td>
<td>2.832</td>
<td>0.450</td>
<td>Vehicle Capacity (seated) : 30</td>
</tr>
<tr>
<td></td>
<td>106 ↔ 111</td>
<td>3.132</td>
<td>0.900</td>
<td>Speed : 50km/h</td>
</tr>
<tr>
<td></td>
<td>111 ↔ 116</td>
<td>2.892</td>
<td>0.550</td>
<td></td>
</tr>
<tr>
<td></td>
<td>116 ↔ 121</td>
<td>2.604</td>
<td>1.175</td>
<td></td>
</tr>
<tr>
<td></td>
<td>121 ↔ 122</td>
<td>4.164</td>
<td>0.650</td>
<td></td>
</tr>
<tr>
<td></td>
<td>122 ↔ 123</td>
<td>2.460</td>
<td>0.850</td>
<td></td>
</tr>
<tr>
<td></td>
<td>123 ↔ 124</td>
<td>2.856</td>
<td>0.250</td>
<td></td>
</tr>
<tr>
<td>Line 2</td>
<td>101 ↔ 102</td>
<td>6.480</td>
<td>0.425</td>
<td>Vehicle Capacity (seated) : 25</td>
</tr>
<tr>
<td></td>
<td>102 ↔ 107</td>
<td>4.944</td>
<td>0.450</td>
<td>Speed : 25km/h</td>
</tr>
<tr>
<td></td>
<td>107 ↔ 112</td>
<td>6.262</td>
<td>0.900</td>
<td></td>
</tr>
<tr>
<td>Line 3</td>
<td>112 ↔ 113</td>
<td>3.420</td>
<td>0.600</td>
<td>Vehicle Capacity (seated) : 35</td>
</tr>
<tr>
<td></td>
<td>113 ↔ 114</td>
<td>2.580</td>
<td>0.475</td>
<td>Speed : 50km/h</td>
</tr>
<tr>
<td></td>
<td>114 ↔ 115</td>
<td>3.084</td>
<td>1.175</td>
<td></td>
</tr>
<tr>
<td></td>
<td>115 ↔ 120</td>
<td>3.156</td>
<td>1.200</td>
<td></td>
</tr>
<tr>
<td></td>
<td>120 ↔ 125</td>
<td>2.676</td>
<td>0.650</td>
<td></td>
</tr>
<tr>
<td></td>
<td>125 ↔ 124</td>
<td>2.412</td>
<td>0.375</td>
<td></td>
</tr>
<tr>
<td>Line 4</td>
<td>101 ↔ 106</td>
<td>5.664</td>
<td>0.250</td>
<td>Vehicle Capacity (seated) : 45</td>
</tr>
<tr>
<td></td>
<td>106 ↔ 111</td>
<td>6.264</td>
<td>1.175</td>
<td>Speed : 25km/h</td>
</tr>
<tr>
<td></td>
<td>111 ↔ 112</td>
<td>7.392</td>
<td>0.700</td>
<td></td>
</tr>
<tr>
<td></td>
<td>112 ↔ 117</td>
<td>5.832</td>
<td>0.300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>117 ↔ 118</td>
<td>5.400</td>
<td>0.275</td>
<td></td>
</tr>
<tr>
<td></td>
<td>118 ↔ 119</td>
<td>5.856</td>
<td>0.300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>119 ↔ 124</td>
<td>5.616</td>
<td>0.800</td>
<td></td>
</tr>
</tbody>
</table>

- Origin Node: 101, Destination Node: 124, Starting time of trip: 7:00, Demand: 20
- Parameter in GA: population size(pop_size)=10, max generation(max_gen)=30, crossover probability ($p_c$)=0.3, mutation probability ($p_m$)=0.2, rate of elite chromosome(x)=10%
- Timetable of line 1, line 2, line 4 at node 101 is given in Table 2. The line 1 starts at 7:00 and then in every 20 minutes, the line 1 is operated. In line 2, it starts at 7:05 and then in 10 minutes, another service is operated. Line 4 starts at 7:15 and then at 7:25 the next service begins.
As a result of the optimal path calculation using the algorithm, path 1 and path 2 have been founded since the predetermined number of the shortest path are set to two. The optimal path searches are carried out in the aspects of the capacity consideration when it calculates the travel time. The detailed components of the path is as follows:

When the vehicle capacity is considered, the optimal paths are calculated in the detailed components of the total travel time as:

- **Optimal Paths: upper 2 paths**
  - **Path 1: 101 (1) 106 (1) 111 (1) 116 (1) 121 (1) 122 (1) 123 (1) 124**
    - waiting time : “0” (Starting time of trip is 7:00 and starting time of Line 1 at node 101 is 7:00)
    - travel time : 22.23min (Calculated from network)
    - transfer time : “0” (No transfer occurs.)
    - variable time : 10.89min (Calculated from network)
    - total travel time : 33.12min
  - **Path 2: 101 (2) 102 (2) 107 (2) 112 (3) 113 (3) 114 (3) 115 (3) 120 (3) 125 (3) 124**
    - waiting time : 5 min (Starting time of trip is 7:00 and starting time of Line 1 at node 101 is 7:05)
    - travel time : 36.51 min (Calculated from network)
    - transfer time : 5 min (One transfer occurs.)
    - variable time : -10.95 min (Calculated from network)
    - total travel time : 35.56min

When the vehicle capacity is not considered, the optimal paths are calculated in the detailed components of the total travel time as:

- **Path 1: 101 (1) 106 (1) 111 (1) 116 (1) 121 (1) 122 (1) 123 (1) 124**
  - waiting time : “0” (Starting time of trip is 7:00 and starting time of Line 1 at node 101 is 7:00)
  - travel time : 20.94min (Calculated from network)
  - transfer time : “0” (No transfer occurs.)
  - variable time : 10.89min (Calculated from network)
  - total travel time : 31.83min
- **Path 2: 101 (2) 102 (2) 107 (2) 112 (3) 113 (3) 114 (3) 115 (3) 120 (3) 125 (3) 124**
  - starting time “0” means “7:00”
  - so, starting time “78” means “8:18”

\[\text{Table 2. Timetable of line 1, line 2, line 4 at node 101}\]

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline
\text{Node} & \text{Line} & 0 & 20 & 41 & 60 & 78 & 100 & 120 & 145 & 162 \\
\hline
101 & 1 & 0 & 20 & 41 & 60 & 78 & 100 & 120 & 145 & 162 \\
2 & 5 & 15 & 24 & 35 & 44 & 55 & 64 & 75 & 83 \\
95 & 104 & 115 & 124 & 135 & 144 & 154 & 165 & 175 \\
4 & 15 & 25 & 34 & 46 & 55 & 65 & 76 & 85 & 94 \\
105 & 114 & 126 & 135 & 145 & 156 & 166 & 175 & 187 \\
\hline
\end{array}
\]

*Start time “0” means “7:00”
So, start time “78” means “8:18”*
Assignment Result

According to the travel times calculated in the shortest path search algorithm, the travel demand is assigned between the path 1 and 2 by the logit model as in Figure 2.

As above the figure shows, trips are assigned not to single path but two paths due to predetermined setting of the number of the shortest path calculation in the model. This is the main advantage of the genetic algorithm to find alternative shortest path set and these characteristics of the genetic algorithm lead the stochastic network loading easily.

4. SUMMARY

In this paper, a transit assignment model has been developed on stochastic networks, with considering vehicle capacity and scheduled timetable. The stochastic network based transit assignment model is appropriate to apply for bus and metro networks altogether since it can distinguish between uncertain and reliable travel times characteristics by imposing stochastic parameters on links or lines. The vehicle capacity constraint based model is useful to evaluate some alternative operational plans and policies in transit networks. On the other hand, the scheduled timetable based model is necessary to evaluate detailed transit operations by calculating personal route choice based on the exact timetables of the transit operations. As a result of this we can assign the passengers in more realistic representation.

We have used a genetic algorithm in order to develop the transit assignment model. The characteristics of the genetic algorithm makes easy to consider the properties of public transport. The genetic algorithm has been developed for the shortest paths finding problem under the stochastic networks with vehicle capacity and scheduled timetable. After the shortest paths have been obtained in the genetic algorithm, the multinomial logit model has
been used to obtain the assigned volumes on the transit lines and networks. In the utility of the alternative paths is composed of a sequence of link travel times from origin to destination.

We used the small network to evaluate this transit assignment model in this paper. In the small network, the results show that this transit assignment model describes the properties of public transport well. In particular, the model calculates some measures using the important aspects in public transport modeling such as timetable information, in-vehicle travel time, exact waiting and transfer time, vehicle capacity and travel time variations. In practice, most of the public transport networks are large-scale network. Therefore, we should apply this transit assignment model to large-scale network and verify the results.

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