NON-LINEAR CAR FOLLOWING MODELS INCORPORATION SECOND LEADING CAR AND EXCESS CRITICAL SPEED

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Abstract: Generally the conventional car-following model presumes that respond is a function of stimulus and sensitivity term. The stimulus comes from relative speed between two successive vehicles while the sensitivity depends on speed and headway. In this study the extended model which includes the second leading car, the excess critical speed and the acceleration of the car ahead gives a better regression fit than the conventional model. The significant difference in reaction time of three dissimilar conditions in driving process; acceleration, deceleration and cruising, causes erroneous estimation when using an average reaction time. The model result gives the realistic reaction time with smaller variance in the modified excess critical speed model than in modified second leading cars model.

Key Words: car following theory, second leading car, excess critical speed

1. INTRODUCTION

A microscopic simulation plays an important role in transportation planning and traffic operation management. Generally it is used for verifying new traffic management strategies, evaluating the effectiveness of proposed policies for transportation management, and developing advanced traffic control system such as traffic-responsive control system and dynamic route guidance system.

As a key element of a microscopic simulation, a car following model is used for describing the driving maneuvers which eventually happen on a single lane or multiple lanes without any lane-changing condition. The car following model has been continuously evolved to explain the inter-relation between leaders and successive vehicles. The outstanding car following models can be described in three different concepts; stimulus-response model, collision avoidance model, and cellular automation model. This study mainly focuses on stimulus-respond model; however, some extended models incorporate the concept of collision avoidance model.

The novel car following theory was initiated by Pipe (1953) and Forbes et al. (1958) which was derived from the minimum safety distance concept. Subsequently the speed-spacing model was originally created by Hermann et al. (1959), drivers were assumed to change their driving speed in related to the spacing from vehicles ahead. Many explanatory variables are included in the linear model to govern the unexplainably high variations in calibrated parameters. Accordingly many nonlinear formulas were proposed with relatively high correlation from outstanding field experimental data.

The linkage between the microscopic and macroscopic part was determined by including the driver speed and the spacing into the simple linear model. (Edie, 1961) Finally, the generalized form of stimulus-response model was formed up as in Equation (1). The drivers are supposed to react to change speed in related to that of the car in front after a time lag, known as reaction time. (Gazis et al., 1961) The conventional car following parameters (α, l, m) and reaction time (T) were analyzed on various basis, like traffic flow condition (congested and uncongested) and driving phases (acceleration, deceleration and steady state) (Ceder and May, 1976, Ozaki, 1993)

$$\ddot{x}_n(t+T) = \frac{\alpha\left[\dot{x}_n(t+T)\right]^m}{\left[x_{n-1}(t) - x_n(t)\right]^l}\left[\dot{x}_{n-1}(t) - \dot{x}_n(t)\right] \tag{1}$$

Kometani and Sasaki (1958) and Gurusinghe et al. (2001) introduced a linear car following model incorporating collision avoidance concept, as in Equation (2) to (4). It was assumed that the followers considered safe speed with respect to the maximum rate of deceleration (f) and the available space, known as headway, in making driving decisions. This model results in reduction of acceleration rate whenever the driving speed exceeds the critical speed. The relationship of reaction time and sensitivity parameters were presented in an exponential function.

$$\ddot{x}_n(t+T) = \alpha_0 + \alpha_1[ECS] + \alpha_2\left[\dot{x}_{n-1}(t) - \dot{x}_n(t)\right] \tag{2}$$

$$ECS = v_{CR} - \dot{x}_n(t) \tag{3}$$

$$v_{CR} = \sqrt{2f[x_{n-1}(t) - x_n(t)]} \tag{4}$$

The additional terms founded on the extended car following model are an acceleration of leading vehicle (Kometani and Sasaki, 1958), the variable from the second leading car (Herman, 1959). The acceleration was integrated in the model because some level of acceleration remained even though the relative speed was zero. In addition, when the second leading car was added in the model, the asymptotic stability can be maintained because of smaller value of sensitivity term. (Herman, 1959) With a highly accurate data obtained from Real Time Kinematics Global Positioning Systems (RTK-GPS), the good fitness is expected from recalibrating extended car following models.

Although linearization of nonlinear model satisfied with the approximation in numerical analysis in comparison to the theoretical analysis (Zhang and Jarrett, 1997), some samples are eliminated during the logarithmic transformation because of an arithmetic error. As a shortcoming of conventional model structure, when two vehicles are closing but the relative speed is still positive, the model results in acceleration; on the other hand when the two vehicles are separating but the relative speed is still negative, the model results in deceleration. Subsequently, this kind of problem can be solved by an application of Fuzzy logic (Kikuchi et al., 1992). In order to simplify solving method, another approach proposed in this study is nonlinear optimization solver.

Since the difficulty in calibration process was found in complicated model, the inter-relationship between reaction time, parameters and some variables were examined. (Gurusinghe et al., 2001) Therefore the parameters can be reproduced when randomly generating from known distribution. G.F. Newell (2000) stated that the time headway and distance headway considerably varied from vehicle to vehicle; however, these variables may conform to some joint probability density function.

The objectives of this study are: (1) to determine the reaction time and parameters simultaneously by nonlinear optimization, (2) to modify the extended car following model, (3) to analyze the reaction time in each driving condition and (4) to evaluate the extended car following model relative to the conventional model.

The experimental data of ten-car platoon was conducted on the test track of 1200 m straight and 300 m semicircular sections. The total of 56 test runs on straight section and 12 test runs on curve section composed various speed patterns which were predetermined for the first driver of the platoon. The RTK-GPS equipped on each car to detect the necessary physical information such as position, velocity and acceleration every 0.1 second interval.

The average reaction time and model parameters are solved concurrently by nonlinear optimization method which the coefficient of correlation is defined as an objective function. The comparison between the conventional model and extended model illustrates that the combination of second leading car (SLC) variables or excess critical speed (ECS) term with acceleration of the vehicle ahead significantly improve the regression fit. The reaction time of different driving conditions directly affects the regression fit of car following model.

2. Methodology

2.1 Data collection

The necessary information of headway, relative speed, speed and acceleration was gained from the field experiment. The test loop was constructed by two parallel 1200 m straight path linked by the approximately 300 m semicircular section at each end. Ten-car platoon moved in accordance with the first driver who was instructed by the predetermined speeds on the test loop.

The RTK-GPS system, Trimble MS750, was used for tracking the movement of vehicles. The data was transmitted every 0.1 second interval. Positioning accuracy is 10mm+2ppm. The speed was measured based on the Doppler Effect which it gave the accuracy of 0.16 km/hr. To calculate the acceleration, the speed curve was approximated by a parabolic curve at every time step with the speed data in the neighborhood: 4 points before and 4 points after the time point. The gradient of the tangent to the parabola at the time point gave the acceleration. No smoothing operations were applied to the GPS output.

The test cars equipped with the GPS receivers on the roof top were arranged in sequence to form an identical platoon, without alteration of the order. The distance between the GPS receivers was defined as distance headway; in other word, all cars were dimensionless. The first driver’s speed was varied to construct the sinusoidal forms of speed-distance relationships. The samples used in this study are limited to the case of constant speed, half wave and one wave due to the sufficient number of contiguous data required in calibration process.

Classified by the geometry of test loop; the test was organized into two sections which are straight section and curve section. The end of the transition curve is set as a beginning of the straight section while the start of the transition curve is set as a start of the curve section or vice versa. Gurusinghe et al. (2002) proved that the data accuracy from the GPS receivers is superior to that of the conventional equipment.

2.2 Data Analysis

Model Calibration

The conventional car following model, describing how the follower based their driving maneuvers on the leading vehicle, assumes that the time of reaction corresponds with the change in relative speed as its magnitude depends on the sensitivity function of speed and distance headway. The sensitivity term represents the fact that the higher speed and the shorter headway should make the driver change the acceleration rate dramatically.

Gurusinghe et al. (2002) analyzed for the reliable values of reaction time from two different methods. The cross-correlation analysis was the first method to determine the average reaction time for each driver. The calculation was made every 0.5 second in range of 0.0-3.0 second. The objective function for the cross-correlation analysis was the product of stimulus and its respond which happens after a reaction time T, as presented in Equation (5). The graphical method was another method to determine the instantaneous reaction time based on the assumption that drivers remain their reaction time of the previous adjustment in driving process until the next adjustment occurs. Then the model was resolved with the known reaction time by using linearization of nonlinear model. The shortcoming of linearization was elimination of some data due to the arithmetic error in logarithmic transformation, particularly where the acceleration rate is approaching zero.

$$MaxR = \frac{1}{N}\sum_{i=1}^{N}[\ddot{x}_n(t+T)][\dot{x}_{n-1}(t) - \dot{x}_n(t)] \quad (5)$$

The numerical analysis in this study is different from the two-step method made by Gurusinghe et al. (2001). This method simultaneously solves for the average reaction time and all parameters by optimization method. Each data pairs are matched in related to a specified reaction time that alters from -0.5 to 5.0 second, every 0.1 second interval. Then each data set of reaction time T is solved by the modeling optimizer LINGO by using coefficient of correlation as an objective function. The benefit of this solving method is an independence from the restriction of arithmetic error around the corner of zero acceleration, which usually encounters in linearization process. By this approach, the average reaction time and parameters can be determined concurrently and the reaction time is calculated specifically to each model structure.

Instead of the cross-correlation analysis, the maximum coefficient of correlation or R^2 is set as a preliminary criterion to select the average reaction time of each model structure. However, the model development is a state of art; the feasible region is defined by the reasonable parameter values in each model structure ($0 < \alpha \leq 1, 0 \leq l \leq 4, 0 \leq m \leq 2$). (Gurusinghe, 2001) After the optimal parameters and reaction time acquire in each case of T, ranging from -0.5 to 4.0 second, the maximum coefficient of correlation is selected to determine the value of optimal average reaction time.

The preliminary analysis confirms that the objective function for nonlinear optimization is convex within the feasible region. The optimal solution is determined by the point of the maximum coefficient of correlation.

3. Model Development

The simple linear and conventional car following model is preliminarily analyzed before the extended car following models are tested with the same data set. The additional factors that probably explain a high variation constant term are included in conventional model.

Many extended car following models included the variables of the second leading car and many variables from the first leading car are tested with the experimental data. The model gives considerably high correlation and realistic results when the SLC variables or the ECS, and the acceleration of the car in front are added. Almost all results from modified models can overcome the simple linear model, conventional model and original ECS model, particularly in case of constant speed, as in Figure 1 and Figure 2. The modified model can take in account of the data sets which lines below the simple linear line in Figure 2. The result shows that the MECS model and MSLC model are superior to the conventional model; however, no one dominates another.

Modified Second Leading Car Model with Acceleration (MSLC)

The similar nonlinear function as in the conventional model, which is composed of sensitivity coefficient, speed and headway, is adapted to describe the effect from the second leading car. Another variable that significantly influences on the respond function is an acceleration rate of the leading vehicle, particularly in deceleration phase. However, in a few cases the constant value is shown up therefore the MSLC model is as in Equation (6)

$$\ddot{x}_n(t+T) = [\dot{x}_n(t+T)]^m \left\{ \frac{\alpha_1[\dot{x}_{n-1}(t) - \dot{x}_n(t)]}{[x_{n-1}(t) - x_n(t)]^{l1}} + \frac{\alpha_2[\dot{x}_{n-2}(t) - \dot{x}_n(t)]}{[x_{n-2}(t) - x_n(t)]^{l2}} \right\} + \alpha_3 \ddot{x}_{n-1}(t) \quad (6)$$

Modified Excess Critical Speed Model with Acceleration (MECS)

An ECS term based on the concept of collision avoidance (CA) is used to describe the effect of speed variation above or below the critical speed. In this study the first linear term is modified to nonlinear one. The maximum deceleration in ECS term is varied in each calculation between 3 – 5 m/s^2 where it was stated that the optimal value is 5 m/s^2. (Gurusinghe, 2001) As in MSLC model, acceleration and constant terms are included in this model.

$$\ddot{x}_n(t+T) = \frac{\alpha_1[\dot{x}_n(t+T)]^m}{[x_{n-1}(t)-x_n(t)]^l}[\dot{x}_{n-1}(t)-\dot{x}_n(t)] + \alpha_2[ECS] + \alpha_3\ddot{x}_{n-1}(t) \qquad (7)$$

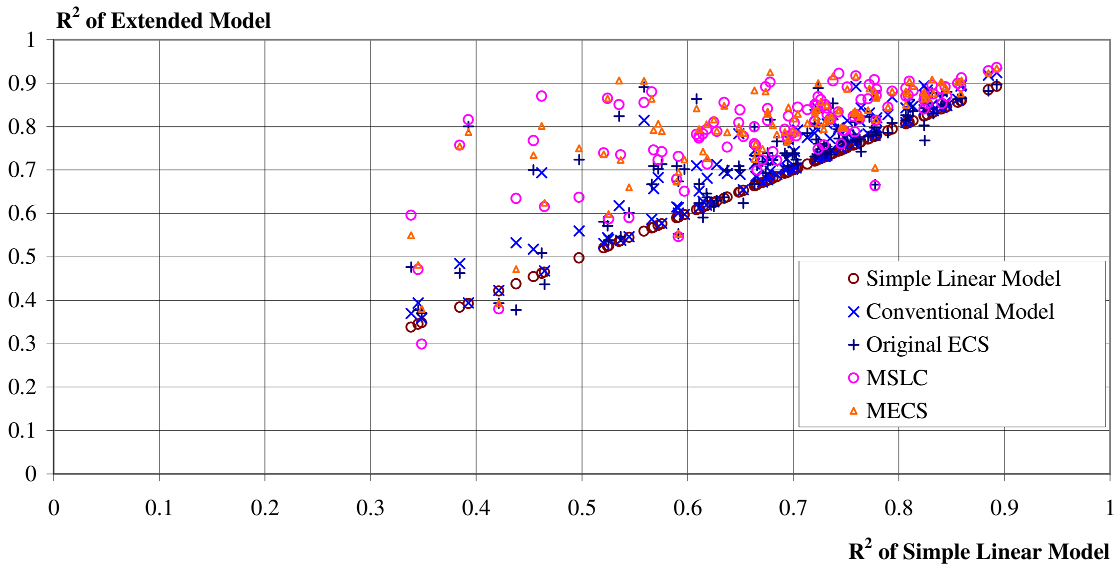

Figure 1 Comparison of Coefficient of Correlation based on Simple Linear Model (Half Wave Speed Pattern)

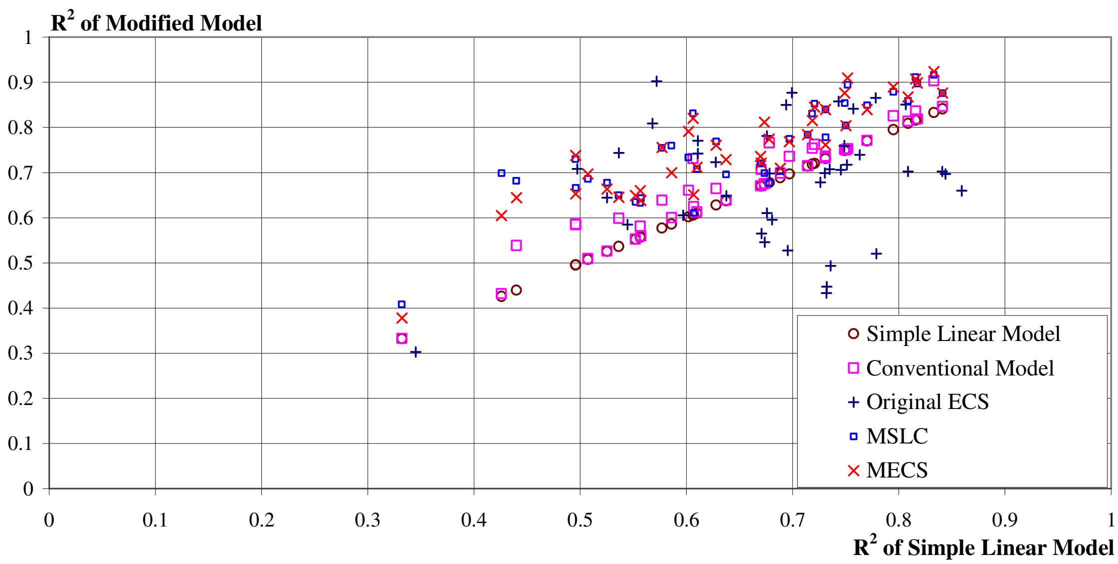

Figure 2 Comparison of Coefficient of Correlation based on Simple Linear Model (Constant Speed Pattern)

Phase Analysis

As expected, there is some error caused by using an average reaction time for whole driving process as illustrated in Figure 3. In case of half wave speed pattern, three different driving tasks; acceleration, deceleration and cruising, are detected. Some loops are formed in case of abrupt change in acceleration to deceleration, particularly in deceleration phase. The gradually increase in acceleration rate does not cause any loop, which implies that the model with average reaction time can take in account of this kind of data, while the gradually change in still deceleration does. Any adjustment of acceleration with in ± 0.5 m/s^2 can be explained as cruising phase. Therefore the deceleration rate performs dramatically different reaction time from that of acceleration phase and cruising phase.

The loops in Figure 3 are corresponding to the changes in headway and speed diagrams. When the leaders start speeding up and the headway is increasing with one or more constant rate, the followers are slowly increasing their speed with the similar rate. Before the predetermined maximum speed is achieved the leaders usually speeds up with higher constant rate, this becomes the start of translation from acceleration to deceleration, and this leads to a big loop across the border line between acceleration and deceleration as seen in Figure3,. After the drivers pass through the maximum speed and slow down, they perform one or more deceleration rates to return to the stable condition. Each change in acceleration, deceleration and particularly translation phases forms Lissajou's loop, which means there is a time lag between the actual and estimated data. (Gurusinghe, 2002)

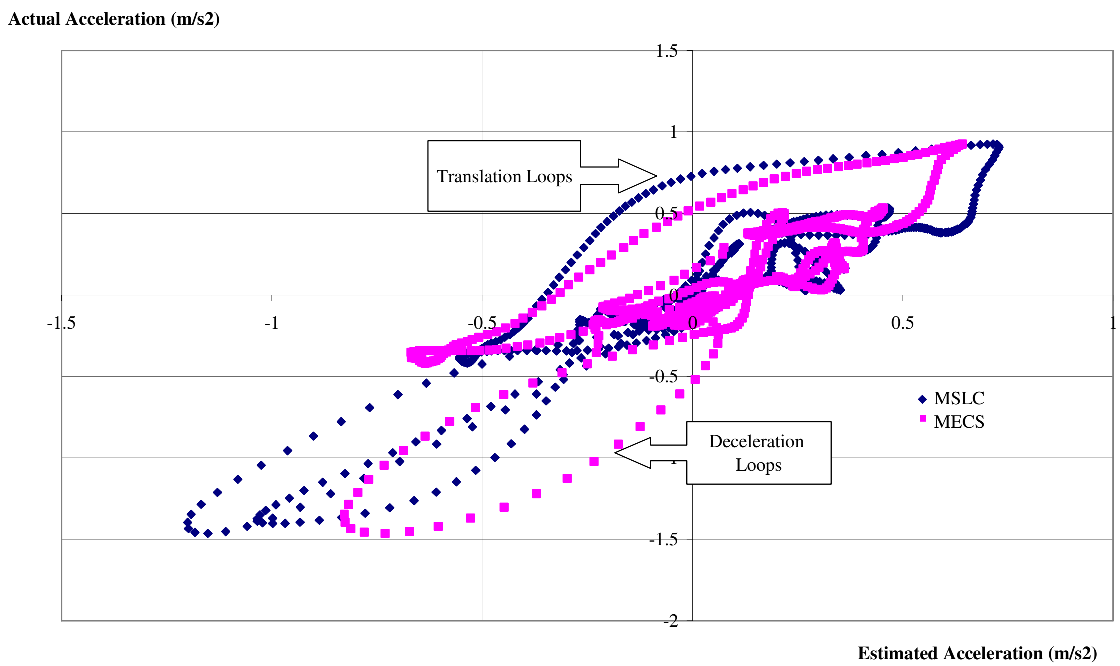

Figure 3 Loop Formation Caused by Using Average Reaction Time

The reaction time analyzed from these two modified models is reasonable. The MECS gives the smaller variation in reaction time than in MSLC, as presented in Figure 4.The distribution of average acceleration-reaction time is indicated that it considerably varies, ranging from 0.5-4.0 second, with an average of 2.5 second from MECS and 2.7 second from MSLC. On the other hand, the average deceleration-reaction time of 1.6 second from MECS and 1.8 second from MSLC, ranging from 0.5-3.0 second, shows that its value is less than the acceleration-reaction time. In addition, the instantaneous reaction time for deceleration phase mostly ranges from 0.5-2.5 second with average of 1.7 second from MECS and 2.0 second from MSLC. It implies that when the high deceleration rate, greater than 0.5 m/s^2, takes place the reaction time is more fluctuated, in other words, the drivers have various reaction to the rapid change in deceleration.

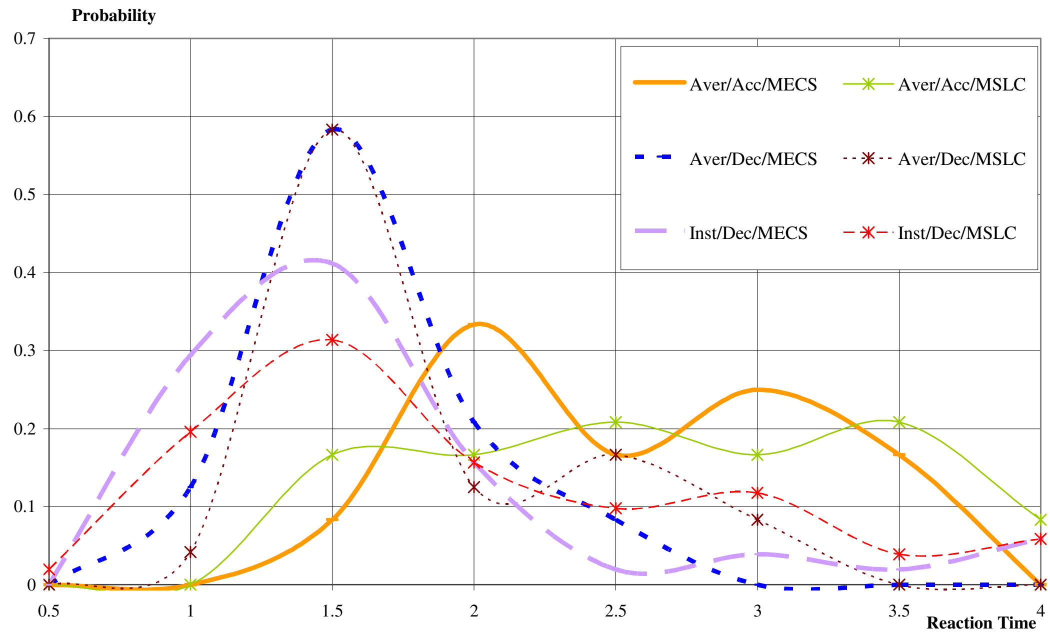

Figure 4 The Average and Instantaneous Reaction Time from MECS/MSLC

4. Conclusion

We confirm that the second leading car and excess critical speed concept have significant effects on the follower by using the accurate data sets. Moreover acceleration of the leader usually plays an important role on the follower, particularly in the case of constant speed. Both models are better of in term of regression fit relative to the conventional and original excess critical speed model. The models give realistic reaction time and model parameters. It is founded that the average reaction time for deceleration rate is significantly different from the acceleration. Due to the high variation of reaction time for acceleration, the different in reaction time between abrupt and gradual change in acceleration is not so significant. The average reaction time can govern that phases of acceleration and cruising, therefore the error from loop formation mostly gains from the deceleration phase.

The further study is to find a condition that can specify the case that suitable to MECS and MSLC model, and the explanation for the loop usually forms up in sudden translation from acceleration to deceleration.

ACKNOWLEDGEMENTS

The authors wish to express their appreciation to Civil Engineering Research Institute of Hokkaido for giving the field data.

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