A NEW METHODOLOGY TO OVERCOME MEMORYLESS PROPERTY OF CAR-FOLLOWING MODELS

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Abstract: A number of different types of car-following models have been developed and continuously refined up to the present time, with various different approaches of describing a relationship between the leader and the following vehicles. However, current car-following models are "memoryless" in the sense that the current state (which consists of instantaneous relative spacings and time-derivatives thereof) between the lead and the following vehicles entirely determines the future state of the following vehicle, with no dependence on the past sequence of events that produced that state. This study aims to propose a new methodology that considers the past sequences of car motions to predict the following vehicle's behavior and to overcome the memoryless property of previous models on car-following behavior.

Key Words: Car-following models, memoryless property

1. BACKGROUND

A variety of theoretical approaches and mathematical descriptions have been applied to the study of traffic flow on a roadway. Car-following is one of them, with an assumption that the motion of each car in the line of traffic depends on that of the car in front of it under high density conditions. The behavior of single-lane no-passing traffic can then be analyzed in terms of the motions of individual cars in the line of traffic. Car-following models have been used, for almost half a century, to describe the process of driver behavior while following each other in the traffic stream. Of course, a complete description of dense traffic requires consideration of other details, such as lane-changing, effects of adjacent vehicles, etc. This paper is concerned mainly with the following component of driver activity, although we discuss later how certain events outside this realm might have an influence.

A car-following model was first proposed by Reuschel (1950) and Pipes (1953) and was greatly extended by Herman et al. (1959~1967) and has been continuously refined up to the present with various approaches of describing a relationship between the leader and the following vehicles. One of the earliest studies is based on the goal of avoiding collisions or maintaining safe following distances (Pipes, 1953). He assumed that the movements of the various vehicles of the line obey a following rule suggested by a "rule of thumb" frequently taught in driver training, which is to allow one additional length of a car in front for every ten miles per hour of speed. By the application of this postulated following rule, he proposed a simple linear equation in which the car spacing is a linear function of speed of the following vehicle.

From the perspective of driving in expectation of a "brick wall stop," where one imagines the lead vehicle to stop instantaneously (a frequent worst case scenario considered in these models), the spacing rule for this model seems dubious, as simple physics suggests that stopping distance should vary with the square of vehicle speed, rather than linearly. Kometani and Sasaki (1959) modified the linear model to accommodate that consideration - the car spacing is expressed by a quadratic relation of speeds of the lead and following vehicles. They introduced a time lag T (reaction time of a driver) in the model and assumed that a driver chooses his velocity at time $t+T$ based on the spacing observed at an earlier time t.

Notwithstanding its obvious flaws, Zhang and Kim (2001) recently modified Pipes' model to reproduce both the so-called "capacity drop" and "traffic hysteresis" phenomena, which some experimenters have deduced from field data. They postulated that cars are always in one of three phases - acceleration, deceleration and coasting. The time gap, which the original model assumed to be constant, was made a function both of vehicle spacing and of which phase a driver was in.

Several different levels of car-following models were developed by Herman et al. (1959~1967), cooperating with the General Motors research team. They all have the same basic form: *Response = function (stimuli, sensitivity)*, where response is acceleration or deceleration of the following vehicle and the stimuli are the relative velocity between the lead and following vehicles. The various modifications of the sensitivity term led to a generalized form of car-following models that is the final form developed by General Motors' research team. Most of the previous models are special cases of this generalized model:

$$\ddot{x}_{n+1}(t+T) = \frac{\alpha_{l,m}[\dot{x}_{n+1}(t+T)]^m}{[x_n(t) - x_{n+1}(t)]^l}[\dot{x}_n(t) - \dot{x}_{n+1}(t)] \tag{1}$$

where $x_{n+1}(t)$ and $\dot{x}_{n+1}(t)$ are the position and speed of the following vehicle at time t, T is a reaction time, α is a sensitivity parameter, and l and m are constants to be calibrated. Note that the magnitude of the response is directly proportional to the relative velocity at the time of observation (before the lag), and to the instantaneous velocity of the following vehicle at the time of actuation (after the lag) with some exponent to be calibrated.

The series of General Motors car-following models do not consider the effect of the inter-vehicle spacing independently of the relative velocity; i.e., drivers will always accelerate if the relative velocity is positive and decelerate if the relative velocity is negative. However, in real driving, it is often seen that when the following vehicle is some reasonable distance behind the leading vehicle, the following vehicle can accelerate even if the relative velocity is zero. One might circumvent this criticism by claiming that this would not happen in high

density traffic; however, due at least in part to lane-changing and a mix of vehicle types, such larger-than-average gaps frequently appear even in rush-hour traffic. Hence, Low et al. (1998) proposed a model adding a nonlinear spacing-dependent term to the GM non-linear car-following model developed by Gazis et al. (1961).

Gipps (1981) claimed that the parameters $\alpha_{l,m}$, l and m in the generalized form of General Motors car-following models have no obvious connection with identifiable characteristics of drivers or vehicles, and argued that the parameters in a model should correspond to obvious characteristics of drivers and vehicles. Hence, he proposed a model for the response of the following vehicle based on the assumption that each driver sets limits to his desired braking and acceleration rates, and then uses these limits to calculate a safe speed with respect to the preceding vehicle. He conducted simulation exercises using the parameters, which correspond to characteristics of driver behavior and showed that when realistic values are assigned to the parameters in a simulation, the model could reproduce the characteristics of real traffic flow.

An obvious critique of the above models is that they assume that all drivers follow the same driving rules. In fact, these rules may differ with different drivers, or even for the same driver and with different conditions, and in fact, possibly with the same driver and nearly identical situations. Subramanian (1996) and Ahmed (1999) extended the GM non-linear car-following model (Gazis et al., 1961) by assuming that the reaction time is a function of factors (e.g., age, gender, weather conditions, geometry, vehicle type and traffic conditions) modeled by a truncated log-normal random variable. We share with these authors the sense that there are significant stochastic components missing from other car-following models. However, there is a difference between injecting some randomness into the model, and doing so in a way that properly captures stochastic effects. For example, simply turning a constant parameter into a random variable, yet still assuming that, once sampled, it will remain the same for that driver throughout the journey, does not capture the incomplete consistency that has been observed within drivers for similar stimuli. Further, this retains the myopic character of the model and does not allow for the kind of longer-term, evolving maneuvers that we feel are a critical component of actual driving. In particular, the use of a simple study site and myopic data collection schemes prevents (unfairly) this variation from even being observed in the first place.

Kikuchi et al. (1992, 1999) also recognized that the reactions of the following vehicle to the lead vehicle might not be based on a deterministic relationship, but rather on a set of approximate driving rules developed through experience. Their approach to modeling these rules consisted of a fuzzy inference system with membership sets that could be used to describe and quantify the behavior of following vehicles. However, the logic to define the membership sets is subjective and depends totally on the judgment and approximation of the researchers. No field experiments were conducted to calibrate and validate these fuzzy membership sets under real driving conditions. While we agree with the premise of the paper, and seek to resolve similar issues, the methodology employed does not warrant any further explanation. Some researchers may argue over semantics, but while fuzzy logic may offer a different way of thinking about a problem, any results could have been derived from classical probability theory as well (Ruspini, 1996).

As mentioned above, a number of different approaches have been adopted and developed to represent the logical process of the following vehicle's response to the action of the lead vehicle. However, current car-following models are "memoryless" in the sense that the

current state (which consists of instantaneous relative spacings and time-derivatives thereof) between the lead and the following vehicles entirely determines the reaction of the following vehicle, with no dependence on the past sequence of events that produced that state. In fact, however, for a given instantaneous state, the most natural following response might differ, depending on how that state was reached. Hence, current car-following models that only consider the current states to predict the following vehicle's behavior might have difficulties representing real car-following behavior under various maneuvers (or scenarios) that might unfold over an interval of time.

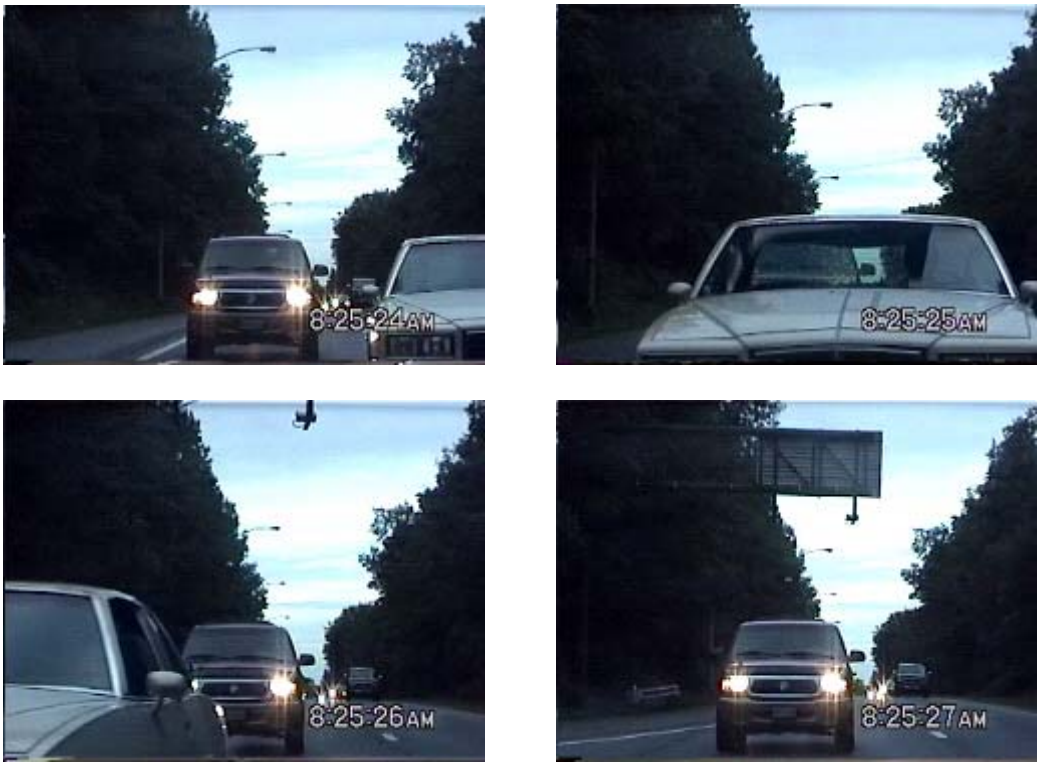
Hence, the objective of this research is to propose a new methodology that considers the past sequences of car motions to predict the following vehicle's behavior, and that overcomes the memoryless property of previous models on car-following behavior. The memoryless property of previous car-following models is described in the second section. A new methodology that adopts a pattern recognition method to represent car-following behavior follows in the third section. The fourth section describes the comparison of performance between the General Motors car-following model and the pattern recognition model with an imaginary time-series data set generated on the basis of driving patterns observed under car-following situations. Finally, some conclusions and future studies are mentioned.

2. MEMORYLESS PROPERTY OF PREVIOUS CAR-FOLLOWING MODELS

A common assumption in car-following models is that the reaction of a following vehicle at time $t+T$ depends only on the situation at an earlier time instant t. Hence, current car-following models treat the dynamic evolution of cars at a given state as a memoryless process; i.e., the current state between the lead and the following vehicle entirely determines the future state of the following vehicle, with no dependence on the past sequences of car motions that produced the current state (in existing car-following models, the state includes only instantaneous conditions, not historic ones).

In fact, for certain instantaneous states, the most natural following responses might differ, depending on how those states were reached. For example, a following driver might not respond promptly to maintain his or her desired distance when the lead vehicle has recently cut into the lane where the following vehicle is driving. If it is clear that the new leader can accelerate and increase the spacing unilaterally, or that an additional lane change may be imminent, the follower may choose simply to wait cautiously without taking any evasive action, whereas current simple state-based models (with a myopic definition of the state) would predict a nearly immediate and probably aggressive deceleration maneuver.

Figures 1 shows typical examples that were observed under real car-following situations. The first example (a) shows a case in which the following vehicle did not respond promptly even when the new lead vehicle cut into the driving lane of the following vehicle and left the lane immediately. The driver of the following vehicle stayed with his own speed without taking any aggressive reaction. The second example (b) shows a similar maneuver of the following vehicle to a different action of the lead vehicle. In this case, the new leader cut into the driving lane of the following vehicle and stayed in the lane. However, the follower did not respond evasively, but drove carefully without making any immediate deceleration.



(a) The lead vehicle cuts in and leaves

(b) The lead vehicle cuts in and stays

Figure 1. Examples under real car-following situations

Actually, Chandler et al. (1958) first introduced the idea that the response of the following vehicle depends not only on what the relative speed was at a certain earlier instant, but rather on its time history. Hence, they extended the simple linear car-following model in which the acceleration of the following vehicle is proportional to the relative speed between the lead and the following vehicles, by adding a weighting function (Lee (1966) called it a "memory function") that is intended to represent the way in which the following driver responds to the information he or she has received from the lead vehicle over some time interval. Therefore, the stimulus at a given time, t, depends on the weighted sum of earlier values of the relative speed.

$$\ddot{x}_{n+1}(t+T) = \int_{t-\Delta t}^{t} [\dot{x}_n(z) - \dot{x}_{n+1}(z)] h(z-t)\, dz \tag{2}$$

where $\ddot{x}_{n+1}(t+T)$ is the acceleration or deceleration of the following vehicle at time (t+T), $\dot{x}_n(t)$ and $\dot{x}_{n+1}(t)$ are the velocity of the lead and the following vehicle at time t respectively, and Δt is the interval over which memory is applied, and $h(t)$ is the weighting function defined on [- Δt ,0].

Lee (1966) investigated the consequences of applying several examples of possible weighting functions such as the Dirac delta function, decaying exponential function, and the Heaviside step function. Obviously, the simple linear car-following model developed by Chandler et al. (1958) is a special case of the more general theory when the Dirac delta function is used as the weighting function and the impulse occurs at time t only. However, no data collection results were shown to determine the appropriate form of the weighting function, or to confirm that this is indeed a reasonable model. Subsequently, Darroach and Rothery (1972) illustrated how one might estimate the memory function, and in the special case when it is proportional to the Dirac delta function, the reaction time and sensitivity parameters of the basic linear model, using Fourier analysis techniques. However, they did not deal with the question of the accuracy of the estimated weighting function and no further research has been made to compare it with real car-following behavior. This type of approach seems to hold some promise, at least to the extent that it attempts to resolve some of the critiques of models we have presented so far. Unfortunately, the idea of using time-series history to predict the response of the following vehicle has had rather little influence on later developments in car-following models.

3. NEW METHODOLOGY: PATTERN RECOGNITION

It has been shown that drivers tend to retain their personalities, in the sense that each driver tends to maintain his driving attributes, and in some instances, drivers return to their nominal attributes after being forced by a traffic disturbance to alter them temporarily (Cassidy et al., 1998). Hence, we might assume that driving patterns such as acceleration and deceleration profiles are expected to remain the same for each driver (of course, there is stochastic variation across drivers). Hence, car following behavior for drivers may not be completely random in nature. There should be some patterns in which the past car-following behavior repeats itself.

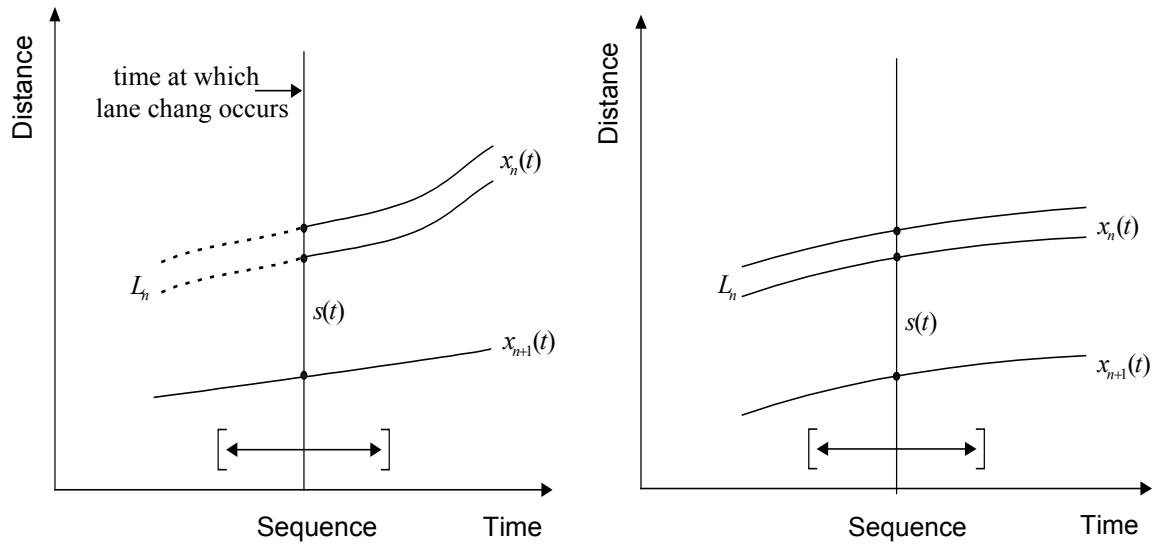
It is possible to represent the following driver's behavior that unfolds over a sequence of time through a pattern recognition (or matching) method that correlates current time series states with historical data for making future predictions. This method is one of the newer methods of forecasting and there are a number of applications which are still being explored, in the

areas of medical diagnosis; syntactic, textile, speech, and face recognition; signal processing; etc.

3.1 Establishment of driving patterns

To begin with, it is essential to classify maneuvers (or scenarios) that might unfold over an interval of time, that are common in congested traffic (i.e, lane changes), and that following vehicles might respond to in unique ways, and then to develop a new (set of) car-following model(s). We use the maneuvers (or scenarios) as patterns here.

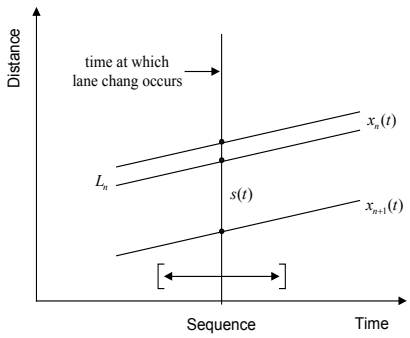
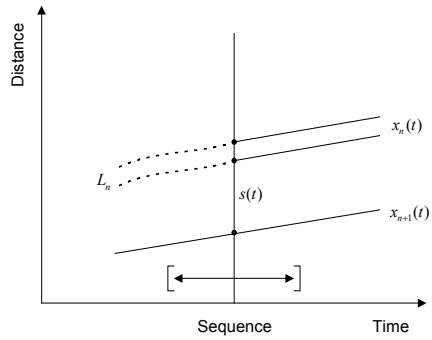
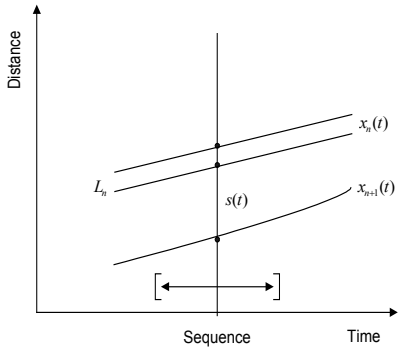
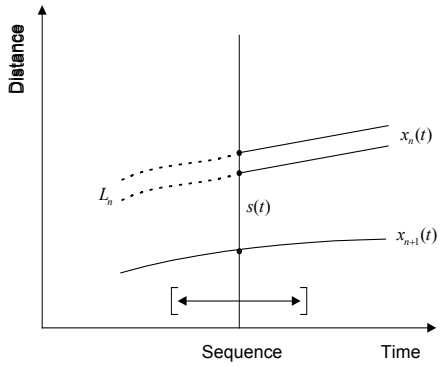
Figure 2 shows two examples where the same state might be produced under very different causes, and hence very different effects might be expected. In the first case, the new lead vehicle labelled n changes lanes in front of the subject car (vehicle labelled n+1), and acquires some relative position and velocity thereby. The vehicle then accelerates, prompting no response from the following vehicle to maintain his or her desired distance. In the second case, if the same spacings and speeds had resulted from a deceleration event, the follower might be inclined to decelerate immediately. This is only a single simple example included for purposes of illustration. Table 1 shows some possible maneuvers which might be observed under real car-following situations. The table is divided into two types of car-following situations; normal and lane-changing situations, where $x_n(t)$ and $x_{n+1}(t)$ are the lead and following vehicles at time t respectively, L_n is the length of the lead vehicle and $s(t)$ is the spacing between vehicles.

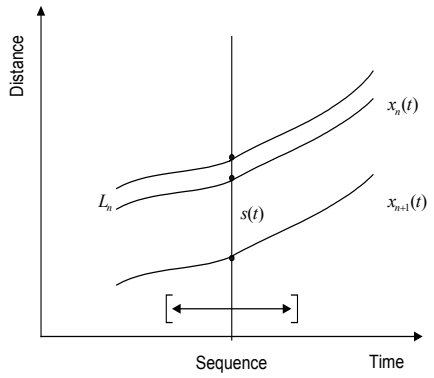
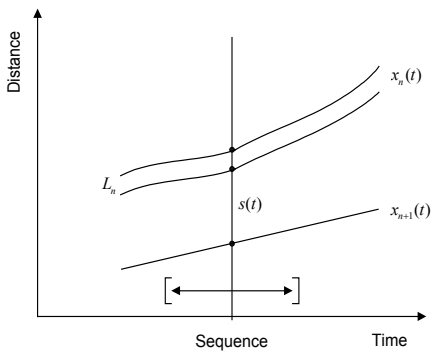
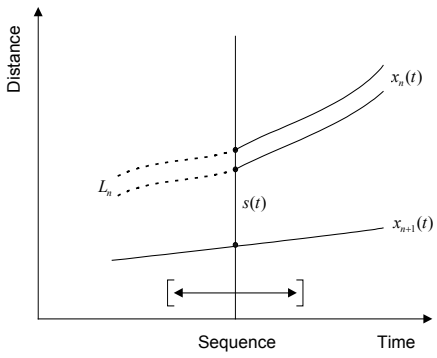
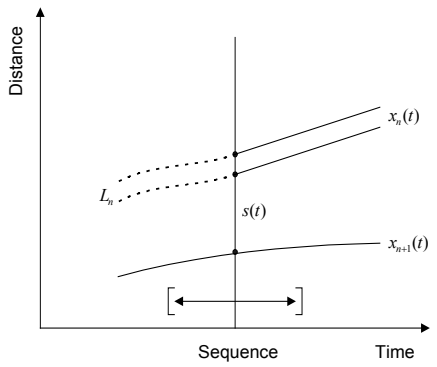


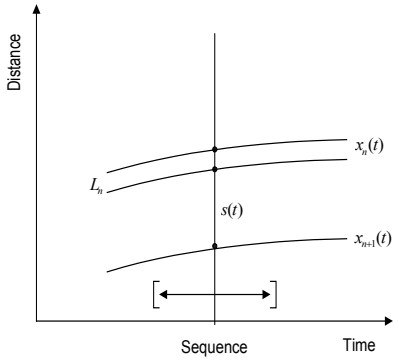
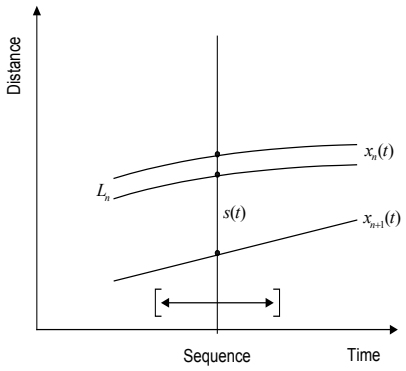
(a) Lane-changing maneuver (b) Deceleration maneuver

Figure 2. Examples of different responses to the same instantaneous state

Table 1. Some possible maneuvers (or scenarios) under car-following situation

Case	Normal situation		Lane-changing situation	
	Lead vehicle Action	Following vehicle reaction	Lead vehicle action	Following vehicle reaction
1	Constant	Constant	Constant	Constant
				
2	Constant	Acceleration	Constant	Acceleration
			-	
3	Constant	Deceleration	Constant	Deceleration
	-			

4	Acceleration	Acceleration	Acceleration	Acceleration
			-	
5	Acceleration	Constant	Acceleration	Constant
				
6	Acceleration	Deceleration	Acceleration	Deceleration
	-			

7	Deceleration	Deceleration	Deceleration	Deceleration
			-	
8	Deceleration	Constant	Deceleration	Constant
			-	
9	Deceleration	Acceleration	Deceleration	Acceleration

Note: shaded cases are very rare cases under real car-following situation.

3.2 Development of pattern recognition algorithm

To develop a pattern recognition algorithm for car-following situations, consider a discrete time-series $y = \{y_1, y_2, ..., y_n\}$ (e.g., relative spacings) and their corresponding time-series $y' = \{y'_1, y'_2, ..., y'_n\}$ (e.g., speed of following vehicle). The main purpose of a pattern recognition algorithm is to find the "closest" value of the current state y_n in the past data, say y_j, and then to predict y_{n+1} on the basis of y_{j+1} and estimate the corresponding value y'_{n+1} on the basis of y_{n+1}. The definition of the current state of the time-series can be extended to include several consecutive values $\{y_k, y_{k+1}, ..., y_{n-1}, y_n\}$ for some k such that $1 \le k \le n$. A segment in the series is defined as a difference vector $s = (s_1, s_2, ..., s_{n-1})$ where $s_i = y_{i+1} - y_i, 1 \le i \le n-1$. The change in the time-series, which is defined as d_i, may be encoded as a 0, 1 or 2 where $d_i = 0$ if $y_i > y_{i+1}$, 1 if $y_i < y_{i+1}$ and 2 if $y_i = y_{i+1}$ to define the direction in any pattern. Hence, a pattern in the time-series can be represented as $p_d = (d_{n-k}, ..., d_{n-1})$, a vector of 0's, 1's, and 2's.

In this paper, we adopted the above method suggested by Singh et al. (1998, 2000) for general pattern recognition to recognize various driving patterns under car-following situations, and modified it according to our research needs. Of course, real traffic situations unfold over continuous time; however, any sampling method is necessarily discrete, which agrees with our use of discrete time-series above. The overall procedure is shown as the following algorithm and a flowchart in Figure 3.

Step 1: Start with a pattern of minimal state size *k* and the pattern direction, i.e., $p_d = (d_{n-k}, ..., d_{n-1})$

Step 2: Search the time-series $(d_1, ..., d_{n-k-1})$ to find the closest match for $p_d = (d_{n-k}, ..., d_{n-1})$. Suppose that the closest match is $p'_d = (d_{j-k}, ..., d_{j-1})$. Hence, corresponding segment lengths for p_d and p'_d are $(s_{n-k}, ..., s_{n-1})$ and $(s_{j-k}, ..., s_{j-1})$ respectively.

Step 3: If d_j = 1 then predict high

$y_{n+1} = y_n + \alpha * s_j$ where α is scaling factor, $\alpha = \frac{1}{k}\sum_{i=1}^{k}\frac{s_{n-i}}{s_{j-i}}$

If d_j = 0 then predict low

$y_{n+1} = y_n - \alpha * s_j$ where α is scaling factor , $\alpha = \frac{1}{k}\sum_{i=1}^{k}\frac{s_{n-i}}{s_{j-i}}$

If d_j = 2 then

$y_{n+1} = y_n$

Step 4: Estimate the corresponding value y'_{n+1} on the basis of y_{n+1} and minimize the mean squared error (MSE) by optimizing pattern size. Repeat step 1 to 4 for patterns of size *k* = *k*+1, *k*+2, … Finally, choose the optimal pattern recognition model which yields minimal error.

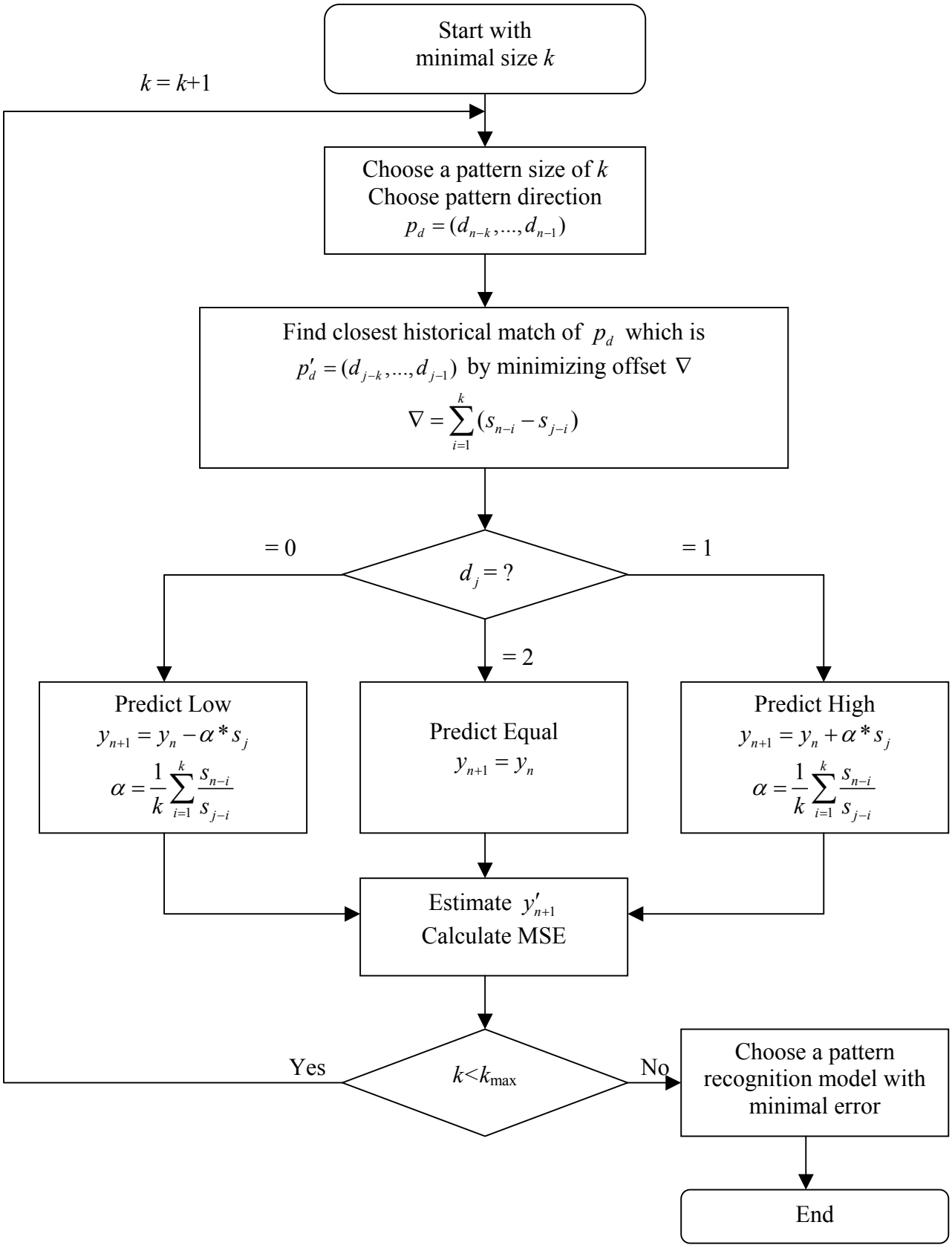


Figure 3. Flowchart of pattern recognition algorithm

4. MODEL PERFORMANCE

We compared the performance of the proposed pattern recognition model with the well-known General Motors car-following model developed by Chandler et al. (1958) by investigating the MSE between actual and predicted response of the following vehicle, for an imaginary time-series data set. This data set was generated on the basis of some possible driving maneuvers shown in Table 1, which might be observed under real car-following situations and the preliminary survey results shown in Figure 1.

The General Motors model is a simple linear model with an assumption that the response of the following vehicle is proportional to the stimuli (spacing or relative speed) between the lead and following vehicles, and has the following form:

$$\dot{x}_{n+1}(t+T) = \alpha[x_n(t) - x_{n+1}(t)] \tag{3}$$

where $x_{n+1}(t)$ and $\dot{x}_{n+1}(t)$ are the position and speed of the following vehicle at time t, T is a reaction time and α is a sensitivity parameter.

The reason we chose the General Motors car-following model to compare results with is that the model has been a starting point of more complicated models and has had significant influence on later developments in simple state-based car-following models (with a myopic definition of the state).

4.1 Comparison of results

As we mentioned above, an imaginary time-series data set was generated with an assumption that the car-following behavior consists of a series of driving patterns and that the past car-following behavior repeats itself. Of course, this is an assumption that ensures that the artificial data lend themselves to pattern recognition, so it should not be surprising if the method works well. What remains to be seen, then, is if such patterns can in fact be discerned in real traffic data, which is a subject of our ongoing research. Figure 4 shows the time-series car-following data, i.e., the following vehicle's speed versus relative spacing between the lead and the following vehicles over 120 seconds. This data set is an example of dynamic driving behavior of the following vehicle (e.g., cut-in or lane changing behavior of a new lead vehicle was assumed to have occurred around the time of 45 and 105 seconds), and it contains similar driving patterns in it.

In order to use the pattern recognition model for forecasting the response (speed) of the following vehicle based on the past sequences of car motions (relative spacings), the car-following time series data set is divided into two parts: estimation data set (0 to 99 seconds) and test data set (100 to 120 seconds). The pattern recognition algorithm uses the estimation data set for matching current patterns with historical patterns. The General Motors model uses 1 second as the reaction time for ease of calculations, 0.5 as the sensitivity parameter value (which is slightly higher than the average value of 0.37 found in the field experiments) and the single relative spacing value at the time of 100 seconds for forecasting the rest of the trajectory of the following vehicle.

Figure 5 shows the actual and predicted speed profile of the following vehicle using the pattern recognition model with optimal size of patterns, $k = 3$, and the General Motors model. We observe from the figure that the pattern recognition model is highly successful in

predicting the response of the following vehicle compared with the General Motors model. Moreover, it was found that the General Motors model, which exhibits the memoryless property predicted an aggressive response to the loss of spacing in front of the following vehicle around the time of 105 second when the lane changing behavior of a new leader vehicle was assumed to have occurred.

Additionally, the calculated Mean Square Error (MSE) between actual and predicted response of the following vehicle for the pattern recognition and General Motors models are 2.16 and 14.12 km^2/h^2, respectively.

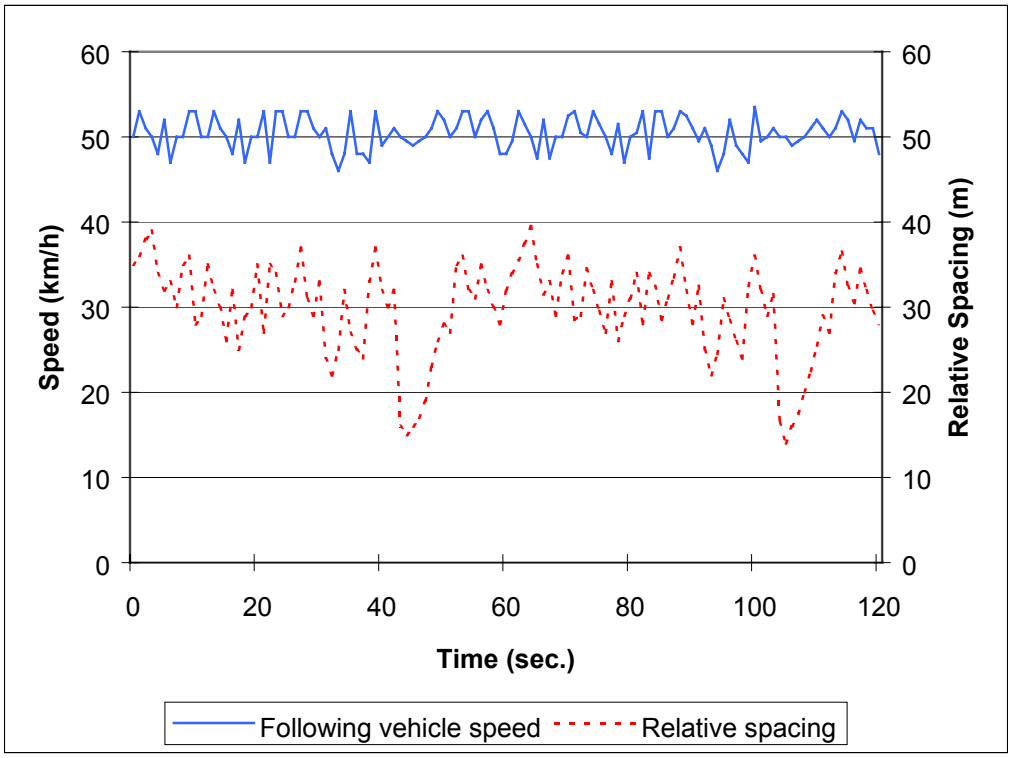


Figure 4. Car-following data on the following vehicle's speed and relative spacing

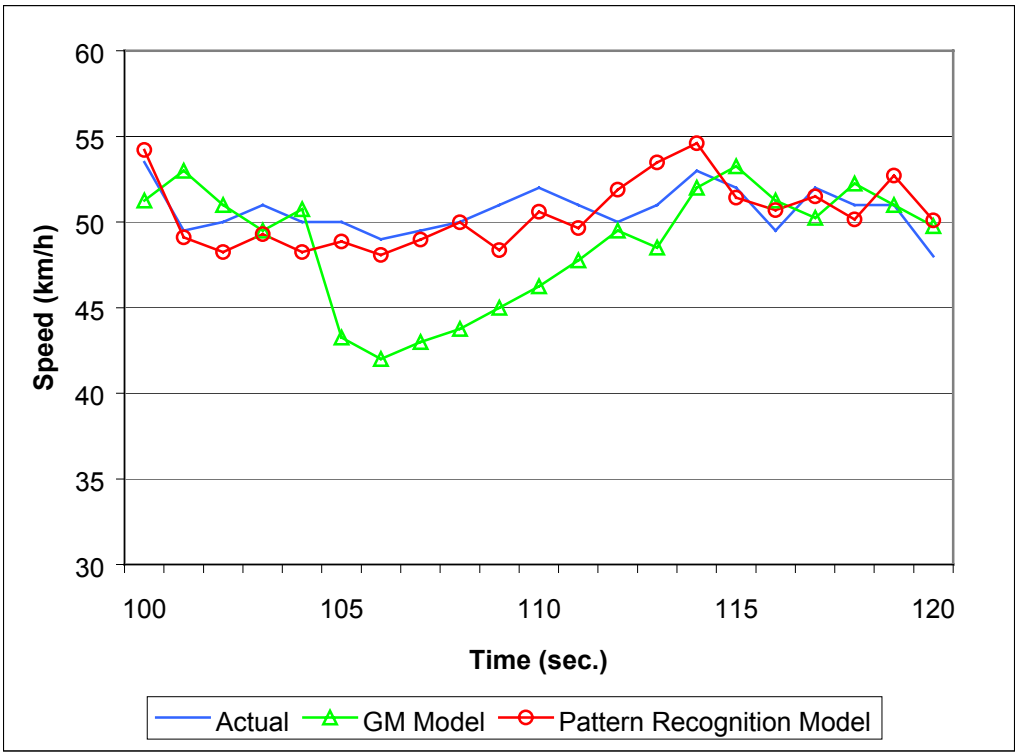


Figure 5. Comparison of speed of the following vehicle

5. CONCLUSIONS AND FUTURE STUDIES

Car-following logic has been a core of all microscopic traffic simulation models. A number of types of car-following models have been developed and continuously refined up to the present with various different approaches of describing a relationship between the leader and the following vehicles. However, current car-following models are "memoryless" in the sense that the current state between the lead and the following vehicles entirely determines the future state, i.e., the reaction of the following vehicle, without considering the past sequence of car motions that produced that state. In this paper, we show how a simple pattern recognition procedure can help to overcome this problem. It was found that the pattern recognition model is highly successful in predicting the response of the following vehicle. Additionally, the MSE between actual and predicted response of the following vehicle is much smaller than the linear General Motors car-following model. In the near future, further studies based on real car-following data under naturalistic driving conditions will be exploited to test these results further with other more complicated models, and more importantly, to attempt to find such patterns in real traffic data. Furthermore, more realistic pattern recognition models incorporating random variations within drivers will be pursued. We hope that this paper is a good platform for the development of more realistic car-following models and that it will help improve the realism of microscopic traffic simulation models, for which car-following logic is the core.

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